## AB Calculus Review Sheet

Legend: A - Precalculus, B - Limits, C - Differential Calculus, D - Applications of Differential Calculus, E - Integral Calculus, F - Applications of Integral Calculus, G - Particle Motion and Rates

When you see the words ...
This is what you think of doing

| 1 (B) | Show that $f(x)$ is continuous. |  |
| :--- | :--- | :--- |
| 2 (C) | Find the equation of the tangent <br> line to $f$ at $\left(x_{1}, y_{1}\right)$. |  |
| 3 (A) | If continuous function $f(x)$ has <br> $f(a)<k$ and $f(b)>k$, explain <br> why there must be a value $c$ such <br> that $a<c<b$ and $f(c)=k$. |  |
| 4 (D) | Find points of relative extrema of <br> $f(x)$. |  |
| 5 (D) | Find range of $f(x)$ on $[a, b]$ <br> (G) | Given the position function $s(t)$, <br> find the average velocity on $\left[t_{1}, t_{2}\right]$. |
| 7 (E) | Approximate $\int_{a} f(x) d x$ using left <br> Riemann sums with $n$ rectangles. |  |
| 8 (D) | Given a graph of $f^{\prime}(x)$, determine <br> intervals where $f(x)$ is <br> increasing/decreasing. |  |
| 9 (C) | Given a piecewise function, show it <br> is differentiable at $x=a$ where the <br> function rule splits. |  |
| 10 | Find critical values of $f(x)$. <br> (D) |  |
| 11 |  |  |
| (A) | Find the zeros of $f(x)$. |  |

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| 12 <br> (B) | Find $\lim _{x \rightarrow a} f(x)$. <br> 13 <br> (C) | Find $x$-values of horizontal tangents <br> to $f$. |
| :--- | :--- | :--- |
| 14 <br> (D) | Show that Rolle's Theorem holds <br> for $f(x)$ on $[a, b]$. |  |
| 15 <br> (D) | Find inflection points of $f(x)$. |  |
| 16 (F) | Find the area between <br> $f(x)$ and $\mathrm{g}(x)$. |  |
| 17 (F) | Given $\frac{d y}{d x}$, draw a slope field. |  |
| 18 | Given the velocity function $v(t)$, <br> determine the difference of position <br> of a particle on $\left[t_{1}, t_{2}\right]$. |  |
| (G) |  |  |

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| $\begin{aligned} & \hline 25 \\ & (\mathrm{G}) \end{aligned}$ | Given the position function $s(t)$, find the instantaneous velocity at $t=k$. |  |
| :---: | :---: | :---: |
| $\begin{aligned} & \hline 26 \\ & (\mathrm{G}) \end{aligned}$ | Given the velocity function $v(t)$ and $s(0)$, find $s(t)$. |  |
| 27 (F) | Find the average value of $f(x)$ on $[a, b]$. |  |
| 28 (E) | Meaning of $\int_{a}^{x} f(t) d t$. |  |
| 29 (E) | Find $\frac{d}{d x} \int_{a}^{x} f(t) d t$ |  |
| $\begin{aligned} & 30 \\ & (\mathrm{C}) \end{aligned}$ | Find $x$-values of horizontal tangents to $f$. |  |
| $\begin{aligned} & 31 \\ & \text { (A) } \end{aligned}$ | Find domain of $f(x)$. |  |
| $\begin{aligned} & 32 \\ & (\mathrm{C}) \end{aligned}$ | Given a chart of $x$ and $f(x)$ and selected values of $x$ between $a$ and $b$, approximate $f^{\prime}(c)$ where $c$ is a value between $a$ and $b$. |  |
| $\begin{aligned} & \hline 33 \\ & (\mathrm{G}) \end{aligned}$ | Given the velocity function $v(t)$ and $s(0)$, find the greatest distance of the particle from the starting position on $\left[0, t_{1}\right]$. |  |
| 34 (F) | $y$ is increasing proportionally to $y$. |  |
| 35 (E) | Given $\int_{a}^{b} f(x) d x$, find $\int^{b}[f(x)+k] d x$ |  |
| $\begin{aligned} & 36 \\ & (\mathrm{C}) \end{aligned}$ | Find the equation of the normal line to $f$ at $\left(x_{1}, y_{1}\right)$. |  |
| $37$ <br> (D) | Show that the Mean Value Theorem holds for $f(x)$ on $[a, b]$. |  |

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| $\begin{array}{\|l\|} \hline 38 \\ \text { (D) } \end{array}$ | Find range of $f(x)$ on $(-\infty, \infty)$. |  |
| :---: | :---: | :---: |
| $\begin{array}{\|l\|} \hline 39 \\ (\mathrm{C}) \end{array}$ | Find the instantaneous rate of change of $f$ at $x=a$. |  |
| 40 (F) | Find the line $x=c$ that divides the area under $f(x)$ on $[a, b]$ into two equal areas. |  |
| 41 (E) | Approximate $\int_{a}^{b} f(x) d x$ using trapezoidal summation. |  |
| 42 (F) | Solve the differential equation $\frac{d y}{d x}=f(x) g(y)$. |  |
| 43 (F) | Given a base bounded by $f(x)$ and $g(x)$ on $[a, b]$ the cross sections of the solid perpendicular to the $x$-axis are squares. Find the volume. |  |
| 44 (E) | Approximate $\int_{a}^{b} f(x) d x$ using midpoint Riemann sums. |  |
| 45 <br> (D) | Find the interval(s) where $f(x)$ is increasing/decreasing. |  |
| $\begin{aligned} & \hline 46 \\ & (\mathrm{~B}) \end{aligned}$ | Find horizontal asymptotes of $f(x)$. |  |
| 47 <br> (C) | The line $y=m x+b$ is tangent to the graph of $f(x)$ at $\left(x_{1}, y_{1}\right)$. |  |
| $\begin{aligned} & 48 \\ & \text { (A) } \end{aligned}$ | Find vertical asymptotes of $f(x)$. |  |
| $\begin{aligned} & 49 \\ & (\mathrm{C}) \end{aligned}$ | Find $x$-values of vertical tangents to $f$. |  |

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| 50 (E) | Given the value of $F(a)$ where the <br> antiderivative of $f$ is $F$, find $F(b)$. |  |
| :--- | :--- | :--- |
| 51 (F) | Find the area under the curve $f(x)$ <br> on the interval $[a, b]$. |  |
| 52 |  |  |
| (D) | Find the minimum slope of $f(x)$ <br> on $[a, b]$. |  |
| 53 (E) | Find $\int_{b}^{a} f(x) d x$ where $a<b$. |  |
| 54 | Given the position function $s(t)$ of <br> a particle moving along a straight <br> line, find the velocity and <br> acceleration. |  |
| 55 (F) | Find the volume when the area <br> between $f(x)$ and $g(x)$ is rotated <br> about the $x$-axis. |  |
| 56 | Given the velocity function $v(t)$, <br> determine the distance a particle <br> travels on $\left[t_{1}, t_{2}\right]$. |  |
| 57 | Given the velocity function $v(t)$, <br> determine if a particle is speeding <br> up or slowing down at <br> $t=k$. | Find intervals where the slope of <br> $f(x)$ is increasing. <br> (Giemann sums with $n$ rectangles. |
| Approximate $\int_{a} f(x) d x$ using right <br> (G) <br> (G) |  |  |

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| $\begin{array}{\|l\|} \hline 60 \\ (C) \end{array}$ | Approximate the value of $f\left(x_{1}+a\right)$ if you know the function goes through point $\left(x_{1}, y_{1}\right)$. |  |
| :---: | :---: | :---: |
| 61 <br> (G) | Calculate $\int_{t_{1}}^{t_{2}}\|v(t)\| d t$ without a calculator. |  |
| $62$ <br> (G) | Given the velocity function $v(t)$ on $\left[t_{1}, t_{2}\right]$, find the minimum acceleration of a particle. |  |
| 63 <br> (D) | Find the absolute maximum or minimum of $f(x)$ on $[a, b]$. |  |
| 64 <br> (A) | Show that $f(x)$ is even. |  |
| 65 (F) | Find the average rate of change of $F^{\prime}(x)$ on $\left[t_{1}, t_{2}\right]$. |  |
| 66 <br> (G) | The meaning of $\int_{a}^{b} R^{\prime}(t) d t$. |  |
| 67 <br> (G) | Given the velocity function $v(t)$, find the average velocity on $\left[t_{1}, t_{2}\right]$. |  |
| $68$ <br> (A) | Find the intersection of $f(x)$ and $g(x)$. |  |
| $\begin{array}{\|l\|} \hline 69 \\ \text { (G) } \end{array}$ | The volume of a solid is changing at the rate of ... |  |
| $\begin{array}{\|l\|} \hline 70 \\ \text { (G) } \end{array}$ | Given the acceleration function $a(t)$ of a particle at rest and $s(0)$, find $s(t)$. |  |
| 71 (E) | Find $\frac{d}{d x} \int_{a}^{g(x)} f(t) d t$. |  |

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| 72 |  |  |
| :--- | :--- | :--- |
| (D) | Determine whether the linear <br> approximation for $f\left(x_{1}+a\right)$ over- <br> estimates or under-estimates <br> $f\left(x_{1}+a\right)$. |  |
| 73 |  |  |
| (B) | Find $\lim _{x \rightarrow a} f(x)$ where $f(x)$ is a <br> piecewise function. |  |
| 73 | Find the derivative of the inverse to <br> $f(x)$ at $x=a$. |  |
| 74 | Given a water tank with $g$ gallons <br> initially, filled at the rate of $F(t)$ <br> gallons/min and emptied at the rate <br> of $E(t)$ gallons/min on $\left[t_{1}, t_{2}\right]$ a) <br> The amount of water in the tank at $t$ <br> $=m$ minutes. b) the rate the water <br> amount is changing at $t=m$ <br> minutes and c) the time $t$ when the <br> water in the tank is at a minimum <br> or maximum. |  |

## AB Calculus Review Sheet Solutions

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When you see the words ...
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$\left.\begin{array}{|l|l|l|}\hline 1 \text { (B) } & \begin{array}{l}\text { Show that } f(x) \text { is continuous. } \\ 2 \text { (C) } \\ \text { line to } f \text { at }\left(x_{1}, y_{1}\right) .\end{array} & \begin{array}{l}\text { Show that 1) } \lim _{x \rightarrow a} f(x) \text { exists } \\ \text { 2) } f(a) \text { exists } \\ \text { 3) } \lim _{x \rightarrow a} f(x)=f(a)\end{array} \\ \hline 3 \text { (A) } & \begin{array}{l}\text { If continuous function } f(x) \text { has } \\ f(a)<k \text { and } f(b)>k, \text { explain } \\ \text { why there must be a value } c \text { such } \\ \text { that } a<c<b \text { and } f(c)=k .\end{array} & \begin{array}{l}\text { This is the Intermediate Value Theorem. } \\ y-y_{1}=m\left(x-x_{1}\right)\end{array} \\ \hline 4 \text { (D) } & \begin{array}{l}\text { Find points of relative extrema of } \\ f(x) .\end{array} & \begin{array}{l}\text { Make a sign chart of } f^{\prime}(x) \text {. At } x=c \text { where the derivative } \\ \text { switches from negative to positive, there is a relative } \\ \text { minimum. When the derivative switches from positive to }\end{array} \\ \text { negative, there is a relative maximum. To actually find the } \\ \text { point, evaluate } f(c) . \text { OR if } f^{\prime}(c)=0, \text { then if } f^{\prime \prime}(c)>0, \text { there } \\ \text { is a relative minimum at } x=c . \text { If } f^{\prime \prime}(c)<0, \text { there is a relative } \\ \text { maximum at } x=c .\left(2^{\text {nd }} \text { Derivative test). }\right.\end{array}\right\}$

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| $12$ (B) | Find $\lim _{x \rightarrow a} f(x)$. | Step 1: Find $f(a)$. If you get a zero in the denominator, Step 2: Factor numerator and denominator of $f(x)$. Do any cancellations and go back to Step 1. If you still get a zero in the denominator, the answer is either $\infty,-\infty$, or does not exist. Check the signs of $\lim _{x \rightarrow a^{-}} f(x)$ and $\lim _{x \rightarrow a^{+}} f(x)$ for equality. |
| :---: | :---: | :---: |
| $\begin{aligned} & \hline 13 \\ & (\mathrm{C}) \\ & \hline \end{aligned}$ | Find $x$-values of horizontal tangents to $f$. | Write $f^{\prime}(x)$ as a fraction. Set numerator of $f^{\prime}(x)=0$. |
| 14 <br> (D) | Show that Rolle's Theorem holds for $f(x)$ on $[a, b]$. | Show that $f$ is continuous and differentiable on $[a, b]$. If $f(a)=f(b)$, then find some $c$ on $[a, b]$ such that $f^{\prime}(c)=0$. |
| $\begin{aligned} & \hline 15 \\ & \text { (D) } \end{aligned}$ | Find inflection points of $f(x)$. | Find and express $f^{\prime \prime}(x)$ as a fraction. Set both numerator and denominator equal to zero and solve. Make a sign chart of $f^{\prime \prime}(x)$. Inflection points occur when $f^{\prime \prime}(x)$ witches from positive to negative or negative to positive. |
| 16 (F) | Find the area between $f(x)$ and $g(x)$. | Find the intersections, $a$ and $b$ of $f(x)$ and $g(x)$. If $f(x) \geq g(x)$ on $[\mathrm{a}, \mathrm{b}]$, then area $A=\int_{a}^{b}[f(x)-g(x)] d x$. |
| 17 (F) | Given $\frac{d y}{d x}$, draw a slope field. | Use the given points and plug them into $\frac{d y}{d x}$, drawing little lines with the calculated slopes at the point. |
| $\begin{aligned} & \hline 18 \\ & (\mathrm{G}) \end{aligned}$ | Given the velocity function $v(t)$, determine the difference of position of a particle on $\left[t_{1}, t_{2}\right]$. | $\text { Displacement }=\int_{t_{1}}^{t_{2}} v(t) d t$ |
| 19 (F) | Find the volume when the area under $f(x)$ is rotated about the $x$ axis on the interval $[a, b]$. | Disks: Radius $=f(x): V=\pi \int_{a}^{b}[f(x)]^{2} d x$ |
| $\begin{array}{\|l\|} \hline 20 \\ (\mathrm{C}) \\ \hline \end{array}$ | Find the average rate of change of $f$ on $[a, b]$. | $\text { Find } \frac{f(b)-f(a)}{b-a}$ |
| $\begin{aligned} & \hline 21 \\ & (\mathrm{C}) \\ & \hline \end{aligned}$ | Find the derivative of $f(g(x))$. | This is the chain rule. You are finding $f^{\prime}(g(x)) \cdot g^{\prime}(x)$. |
| $\begin{aligned} & 22 \\ & (\mathrm{C}) \end{aligned}$ | Find the derivative of a function using the derivative definition. | Find $\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ or $\lim _{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ |
| $\begin{array}{\|l} \hline 23 \\ \text { (A) } \\ \hline \end{array}$ | Show that $f(x)$ is odd. | Show that $f(-x)=-f(x)$. This shows that the graph of $f$ is symmetric to the origin. |
| $24$ <br> (B) | Find $\lim _{x \rightarrow \infty} f(x)$ and $\lim _{x \rightarrow-\infty} f(x)$. | Express $f(x)$ as a fraction. Determine location of the highest power: <br> Denominator: $\lim _{x \rightarrow \infty} f(x)=\lim _{x \rightarrow-\infty} f(x)=0$ <br> Both Num and Denom: ratio of the highest power coefficients <br> Numerator: $\lim _{x \rightarrow \infty} f(x)= \pm \infty$ (plug in large number) |

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| $\begin{array}{\|l\|} \hline 25 \\ (\mathrm{G}) \end{array}$ | Given the position function $s(t)$, find the instantaneous velocity at $t=k$. | Inst. vel. $=s^{\prime}(k)$. |
| :---: | :---: | :---: |
| $\begin{array}{\|l\|} \hline 26 \\ (\mathrm{G}) \\ \hline \end{array}$ | Given the velocity function $v(t)$ and $s(0)$, find $s(t)$. | $s(t)=\int v(t) d t+C$. Plug in $s(0)$ to find $C$. |
| 27 (F) | Find the average value of $f(x)$ on $[a, b]$. | $F_{a v g}=\frac{\int_{a}^{b} f(x) d x}{b-a}$ |
| 28 (E) | Meaning of $\int_{a}^{x} f(t) d t$ | The accumulation function - accumulated area under function $f$ starting at some constant $a$ and ending at some variable $x$. |
| 29 (E) | Find $\frac{d}{d x} \int_{a}^{x} f(t) d t$. | $\frac{d}{d x} \int_{a}^{x} f(t) d t=f(x)$. The 2nd Fundamental Theorem. |
| $\begin{array}{\|l\|} \hline 30 \\ (\mathrm{C}) \\ \hline \end{array}$ | Find $x$-values of horizontal tangents to $f$. | Write $f^{\prime}(x)$ as a fraction. Set numerator of $f^{\prime}(x)=0$. |
| 31 <br> (A) | Find domain of $f(x)$. | Assume domain is $(-\infty, \infty)$. Restrict domains: denominators $\neq$ 0 , square roots of only non-negative numbers, logarithm or natural log of only positive numbers. |
| $\begin{array}{\|l\|} \hline 32 \\ (\mathrm{C}) \\ \hline \end{array}$ | Given a chart of $x$ and $f(x)$ and selected values of $x$ between $a$ and $b$, approximate $f^{\prime}(c)$ where $c$ is a value between $a$ and $b$. | Straddle $c$, using a value of $k \geq c$ and a value of $h \leq c . f^{\prime}(c) \approx \frac{f(k)-f(h)}{k-h}$ |
| $\begin{array}{\|l\|} \hline 33 \\ (\mathrm{G}) \end{array}$ | Given the velocity function $v(t)$ and $s(0)$, find the greatest distance of the particle from the starting position on $\left[0, t_{1}\right]$. | Generate a sign chart of $v(t)$ to find turning points. $s(t)=\int v(t) d t+C$. Plug in $s(0)$ to find $C$. <br> Evaluate $s(t)$ at all turning points and find which one gives the maximum distance from $s(0)$. |
| 34 (F) | $y$ is increasing proportionally to $y$. | $\frac{d y}{d t}=k y$ which translates to $y=C e^{k t}$ |
| 35 (E) | Given $\int_{a}^{b} f(x) d x$, find $\int_{a}^{b}[f(x)+k] d x$. | $\int_{a}^{b}[f(x)+k] d x=\int_{a}^{b} f(x) d x+\int_{a}^{b} k d x$ |
| $\begin{array}{\|l\|} \hline 36 \\ (\mathrm{C}) \end{array}$ | Find the equation of the normal line to $f$ at $\left(x_{1}, y_{1}\right)$. | Find slope $m \perp=\frac{-1}{f^{\prime}\left(x_{i}\right)}$. Then use point slope equation: $y-y_{1}=m\left(x-x_{1}\right)$ |
| $37$ <br> (D) | Show that the Mean Value Theorem holds for $f(x)$ on $[a, b]$. | Show that $f$ is continuous and differentiable on $[a, b]$. If $f(a)=f(b)$, then find some $c$ on $[a, b]$ such that $f^{\prime}(c)=\frac{f(b)-f(a)}{b-a}$ |

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| $\begin{aligned} & \hline 38 \\ & \text { (D) } \end{aligned}$ | Find range of $f(x)$ on $(-\infty, \infty)$. | Use relative extrema techniques to find relative max/mins. Evaluate $f$ at these values. Then examine $f(a)$ and $f(b)$. Then examine $\lim _{x \rightarrow \infty} f(x)$ and $\lim _{x \rightarrow-\infty} f(x)$. |
| :---: | :---: | :---: |
| $\begin{aligned} & \hline 39 \\ & (\mathrm{C}) \\ & \hline \end{aligned}$ | Find the instantaneous rate of change of $f$ at $x=a$. | Find $f^{\prime}(a)$ |
| 40 (F) | Find the line $x=c$ that divides the area under $f(x)$ on $[a, b]$ into two equal areas. | $\int_{a}^{c} f(x) d x=\int_{c}^{b} f(x) d x \text { or } \int_{a}^{b} f(x) d x=2 \int_{a}^{c} f(x) d x$ |
| 41 (E) | Approximate $\int_{a}^{b} f(x) d x$ using trapezoidal summation. | $A=\left(\frac{b-a}{2 n}\right)\left[f\left(x_{0}\right)+2 f\left(x_{1}\right)+2 f\left(x_{2}\right)+\ldots+2 f\left(x_{n-1}\right)+f\left(x_{n}\right)\right]$ <br> This formula only works when the base of each trapezoid is the same. If not, calculate the areas of individual trapezoids. |
| 42 (F) | Solve the differential equation $\frac{d y}{d x}=f(x) g(y)$. | Separate the variables: $x$ on one side, $y$ on the other with the $d x$ and $d y$ in the numerators. Then integrate both sides, remembering the $+C$, usually on the $x$-side. |
| 43 (F) | Given a base bounded by $f(x)$ and $g(x)$ on $[a, b]$ the cross sections of the solid perpendicular to the $x$-axis are squares. Find the volume. | $\begin{aligned} & \text { Base }=f(x)-g(x) . \text { Area }=\text { base }^{2}=[f(x)-g(x)]^{2} \\ & \text { Volume }=\int_{a}^{b}[f(x)-g(x)]^{2} d x \end{aligned}$ |
| 44 (E) | Approximate $\int_{a}^{b} f(x) d x$ using midpoint Riemann sums. | Typically done with a table of points. Be sure to use only values that are given. If you are given 7 points, you can only calculate 3 midpoint rectangles. |
| $\begin{aligned} & \hline 45 \\ & \text { (D) } \end{aligned}$ | Find the interval(s) where $f(x)$ is increasing/decreasing. | Find critical values of $f^{\prime}(x)$. Make a sign chart to find sign of $f^{\prime}(x)$ in the intervals bounded by critical values. Positive means increasing, negative means decreasing. |
| $46$ <br> (B) | Find horizontal asymptotes of $f(x)$. | $\lim _{x \rightarrow \infty} f(x)$ and $\lim _{x \rightarrow-\infty} f(x)$ |
| 47 <br> (C) | The line $y=m x+b$ is tangent to the graph of $f(x)$ at $\left(x_{1}, y_{1}\right)$. | Two relationships are true: <br> 1) The function $f$ and the line share the same slope at $x_{1}$ : $m=f^{\prime}\left(x_{1}\right)$ <br> 2) The function $f$ and the line share the same $y$-value at $x_{1}$. |
| $\begin{aligned} & \hline 48 \\ & \text { (A) } \end{aligned}$ | Find vertical asymptotes of $f(x)$. | Express $f(x)$ as a fraction, express numerator and denominator in factored form, and do any cancellations. Set denominator equal to 0 . |
| $\begin{aligned} & 49 \\ & (\mathrm{C}) \end{aligned}$ | Find $x$-values of vertical tangents to $f$. | Write $f^{\prime}(x)$ as a fraction. Set denominator of $f^{\prime}(x)=0$. |

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| 50 (E) | Given the value of $F(a)$ where the antiderivative of $f$ is $F$, find $F(b)$. | Use the fact that $\int_{a}^{b} f(x) d x=F(b)-F(a)$ so $F(b)=F(a)+\int_{a}^{b} f(x) d x$. Use the calculator to find the definite integral. |
| :---: | :---: | :---: |
| 51 (F) | Find the area under the curve $f(x)$ on the interval $[a, b]$. | $\int_{a}^{b} f(x) d x$ |
| $\begin{aligned} & \hline 52 \\ & \text { (D) } \end{aligned}$ | Find the minimum slope of $f(x)$ on $[a, b]$. | Find the derivative of $f^{\prime}(x)$ which is $f^{\prime \prime}(x)$. Find critical values of $f^{\prime \prime}(x)$ and make a sign chart of $f^{\prime \prime}(x)$. Values of $x$ where $f^{\prime \prime}(x)$ switches from negative to positive are potential locations for the minimum slope. Evaluate $f^{\prime}(x)$ at those values and also $f^{\prime}(a)$ and $f^{\prime}(b)$ and choose the least of these values. |
| 53 (E) | Find $\int_{b}^{a} f(x) d x$ where $a<b$ | $\int_{b}^{a} f(x) d x=-\int_{a}^{b} f(x) d x$ |
| $\begin{array}{\|l\|} \hline 54 \\ (\mathrm{G}) \end{array}$ | Given the position function $s(t)$ of a particle moving along a straight line, find the velocity and acceleration. | $v(t)=s^{\prime}(t) \quad a(t)=v^{\prime}(t)=s^{\prime \prime}(t)$ |
| 55 (F) | Find the volume when the area between $f(x)$ and $g(x)$ is rotated about the $x$-axis. | Washers: Outside radius $=f(x)$. Inside radius $=g(x)$. Establish the interval where $f(x) \geq g(x)$ and the values of $a$ and $b$, where $f(x)=g(x) . V=\pi \int^{b}\left([f(x)]^{2}-[g(x)]^{2}\right) d x$ |
| $\begin{array}{\|l\|} \hline 56 \\ (\mathrm{G}) \end{array}$ | Given the velocity function $v(t)$, determine the distance a particle travels on $\left[t_{1}, t_{2}\right]$. | $\text { Distance }=\int_{t_{1}}^{t_{2}}\|v(t)\| d t$ |
| $\begin{aligned} & \hline 57 \\ & (\mathrm{G}) \end{aligned}$ | Given the velocity function $v(t)$, determine if a particle is speeding up or slowing down at $t=k$. | Find $v(k)$ and $a(k)$. If both have the same sign, the particle is speeding up. If they have different signs, the particle is slowing down. |
| $\begin{aligned} & \hline 58 \\ & \text { (D) } \end{aligned}$ | Find intervals where the slope of $f(x)$ is increasing. | Find the derivative of $f^{\prime}(x)$ which is $f^{\prime \prime}(x)$. Find critical values of $f^{\prime \prime}(x)$ and make a sign chart of $f^{\prime \prime}(x)$ looking for positive intervals. |
| 59 (E) | Approximate $\int_{a}^{b} f(x) d x$ using right Riemann sums with $n$ rectangles. | $A=\left(\frac{b-a}{n}\right)\left[f\left(x_{1}\right)+f\left(x_{2}\right)+f\left(x_{3}\right)+\ldots+f\left(x_{n}\right)\right]$ |

Legend: A - Precalculus, B - Limits, C - Differential Calculus, D - Applications of Differential Calculus, E - Integral Calculus, F - Applications of Integral Calculus, G - Particle Motion and Rates

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| $\begin{aligned} & 60 \\ & (\mathrm{C}) \end{aligned}$ | Approximate the value of $f\left(x_{1}+a\right)$ if you know the function goes through point $\left(x_{1}, y_{1}\right)$. | Find slope $m=f^{\prime}\left(x_{i}\right)$. Then use point slope equation: $y-y_{1}=m\left(x-x_{1}\right)$. Evaluate this line for $y$ at $x=x_{1}+a$. Note: The closer $a$ is to 0 , the better the approximation will be. Also note that using concavity, it can be determine if this value is an over or under-approximation for $f\left(x_{1}+a\right)$. |
| :---: | :---: | :---: |
| $61$ <br> (G) | Calculate $\int_{t_{1}}^{t_{2}}\|v(t)\| d t$ without a calculator. | Set $v(t)=0$ and make a sign charge of $v(t)=0$ on $\left[t_{1}, t_{2}\right]$. On intervals $[a, b]$ where $v(t)>0, \int_{a}^{b}\|v(t)\| d t=\int_{a}^{b} v(t) d t$ On intervals $[a, b]$ where $v(t)<0, \int_{a}^{b}\|v(t)\| d t=\int_{b}^{a} v(t) d t$ |
| $\begin{aligned} & \hline 62 \\ & (\mathrm{G}) \end{aligned}$ | Given the velocity function $v(t)$ on [ $t_{1}, t_{2}$ ], find the minimum acceleration of a particle. | Find $a(t)$ and set $a^{\prime}(t)=0$. Set up a sign chart and find critical values. Evaluate the acceleration at critical values and also $t_{1}$ and $t_{2}$ to find the minimum. |
| $\begin{aligned} & \hline 63 \\ & \text { (D) } \end{aligned}$ | Find the absolute maximum or minimum of $f(x)$ on $[a, b]$. | Use relative extrema techniques to find relative max/mins. Evaluate $f$ at these values. Then examine $f(a)$ and $f(b)$. The largest of these is the absolute maximum and the smallest of these is the absolute minimum. |
| 64 <br> (A) | Show that $f(x)$ is even. | Show that $f(-x)=f(x)$. This shows that the graph of $f$ is symmetric to the $y$-axis. |
| 65 (F) | Find the average rate of change of $F^{\prime}(x)$ on $\left[t_{1}, t_{2}\right]$. | $\frac{\frac{d}{d t} \int_{t_{1}}^{t_{2}} F^{\prime}(x) d x}{t_{2}-t_{1}}=\frac{F^{\prime}\left(t_{2}\right)-F^{\prime}\left(t_{1}\right)}{t_{2}-t_{1}}$ |
| $\begin{aligned} & \hline 66 \\ & (\mathrm{G}) \end{aligned}$ | The meaning of $\int_{a}^{b} R^{\prime}(t) d t$. | This gives the accumulated change of $R(t)$ on $[a, b]$. $\int_{a}^{b} R^{\prime}(t) d t=R(b)-R(a)$ or $R(b)=R(a)+\int_{a}^{b} R^{\prime}(t) d t$ |
| $67$ <br> (G) | Given the velocity function $v(t)$, find the average velocity on $\left[t_{1}, t_{2}\right]$. | $\text { Avg. vel. }=\frac{\int_{t_{1}}^{t_{2}} v(t) d t}{t_{2}-t_{1}}$ |
| $\begin{aligned} & \hline 68 \\ & \text { (A) } \end{aligned}$ | Find the intersection of $f(x)$ and $g(x)$. | Set the two functions equal to each other. Find intersection on calculator. |
| $\begin{aligned} & 69 \\ & \text { (G) } \\ & \hline \end{aligned}$ | The volume of a solid is changing at the rate of ... | $\frac{d V}{d t}=\ldots$ |
| $\begin{aligned} & \hline 70 \\ & \text { (G) } \end{aligned}$ | Given the acceleration function $a(t)$ of a particle at rest and $s(0)$, find $s(t)$. | $\begin{aligned} & v(t)=\int a(t) d t+C_{1} . \text { Plug in } v(0)=0 \text { to find } C_{1} . \\ & s(t)=\int v(t) d t+C_{2} . \text { Plug in } s(0) \text { to find } C_{2} . \end{aligned}$ |
| 71 (E) | Find $\frac{d}{d x} \int_{a}^{g(x)} f(t) d t$ | $\frac{d}{d x} \int_{a}^{g(x)} f(t) d t=f(g(x)) \cdot g^{\prime}(x)$. The 2nd Fundamental Theorem. |

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| $72$ <br> (D) | Determine whether the linear approximation for $f\left(x_{1}+a\right)$ overestimates or under-estimates $f\left(x_{1}+a\right)$. | Find slope $m=f^{\prime}\left(x_{i}\right)$. Then use point slope equation: $y-y_{1}=m\left(x-x_{1}\right)$. Evaluate this line for $y$ at $x=x_{1}+a$. If $f^{\prime \prime}\left(x_{1}\right)>0, f$ is concave up at $x_{1}$ and the linear approximation is an underestimation for $f\left(x_{1}+a\right) . f^{\prime \prime}\left(x_{1}\right)<0$, $f$ is concave down at $x_{1}$ and the linear approximation is an overestimation for $f\left(x_{1}+a\right)$. |
| :---: | :---: | :---: |
| $\begin{array}{\|l\|} \hline 73 \\ \text { (B) } \end{array}$ | Find $\lim _{x \rightarrow a} f(x)$ where $f(x)$ is a piecewise function. | Determine if $\lim _{x \rightarrow a^{-}} f(x)=\lim _{x \rightarrow a^{+}} f(x)$ by plugging in $a$ to $f(x), x<a$ and $f(x), x>a$ for equality. If they are not equal, the limit doesn't exist. |
| $\begin{array}{\|l\|} \hline 73 \\ \text { (C) } \end{array}$ | Find the derivative of the inverse to $f(x)$ at $x=a$. | Follow this procedure: <br> 1) Interchange $x$ and $y$ in $f(x)$. <br> 2) Plug the $x$-value into this equation and solve for $y$ (you may need a calculator to solve graphically) <br> 3) Using the equation in 1) find $\frac{d y}{d x}$ implicitly. <br> 4) Plug the $y$-value you found in 2) to $\frac{d y}{d x}$ |
| $\begin{array}{\|l\|} \hline 74 \\ (\mathrm{G}) \end{array}$ | Given a water tank with $g$ gallons initially, filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons $/$ min on $\left[t_{1}, t_{2}\right]$ a) The amount of water in the tank at $t$ $=m$ minutes. b) the rate the water amount is changing at $t=m$ minutes and c) the time $t$ when the water in the tank is at a minimum or maximum. | a) $g+\int_{0}^{m}[F(t)-E(t)] d t$ <br> b) $\frac{d}{d t} \int_{0}^{m}[F(t)-E(t)] d t=F(m)-E(m)$ <br> c) set $F(m)-E(m)=0$, solve for $m$, and evaluate $g+\int_{0}^{m}[F(t)-E(t)] d t$ at values of $m$ and also the endpoints. |

