

# Syllabus overview

This book covers the whole syllabus for the DP Mathematics: applications and interpretation SL course. Here is an overview of the syllabus content covered in each chapter.

## 1 Measuring space: accuracy and 2D geometry

Syllabus reference	Syllabus content
SL1.5*	Laws of exponents with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.1*	Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.
SL1.6	Approximation: decimal places, significant figures. Upper and lower bounds of rounded numbers. Percentage errors. Estimation.
SL3.2*	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ . The cosine rule $c^2 = a^2 + b^2 - 2ab \cos C$ ; $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ . Area of a triangle as $\frac{1}{2}ab \sin C$ .
SL3.3*	Applications of right and non-right angled trigonometry, including Pythagoras theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.
SL3.4	The circle: length of an arc; area of a sector.

\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.

## 2 Representing space: non-right angled trigonometry and volumes

Syllabus reference	Syllabus content
SL3.2*	<p>Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.</p> <p>The sine rule: <math>\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}</math>.</p> <p>The cosine rule <math>c^2 = a^2 + b^2 - 2ab \cos C</math>; <math>\cos A = \frac{b^2 + c^2 - a^2}{2bc}</math>.</p> <p>Area of a triangle as <math>\frac{1}{2}ab \sin C</math>.</p>
SL3.3*	<p>Applications of right and non-right angled trigonometry, including Pythagoras theorem.</p> <p>Angles of elevation and depression.</p> <p>Construction of labelled diagrams from written statements.</p>
SL3.1*	<p>The distance between two points in three-dimensional space, and their midpoint.</p> <p>Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids.</p> <p>The size of an angle between two intersecting lines or between a line and a plane.</p>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

### 3 Representing and describing data: descriptive statistics

Syllabus reference	Syllabus content
SL1.2*	<p>Arithmetic sequences and series.</p> <p>Use of the formulae for the <math>n</math>th term and the sum of the first <math>n</math> terms of the sequence.</p> <p>Use of sigma notation for sums of arithmetic sequences.</p> <p>Applications.</p> <p>Analysis, interpretation and prediction where a model is not perfectly arithmetic in real-life.</p>
SL4.1*	<p>Concepts of population, sample, random sample, discrete and continuous data.</p> <p>Reliability of data sources and bias in sampling.</p> <p>Interpretation of outliers.</p> <p>Sampling techniques and their effectiveness.</p>
SL4.2*	<p>Presentation of data (discrete and continuous): frequency distributions (tables).</p> <p>Histograms.</p> <p>Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR).</p> <p>Production and understanding of box and whisker diagrams.</p>
SL4.3*	<p>Measures of central tendency (mean, median and mode).</p> <p>Estimation of mean from grouped data.</p> <p>Modal class.</p> <p>Measures of dispersion (interquartile range, standard deviation and variance).</p> <p>Effect of constant changes on the original data.</p> <p>Quartiles of discrete data.</p>
SL4.4*	<p>Linear correlation of bivariate data.</p> <p>Pearson's product-moment correlation coefficient, <math>r</math>.</p> <p>Scatter diagrams; lines of best fit, by eye, passing through the mean point.</p> <p>Equation of the regression line of <math>y</math> on <math>x</math>.</p> <p>Use of the equation of the regression line for prediction purposes.</p> <p>Interpret the meaning of the parameters, <math>a</math> and <math>b</math>, in a linear regression <math>y = ax + b</math>.</p>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

## 4 Dividing up space: coordinate geometry, lines, Voronoi diagrams

Syllabus reference	Syllabus content
SL2.1*	<p>Different forms of the equation of a straight line.</p> <p>Gradient; intercepts</p> <p>Lines with gradients, <math>m_1</math> and <math>m_2</math></p> <p>Parallel lines <math>m_1 = m_2</math></p> <p>Perpendicular lines, <math>m_1 \times m_2 = -1</math></p>
SL2.3*	<p>The graph of a function; its equation <math>y = f(x)</math>.</p> <p>Creating a sketch from information given or a context, including transferring a graph from screen to paper.</p> <p>Using technology to graph functions including their sums and differences.</p>
SL2.4*	<p>Determine key features of graphs.</p> <p>Finding the point of intersection of two curves or lines using technology.</p>
SL3.1*	<p>The distance between two points in three-dimensional space, and their midpoint.</p> <p>Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids.</p> <p>The size of an angle between two intersecting lines or between a line and a plane.</p>
SL3.5	Equations of perpendicular bisectors.
SL3.6	<p>Voronoi diagrams; sites, vertices, edges, cells.</p> <p>Addition of a site to an existing Voronoi diagram.</p> <p>Nearest neighbour interpolation.</p> <p>Applications of 'the toxic waste dump' problem.</p>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

## 5 Modelling constant rates of change: linear functions

Syllabus reference	Syllabus content
SL2.2*	Concept of a function, domain, range and graph. Function notation, eg $f(x), v(t), C(n)$ The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$ , and the notation $f^{-1}(x)$ .
SL2.3*	The graph of a function; its equation $y = f(x)$ . Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4*	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL2.5	Modelling with the following functions: <ul style="list-style-type: none"> <li>• Linear models: <math>f(x) = mx + c</math></li> <li>• Quadratic models: <math>f(x) = ax^2 + bx + c; a \neq 0</math>. Axis of symmetry, vertex, zeros and roots, intercepts on the <math>x</math>-axis and <math>y</math>-axis.</li> <li>• Exponential growth and decay models:               <math display="block">f(x) = ka^x + c</math> <math display="block">f(x) = ka^{-x} + c</math>               (for <math>a &gt; 0</math>)  <math display="block">f(x) = ke^{rx} + c</math> </li> </ul> Equation of a horizontal asymptote. <ul style="list-style-type: none"> <li>• Direct/inverse variation: <math>f(x) = ax^n, n \in \mathbb{Z}</math>                The <math>y</math>-axis as a vertical asymptote when <math>n &lt; 0</math>.</li> <li>• Cubic models: <math>f(x) = ax^3 + bx^2 + cx + d</math></li> <li>• Sinusoidal models: <math>f(x) = a \sin(bx) + d, f(x) = a \cos(bx) + d</math></li> </ul>
SL1.2*	Arithmetic sequences and series. Use of the formulae for the $n$ th term and the sum of the first $n$ terms of the sequence. Use of sigma notation for sums of arithmetic sequences. Applications. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real-life.
SL1.8	Use technology to solve:

	<ul style="list-style-type: none"> <li>• Systems of linear equations in up to 3 variables</li> <li>• Polynomial equations</li> </ul>
SL2.6	<p>Modelling skills:</p> <ul style="list-style-type: none"> <li>• Use the modelling process described in the “mathematical modelling” section to create, fit and use the theoretical models in section SL2.5, and their graphs.</li> </ul> <p>Develop and fit the model:</p> <ul style="list-style-type: none"> <li>• Given a context recognize and choose an appropriate model and possible parameters.</li> <li>• Determine a reasonable domain for a model.</li> <li>• Find the parameters of a model.</li> </ul> <p>Test and reflect upon the model:</p> <ul style="list-style-type: none"> <li>• Comment on the appropriateness and reasonableness of a model.</li> <li>• Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation.</li> </ul> <p>Use the model:</p> <ul style="list-style-type: none"> <li>• Reading, interpreting and making predictions based on the model.</li> </ul>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

## 6 Modelling relationships: linear correlation of bivariate data

Syllabus reference	Syllabus content
SL4.4*	Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, $r$ . Scatter diagrams; lines of best fit, by eye, passing through the mean point. Equation of the regression line of $y$ on $x$ . Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, $a$ and $b$ , in a linear regression $y = ax + b$
SL4.10	Spearman's rank correlation coefficient, $r_s$ . Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

## 7 Quantifying uncertainty: probability, binomial and normal distributions

Syllabus reference	Syllabus content
SL4.2*	<p>Presentation of data (discrete and continuous): frequency distributions (tables).</p> <p>Histograms.</p> <p>Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR).</p> <p>Production and understanding of box and whisker diagrams.</p>
SL4.5*	<p>Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (<math>U</math>) and event.</p> <p>The probability of an event <math>A</math> is <math>P(A) = \frac{n(A)}{n(U)}</math>.</p> <p>The complementary events <math>A</math> and <math>A'</math> (not <math>A</math>).</p> <p>Expected number of occurrences.</p>
SL4.6*	<p>Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities.</p> <p>Combined events: <math>P(A \cup B) = P(A) + P(B) - P(A \cap B)</math>.</p> <p>Mutually exclusive events: <math>P(A \cap B) = 0</math></p> <p>Conditional probability: <math>P(A B) = \frac{P(A \cap B)}{P(B)}</math></p> <p>Independent events: <math>P(A \cap B) = P(A)P(B)</math>.</p>
SL4.7*	<p>Concept of discrete random variables and their probability distributions.</p> <p>Expected value (mean), <math>E(X)</math> for discrete data.</p> <p>Applications.</p>
SL4.8*	<p>Binomial distribution.</p> <p>Mean and variance of the binomial distribution.</p>
SL4.9*	<p>The normal distribution and curve.</p> <p>Properties of the normal distribution.</p> <p>Diagrammatic representation.</p> <p>Normal probability calculations.</p> <p>Inverse normal calculations.</p>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*



## 8 Testing for validity: Spearman's, hypothesis testing and $\chi^2$ test for independence

Syllabus reference	Syllabus content
SL4.11	Formulation of null and alternative hypotheses, $H_0$ and $H_1$ . Significance levels. $p$ -values. Expected and observed frequencies. The $\chi^2$ test for independence: contingency tables; degrees of freedom, critical value. The $\chi^2$ goodness of fit test.
SL4.10	Spearman's rank correlation coefficient, $r_s$ . Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.

## 9 Modelling relationships with functions: power functions

Syllabus reference	Syllabus content
SL2.2*	Concept of a function, domain, range and graph. Function notation, eg $f(x), v(t), C(n)$ The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$ , and the notation $f^{-1}(x)$ .
SL2.3*	The graph of a function; its equation $y = f(x)$ . Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.
SL2.4	Determine key features of graphs. Finding the point of intersection of two curves or lines using technology.
SL2.5	Modelling with the following functions: <ul style="list-style-type: none"> <li>• Linear models: <math>f(x) = mx + c</math></li> <li>• Quadratic models: <math>f(x) = ax^2 + bx + c; a \neq 0</math>. Axis of symmetry, vertex, zeros and roots, intercepts on the <math>x</math>-axis and <math>y</math>-axis.</li> <li>• Exponential growth and decay models:               <math display="block">f(x) = ka^x + c</math> <math display="block">f(x) = ka^{-x} + c</math>               (for <math>a &gt; 0</math>)  <math display="block">f(x) = ke^{rx} + c</math> </li> </ul> Equation of a horizontal asymptote. <ul style="list-style-type: none"> <li>• Direct/inverse variation: <math>f(x) = ax^n, n \in \mathbb{Z}</math>                The <math>y</math>-axis as a vertical asymptote when <math>n &lt; 0</math>.</li> <li>• Cubic models: <math>f(x) = ax^3 + bx^2 + cx + d</math></li> <li>• Sinusoidal models: <math>f(x) = a \sin(bx) + d, f(x) = a \cos(bx) + d</math></li> </ul>
SL2.6	Modelling skills: <ul style="list-style-type: none"> <li>• Use the modelling process described in the “mathematical modelling” section to create, fit and use the theoretical models in section SL2.5, and their graphs.</li> </ul> Develop and fit the model: <ul style="list-style-type: none"> <li>• Given a context recognize and choose an appropriate model and possible parameters.</li> <li>• Determine a reasonable domain for a model.</li> </ul>

	<ul style="list-style-type: none"> <li>Find the parameters of a model.</li> </ul> <p>Test and reflect upon the model:</p> <ul style="list-style-type: none"> <li>Comment on the appropriateness and reasonableness of a model.</li> <li>Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation.</li> </ul> <p>Use the model:</p> <ul style="list-style-type: none"> <li>Reading, interpreting and making predictions based on the model.</li> </ul>
SL1.8	<p>Use technology to solve:</p> <ul style="list-style-type: none"> <li>Systems of linear equations in up to 3 variables</li> <li>Polynomial equations</li> </ul>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

## 10 Modelling rates of change: exponential and logarithmic functions

Syllabus reference	Syllabus content
SL1.3*	<p>Geometric sequences and series</p> <p>Use of the formulae for the <math>n</math>th term and the sum of the first <math>n</math> terms of the sequence.</p> <p>Use of sigma notation for the sums of geometric sequences.</p> <p>Applications.</p>
SL1.4*	<p>Financial applications of geometric sequences and series:</p> <ul style="list-style-type: none"> <li>• Compound interest</li> <li>• Annual depreciation</li> </ul>
SL1.5*	<p>Laws of exponents with integer exponents.</p> <p>Introduction to logarithms with base 10 and e.</p> <p>Numerical evaluation of logarithms using technology.</p>
SL1.7	Amortization and annuities using technology.
SL2.2*	<p>Concept of a function, domain, range and graph.</p> <p>Function notation, eg <math>f(x), v(t), C(n)</math></p> <p>The concept of a function as a mathematical model.</p> <p>Informal concept that an inverse function reverses or undoes the effect of a function.</p> <p>Inverse function as a reflection in the line <math>y = x</math>, and the notation <math>f^{-1}(x)</math>.</p>
SL2.3*	<p>The graph of a function; its equation <math>y = f(x)</math>.</p> <p>Creating a sketch from information given or a context, including transferring a graph from screen to paper.</p> <p>Using technology to graph functions including their sums and differences.</p>
SL2.4	<p>Determine key features of graphs.</p> <p>Finding the point of intersection of two curves or lines using technology.</p>
SL2.5	<p>Modelling with the following functions:</p> <ul style="list-style-type: none"> <li>• Linear models: <math>f(x) = mx + c</math></li> <li>• Quadratic models: <math>f(x) = ax^2 + bx + c; a \neq 0</math>. Axis of symmetry, vertex, zeros and roots, intercepts on the <math>x</math>-axis and <math>y</math>-axis.</li> <li>• Exponential growth and decay models: <ul style="list-style-type: none"> <li><math>f(x) = ka^x + c</math></li> <li><math>f(x) = ka^{-x} + c</math></li> <li>(for <math>a &gt; 0</math>)</li> <li><math>f(x) = ke^{rx} + c</math></li> </ul> </li> </ul>

	<p>Equation of a horizontal asymptote.</p> <ul style="list-style-type: none"> <li>• Direct/inverse variation: <math>f(x) = ax^n, n \in \mathbb{Z}</math></li> </ul> <p>The <math>y</math>-axis as a vertical asymptote when <math>n &lt; 0</math>.</p> <ul style="list-style-type: none"> <li>• Cubic models: <math>f(x) = ax^3 + bx^2 + cx + d</math></li> <li>• Sinusoidal models: <math>f(x) = a \sin(bx) + d, f(x) = a \cos(bx) + d</math></li> </ul>
SL2.6	<p>Modelling skills:</p> <ul style="list-style-type: none"> <li>• Use the modelling process described in the “mathematical modelling” section to create, fit and use the theoretical models in section SL2.5, and their graphs.</li> </ul> <p>Develop and fit the model:</p> <ul style="list-style-type: none"> <li>• Given a context recognize and choose an appropriate model and possible parameters.</li> <li>• Determine a reasonable domain for a model.</li> <li>• Find the parameters of a model.</li> </ul> <p>Test and reflect upon the model:</p> <ul style="list-style-type: none"> <li>• Comment on the appropriateness and reasonableness of a model.</li> <li>• Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation.</li> </ul> <p>Use the model:</p> <ul style="list-style-type: none"> <li>• Reading, interpreting and making predictions based on the model.</li> </ul>

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

## 11 Modelling periodic phenomena: trigonometric functions

Syllabus reference	Syllabus content
SL2.5	<p>Modelling with the following functions:</p> <ul style="list-style-type: none"> <li>Linear models: <math>f(x) = mx + c</math></li> <li>Quadratic models: <math>f(x) = ax^2 + bx + c; a \neq 0</math>. Axis of symmetry, vertex, zeros and roots, intercepts on the <math>x</math>-axis and <math>y</math>-axis.</li> <li>Exponential growth and decay models:           <math display="block">f(x) = ka^x + c</math> <math display="block">f(x) = ka^{-x} + c</math>           (for <math>a &gt; 0</math>)           <math display="block">f(x) = ke^{rx} + c</math> </li> </ul> <p>Equation of a horizontal asymptote.</p> <ul style="list-style-type: none"> <li>Direct/inverse variation: <math>f(x) = ax^n, n \in \mathbb{Z}</math></li> </ul> <p>The <math>y</math>-axis as a vertical asymptote when <math>n &lt; 0</math>.</p> <ul style="list-style-type: none"> <li>Cubic models: <math>f(x) = ax^3 + bx^2 + cx + d</math></li> <li>Sinusoidal models: <math>f(x) = a \sin(bx) + d, f(x) = a \cos(bx) + d</math></li> </ul>
SL2.6	<p>Modelling skills:</p> <p>Use the modelling process described in the “mathematical modelling” section to create, fit and use the theoretical models in section SL2.5, and their graphs.</p> <p>Develop and fit the model:</p> <p>Given a context recognize and choose an appropriate model and possible parameters.</p> <p>Determine a reasonable domain for a model.</p> <p>Find the parameters of a model.</p> <p>Test and reflect upon the model:</p> <p>Comment on the appropriateness and reasonableness of a model.</p> <p>Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation.</p> <p>Use the model:</p> <p>Reading, interpreting and making predictions based on the model.</p>

## 12 Analyzing rates of change: differential calculus

Syllabus reference	Syllabus content
SL5.1*	Introduction to the concept of a limit. Derivative interpreted as gradient function and as rate of change.
SL5.2*	Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0, f'(x) = 0, f'(x) < 0$ .
SL5.3*	Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}, n \in \mathbb{Z}$ The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ , where all exponents are integers.
SL5.4*	Tangents and normals at a given point, and their equations.
SL5.6	Values of $x$ where the gradient of a curve is zero. Solution of $f'(x) = 0$ . Local maximum and minimum points.
SL5.7	Optimisation problems in context.

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*

### 13 Approximating irregular spaces: integration

Syllabus reference	Syllabus content
SL5.5*	Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ , where $n \in \mathbb{Z}, n \neq -1$ Definite integrals using technology. Anti-differentiation with a boundary condition to determine the constant term. Areas between a curve $y = f(x)$ and the $x$ -axis, where $f(x) > 0$ .
SL5.8	Approximating areas using the trapezoidal rule.

*\*Shows content that is common to both the Mathematics: analysis and approaches and the Mathematics: applications and interpretation courses.*