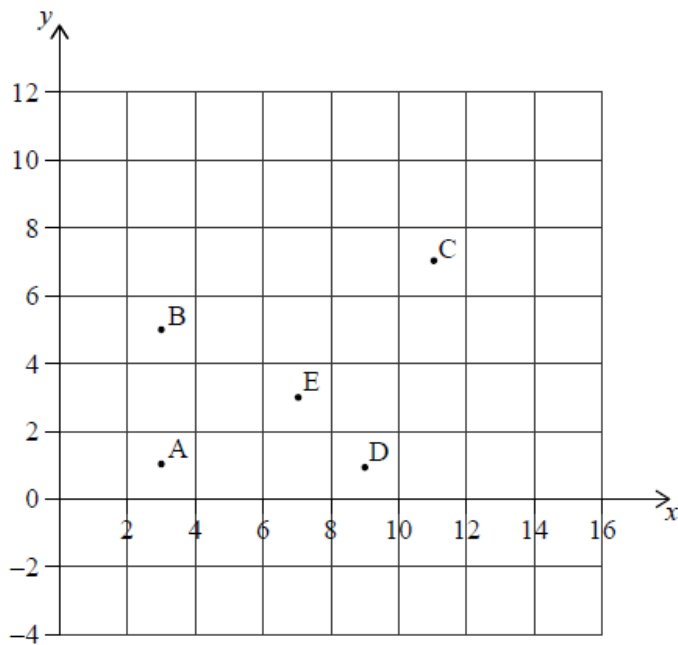


# Voronoi [33 marks]

Points A(3, 1), B(3, 5), C(11, 7), D(9, 1) and E(7, 3) represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

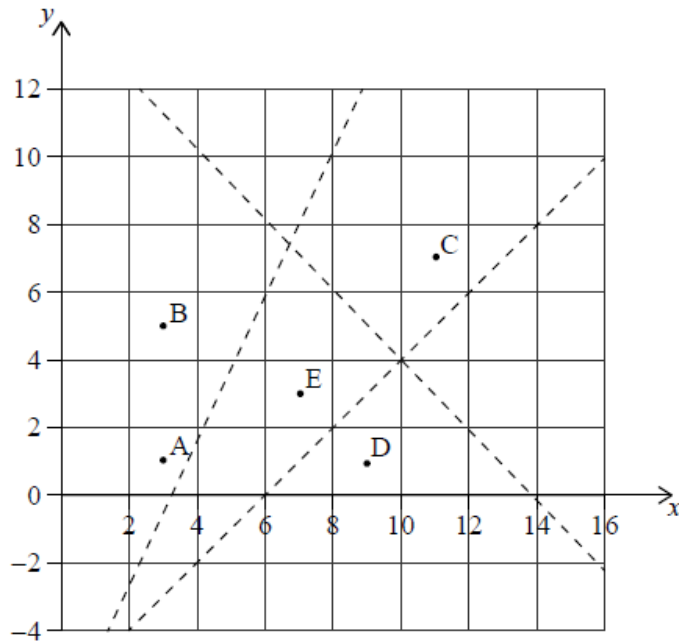
Vertical scale: 1 unit represents 1 km.



1a. Calculate the gradient of the line segment AE.

[2 marks]

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.

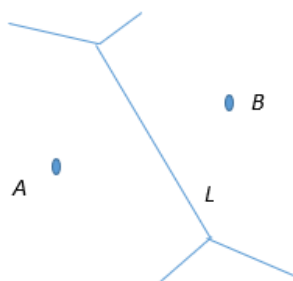


- 1b. Find the equation of the line which would complete the Voronoi cell containing site E. [3 marks]

Give your answer in the form  $ax + by + d = 0$  where  $a, b, d \in \mathbb{Z}$ .

- 1c. In the context of the question, explain the significance of the Voronoi cell containing site E. [1 mark]

2. The diagram below is part of a Voronoi diagram. [9 marks]



**Diagram not to scale**

$A$  and  $B$  are sites with  $B$  having the co-ordinates of  $(4, 6)$ .  $L$  is an edge; the equation of this perpendicular bisector of the line segment from  $A$  to  $B$  is  $y = -2x + 9$

Find the co-ordinates of the point  $A$ .

A king rules a small mountain kingdom which is in the form of a square of length 4 kilometres. The square is described by the co-ordinate system

$$0 \leq x \leq 4, 0 \leq y \leq 4.$$

The king has four adult children, each of which has a luxury palace located at the points  $(1, 1)$ ,  $(3, 1)$ ,  $(1, 3)$ ,  $(3, 3)$ . Each child owns all the land that is nearer their palace than any other palace.

3a. Sketch a Voronoi diagram to represent this information. *[2 marks]*

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The king has a fifth (youngest) child who is now just growing up. He installs her in a new palace situated at point  $(2, 2)$ . The rule about ownership of land is then reapplied.

3b. Sketch a new Voronoi diagram to represent this new situation. *[2 marks]*

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3c. State what the shape of the land, owned by the youngest child, is. *[1 mark]*

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3d. Find the area of the youngest child's land. *[4 marks]*

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3e. Find how much land an older child has lost. *[1 mark]*

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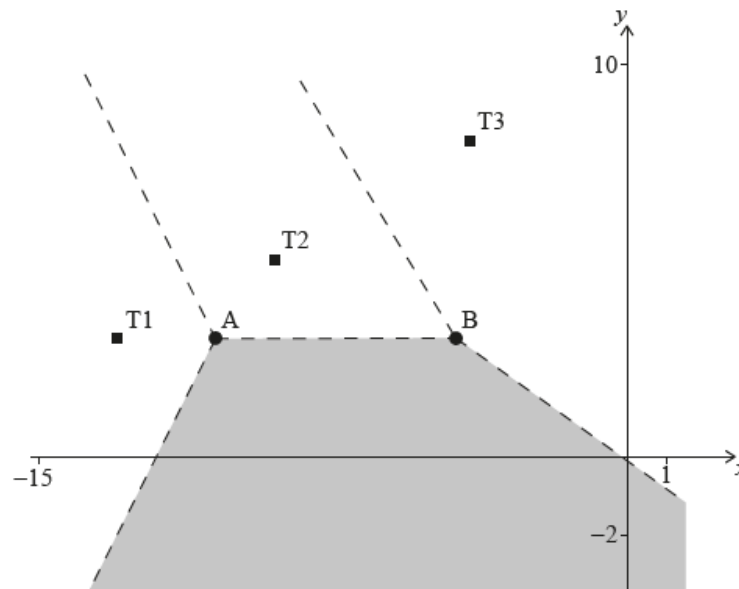
3f. State, with a reason, if all five children now own an equal amount of land. *[2 marks]*

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The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth identical cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

4a. Explain why Tim will receive the strongest signal from tower T4. [1 mark]

Tower T2 has coordinates  $(-9, 5)$  and the edge connecting vertices A and B has equation  $y = 3$ .

4b. Write down the coordinates of tower T4. [2 marks]

4c. Tower T1 has coordinates  $(-13, 3)$ . [3 marks]

Find the gradient of the edge of the Voronoi diagram between towers T1 and T2.