Finding the derivative

1. If $y = \left(x^3 + 1\right)^2$, then $\frac{dy}{dx} =$

(A) $\left(3x^2\right)^2$

(B) $2\left(x^3 + 1\right)$

(C) $2\left(3x^2 + 1\right)$

(D) $3x^2\left(x^3 + 1\right)$

(E) $6x^2\left(x^3 + 1\right)$

2. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x) + f''(x)$

(B) $f'(x) + \left(f''(x)\right)^2$

(C) $\left(f'(x) + f''(x)\right)^2$

(D) $\left(f'(x)\right)^2 + f''(x)$

(E) $2f'(x) + f''(x)$

3. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\left(f(\ln x)\right) =$

(A) $\frac{2\ln x + 2}{x}$

(B) $2x \ln x + 2x$

(C) $2\ln x + 2$

(D) $2\ln x + \frac{2}{x}$

(E) $\frac{2x + 2}{x}$

4. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$

(B) $e^{(2/x)}$

(C) $e^{(-2/x^2)}$

(D) $-\frac{2}{x^2}e^{(2/x)}$

(E) $-2x^2e^{(2/x)}$

5. If $f(x) = e^{1/x}$, then *f'(x)*=

(A) $-\frac{e^{1/x}}{x^2}$

(B) $-e^{1/x}$

(C) $\frac{e^{1/x}}{x}$

(D) $\frac{e^{1/x}}{x^2}$

(E) $\frac{1}{x}e^{(1/x)-1}$

Finding the derivative

6. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{3\sqrt{3}}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $-\frac{\sqrt{3}}{2}$

(D) $-\frac{3}{2}$

(E) $-\frac{3\sqrt{3}}{2}$

7. If $y = \cos^2 3x$, then dy/dx=

(A) -6 sin 3*x* cos 3*x*

(B) -2 cos 3*x*

(C) 2 cos 3*x*

(D) 6 cos 3*x*

(E) 2 sin 3*x* cos 3*x*

8. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) One

(B) Two

(C) Three

(D) Five

(E) Nine

9. If $y = \left(x^3 - \cos x\right)^5$ then $y' =$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

10. If $f(x) = \mathrm{e}^x$, then $\ln(f'(2)) =$

(A) 2

(B) 0

(C) $1/\mathrm{e}^2$

(D) 2e

(E) e^2

Finding the derivative

11. $\frac{d}{dx}(2^x)$=

(A) 2^{x-1}

(B) $\left(2^{x-1}\right)x$

(C) $\left(2^x\right)\ln 2$

(D) $\left(2^{x-1}\right)\ln 2$

(E) 2x/ln2

12. d/dx(ln e^{2x}) =

(A) $1/e^{2x}$

(B) $2/e^{2x}$

(C) $2x$

(D) 1

(E) 2

13. Let *f* and *g* be differentiable functions with the following properties

(i) $g(x)>0$ for all x

(ii) $f(0)=1$ If $h(x)=f(x)g(x)$ and $h'(x)=f(x)g'(x)$, then $f(x)$= :

(A) $f'(x)$

(B) $g(x)$

(C) e^x

(D) 0

(E) 1

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x\cos 2x$

(B) $4x\cos 2x$

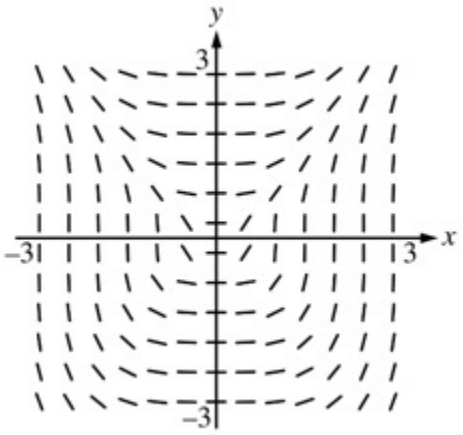
(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x\cos 2x)$

(E) $2x(\sin 2x + x\cos 2x)$

Finding the derivative

15.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

16. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y = -7x + 6$

(B) $y = -6x + 5$

(C) $y = -2x$

(D) $y = 2x - 3$

(E) $y = 7x - 8$

17. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} =$

(A) $\frac{12x+13}{(3x+2)^2}$

(B) $\frac{12x-13}{(3x+2)^2}$

(C) $\frac{5}{(3x+2)^2}$

(D) $\frac{-5}{(3x+2)^2}$

(E) $\frac{2}{3}$

18. What is the instantaneous rate of change at *x*=2 of the function *f* given by $f(x) = \frac{x^2-2}{x-1}$?

Finding the derivative

(A) -2
(B) $\frac{1}{6}$
(C) $\frac{1}{2}$
(D) 2
(E) 6

19. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$
(A) $\frac{1}{x}$
(B) $\frac{1}{x^2}$
(C) $\frac{\ln x - 1}{x^2}$
(D) $\frac{1 - \ln x}{x^2}$
(E) $\frac{1 + \ln x}{x^2}$

20. If *f(x)*=tan(2*x*) , then $f'(\frac{\pi}{6}) =$
(A) $\sqrt{3}$
(B) $2\sqrt{3}$
(C) 4
(D) $4\sqrt{3}$
(E) 8

21. If $y = \sin(3x)$, then $\frac{dy}{dx} =$
(A) $-3\cos(3x)$
(B) $-\cos(3x)$
(C) $-\frac{1}{3}\cos(3x)$
(D) $\cos(3x)$
(E) $3\cos(3x)$

22. If $f(x) = \sin x$, then f′(π/3) =
(A) $-\frac{1}{2}$
(B) $\frac{1}{2}$
(C) $\frac{\sqrt{2}}{2}$
(D) $\frac{\sqrt{3}}{2}$
(E) $\sqrt{3}$

23. If y=tanx−cotx, then dy/dx =

Finding the derivative

(A) sec x csc x
(B) sec x- csc x
(C) sec x + csc x
(D) $\sec^2 x$ - $\csc^2 x$
(E) $\sec^2 x$ + $\csc^2 x$

24. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) –2 cos 3
(B) –2 sin 3 cos 3
(C) 6 cos 3
(D) 2 sin 3 cos 3
(E) 6 sin 3 cos 3

25. If $f(x)=\sin(e^{-x})$, then $f'(x)=$

(A) $-\cos(e^{-x})$
(B) $\cos(e^{-x})+e^{-x}$
(C) $\cos(e^{-x})-e^{-x}$
(D) $e^{-x}\cos(e^{-x})$
(E) $-e^{-x}\cos(e^{-x})$

26. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$
(B) $\frac{1}{5}$
(C) $\frac{1}{4}$
(D) $\frac{2}{5}$
(E) nonexistent

27. If $f(x) = \left(x^2 - 2x - 1\right)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$
(B) 0
(C) $-\frac{2}{3}$
(D) $-\frac{4}{3}$
(E) -2

28. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

Finding the derivative

(A) $-\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{2}{5}$

(E) nonexistent

29. $\frac{d}{dx} \ln|\cos(\frac{\pi}{x})|$ is

(A) $\frac{-\pi}{x^2 \cos(\frac{\pi}{x})}$

(B) $-\tan(\frac{\pi}{x})$

(C) $\frac{1}{\cos(\frac{\pi}{x})}$

(D) $\frac{\pi}{x} \tan(\frac{\pi}{x})$

(E) $\frac{\pi}{x^2}\tan(\frac{\pi}{x})$

30. $d/dx(1/x^3 - 1/x + x^2)$ at $x = -1$ is

(A) -6

(B) -4

(C) 0

(D) 2

(E) 6

31. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) -6

(B) -3

(C) 3

(D) 6

(E) 8

32. If $f(x) = x$, then $f'(5) =$

(A) 0

(B) 1/5

(C) 1

(D) 5

(E) 25/2

Finding the derivative

33. Let f and g be differentiable functions such that

$$f(1) = 2, \quad f'(1) = 3, \quad f'(2) = -4,$$

$$g(1) = 2, \quad g'(1) = -3, \quad g'(2) = 5.$$

If $h(x) = f(g(x))$, then $h'(1) =$

(A) -9

(B) -4

(C) 0

(D) 12

(E) 15

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$

(B) $y = -x^2$

(C) $y = -\frac{1}{3}x^3$

(D) $y = \frac{1}{3}x^3$

(E) $y = \ln x$

35. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $1/x$

(B) $1/\ln x$

(C) $\ln x/x$

(D) x

(E) $1/x \ln x$

36. If $f(x) = (x - 1)(x^2 + 2)^3$, then $f'(x) =$

(A) $6x(x^2+2)^2$

(B) $6x(x - 1)(x^2 + 2)^2$

(C) $(x^2 + 2)^2(x^2 + 3x - 1)$

(D) $(x^2 + 2)^2(7x^2 - 6x + 2)$

(E) $-3(x - 1)(x^2 + 2)^2$

37. Suppose that f is an odd function; i.e., $f(-x)=-f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

Finding the derivative

(A) $f'(x_0)$
(B) $-f'(x_0)$
(C) $\frac{1}{f'(x_0)}$
(D) $-\frac{1}{f'(x_0)}$
(E) None of the above

38. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1
(B) $\frac{3}{2}$
(C) 2
(D) $\frac{7}{2}$
(E) $\frac{3+e}{2}$

39. If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$
(B) $1 - \cos x$
(C) $\cos x$
(D) $\sin x - x\cos x$
(E) $\sin x + x\cos x$

40. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0, 0)$ only
(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only
(C) $(0, 0)$ and $(-4, 2)$
(D) $(0, 0)$ and $\left(4, \frac{2}{3}\right)$
(E) There are no such points.

41. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.
(B) $x < -1$ and $x > 3$
(C) $-3 < x < 1$
(D) $-1 < x < 3$
(E) All values of x

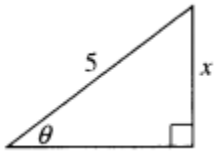
42. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

Finding the derivative

(A) $2xe^x$
(B) $x(x+2e^x)$
(C) $xe^x(x+2)$
(D) $2x+e^x$
(E) $2x+e$

43. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

44.



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is *x* increasing in units per minute when *x* equals 3 units?

(A) 3
(B) $\frac{15}{4}$
(C) 4
(D) 9
(E) 12

45. If *f* and *g* are twice differentiable and if $h(x)=f(g(x))$, then $h''(x)=$
(A) $f''(g(x))[g'(x)]^2+f'(g(x))g''(x)$
(B) $f''(g(x))g'(x)+f'(g(x))g''(x)$
(C) $f''(g(x))[g'(x)]^2$
(D) $f''(g(x))g''(x)$
(E) $f''(g(x))$

46. If $\frac{d}{dx}(f(x))=g(x)$ and $\frac{d}{dx}(g(x))=f(x^2)$, then $\frac{d^2}{dx^2}(f(x^3))=$

Finding the derivative

(A) $f(x^6)$
(B) $g(x^3)$
(C) $3x^2g(x^3)$
(D) $9x^4f(x^6) + 6x\,g(x^3)$
(E) $f(x^6) + g(x^3)$

47. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}(f(h(x))) =$
(A) $g(x^2)$
(B) $2xg(x)$
(C) $g'(x)$
(D) $2xg(x^2)$
(E) $x^2g(x^2)$

48. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point (1, 5) is
(A) $13x-y=8$
(B) $13x+y=18$
(C) $x-13y=64$
(D) $x+13y=66$
(E) $-2x+3y=13$

49. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$
(A) $\frac{1}{e}$
(B) $\frac{2}{e}$
(C) $\frac{1}{2\sqrt{e}}$
(D) $\frac{1}{\sqrt{e}}$
(E) $\frac{2}{\sqrt{e}}$

50. Let h be a differentiable function, and let f be the function defined by $f(x) = h(x^2 - 3)$. Which of the following is equal to $f'(2)$?
(A) $h'(1)$
(B) $4h'(1)$
(C) $4h'(2)$
(D) $h'(4)$
(E) $4h'(4)$

51. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

Finding the derivative

(A) $\frac{7}{\sqrt{5}}$

(B) $\frac{14}{\sqrt{5}}$

(C) $\frac{18}{\sqrt{5}}$

(D) $\frac{15}{\sqrt{21}}$

(E) $\frac{30}{\sqrt{21}}$