8.4 The *t*-test

Maisy noticed that the sun shines more on one side of her garden than the other. She wanted to know whether this had any effect on the heights of the tulips on either side of the garden. She measured 20 tulips from each side of the garden. The data is shown in the table.

Length, in cm, of tulips on right side	Length, in cm, of tulips on left side
21	24
21	25
26	25
25	26
28	32
24	29
22	31
22	27
29	26
28	28
28	22
27	22
21	28
23	28
24	30
24	31
27	29
26	28
26	28
25	32

1 How can Maisy compare the two sets of data?

2 Discuss whether it would be fair to look at the mean values only. If not, give a reason.

3 Discuss whether you think that the spread of the data should also be taken into consideration. Give a reason for your answer.

4 Consider whether the two sets of data must be the same size or not.

The best way to compare these two sets of data is to use a *t*-test.

William Gosset was employed by Guinness to improve the taste and quality of their beer. In order to monitor the quality of the hops that were used in the brewing process, he invented the *t*-test. He published under the pen name "Student". Hence, it is sometimes referred to as Student's *t*-test.

The *t*-test is used for two data sets that are measuring the same thing (like the tulips above), and only applies to normally distributed data. There is a formula that is used to calculate it, but you won't need to use it as your GDC will do the work for you.

First of all, you need to set up your null and alternative hypotheses.

The null hypothesis is that the two means, $\bar{x}_1$ and $\bar{x}_2$, are equal:

$H_0: \bar{x}_1 = \bar{x}_2$

The alternative hypothesis is that the two means are not equal. For a **two-tailed test** this just means checking that $\bar{x}_1 \neq \bar{x}_2$. For a **one-tailed test**, it means checking either that $\bar{x}_1$ is less than $\bar{x}_2$ or that it is greater than $\bar{x}_2$.

For a two-tailed test, $H_1: \bar{x}_1 \neq \bar{x}_2$

For a one-tailed test, $H_1: \bar{x}_1 > \bar{x}_2$ or $\bar{x}_1 < \bar{x}_2$

For Maisy's tulips, the null and alternative hypotheses would be:

H_0: The mean of the tulips on the right side is the same as the mean of the tulips on the left side.

H_1: The mean of the tulips on the right side is not the same as the mean of the tulips on the left side.

Using your GDC you will be able to find the *t*-statistic and the *p*-value. To do the *t*-test with your GDC, put your data into two lists. Go to Statistics, Tests, 2-sample *t* Test, data (input), choose the correct alternative hypothesis, pooled (yes), enter. You will see the *t*-value and the *p*-value. However, in examinations you will only use the *p*-value.

Once you have found the p-value, you can compare it with the significance level just like you did in the χ^2 test.

If the p-value is greater than the significance level then you accept the null hypothesis; if it is smaller, then you do not accept the null hypothesis.

The t-test is mainly conducted at the 5% significance level, though, like the χ^2 test, it can also be conducted at the 1% or 10% significance levels.

The t-value is -3.07 and the p-value is 0.003 92.

$0.003\,92 < 0.05$, so the null hypothesis is rejected: there is a difference in the heights of the tulips on either side of the garden.

Example 7

Mr Arthur gives his two chemistry groups the same test. He wants to find out whether there is any difference between the achievement levels of the two groups.

The results are:

Group 1	54	62	67	43	85	69	73	81	47	92	55	59	68	72
Group 2	73	67	58	46	91	48	82	81	67	74	57	66		

a Write down the null and alternative hypotheses.

b Find the t-value and p-value for a t-test at the 5% significance level.

c Write down the conclusion to the test.

a $H_0: \bar{x}_1 = \bar{x}_2$ (there is no difference between the grades in Group 1 and the grades in Group 2)

$H_1: \bar{x}_1 \neq \bar{x}_2$ (there is a difference between the grades in Group 1 and the grades in Group 2)

b t-value $= -0.235$, p-value $= 0.816$

c $0.816 > 0.05$, so you accept the null hypothesis: there is no significant difference between the two groups.

Notice that the two groups do not need to be the same size.

This will be a two-tailed test as you want to know whether Group 1 is better or worse than Group 2.

Example 8

An oil company claims to have developed a fuel that will increase the distance travelled for every litre of fuel.

Ten scooters are filled with one litre of normal fuel and ten scooters are filled with one litre of the new fuel. The distances, in km, travelled on the one litre by each scooter are as follows:

Original fuel	36	38	44	42	45	39	48	51	38	43
New fuel	43	39	51	49	53	48	52	46	53	49

a Write down the null and alternative hypotheses.

b Find the t-value and p-value for a t-test at the 5% significance level.

c Write down the conclusion to the test.

a $H_0: \bar{x}_1 = \bar{x}_2$ (there is no difference between the distance travelled with the original and new fuels) $H_1: \bar{x}_1 < \bar{x}_2$ (the distance travelled with the original fuel is less than the distance travelled with the new fuel) **b** t-value = −2.83, p-value = 0.005 61 **c** 0.005 61 < 0.05, so you do not accept the null hypothesis: the company's claim is correct.	Note that this is a one-tailed test as you are only considering that the distance travelled with the original fuel is less than the distance travelled with the new fuel.

Reflect How do we test for validity with statistical inferences?

TOK

What counts as understanding in mathematics?

Exercise 8G

1 Petra noticed that one of her apple trees grew in the shade and the others did not. She wanted to find out whether apples from the tree in the shade weighed less than those in the sun. She picked nine apples from each tree and weighed them in grams.

Tree in shade	75	82	93	77	85	78	91	83	92
Tree not in shade	74	81	95	79	95	82	93	88	90

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 10% significance level.

d Write down the conclusion to the test.

2 Fergus heard that babies born in the country weigh more than babies born in the town. He contacted two midwives, one who delivered babies in the country and one who delivered babies in the town, and asked them for the weights, in kg, of the babies that they had delivered during the previous week.

Country babies	2.8	3.2	2.7	3.5	3.0	2.9	4.1	3.9	
Town babies	3.1	3.5	2.8	3.7	4.2	2.6	3.2	2.9	3.8

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 10% significance level.

d Write down the conclusion to the test.

3 Jocasta picked some runner beans from two different plants and measured them, in cm. She wanted to find out whether there was any difference in the lengths of the beans from the two plants.

Length of beans on plant 1	19	23	21	25	24	18	25	18	24	16
Length of beans on plant 2	27	24	25	28	25	27	24	26	22	23

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 5% significance level.

d Write down the conclusion to the test.

4 The lifetimes of two different types of light bulb were tested to find out whether one was better than the other or not. The numbers of hours are listed in the table.

Bulb 1	1236	1350	1489	2052	1986	1875	2134	1985
Bulb 2	1567	1432	1267	2145	1879	1987	1679	1765

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 5% significance level.

d Write down the conclusion to the test.

5 The weights, in kg, of the 11-year-old boys and girls in Grade 6 were recorded to find out whether the boys weighed less than the girls.

Boys' weights	Girls' weights
33	35
32	39
35	43
36	45
41	39
32	44
38	38
34	32
36	
31	

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 5% significance level.

d Write down the conclusion to the test.

6 A pharmaceutical company claims to have invented a new remedy for weight loss. It claims that people using this remedy will lose more weight than people not using the remedy. A total of 20 people are weighed and tested. Ten people are given the new remedy and the other ten are given a placebo. After two months the people are weighed again and any weight loss, in kg, is noted in the table below.

New remedy	1.2	2.4	1.6	3.5	3.2	4.6	2.5	0.8	1.2	3.9
Placebo	0.6	0.0	1.0	1.3	2.1	0.7	1.9	2.4	0.3	1.0

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 1% significance level.

d Write down the conclusion to the test.

7 The lengths, in cm, of sweetcorn cobs in fields on either side of a main road are measured to find out whether there is any difference between them.

Field 1	17	18	15	21	22	24	19	23	25
Field 2	19	21	23	16	18	22	23	16	19

a Write down the null and alternative hypotheses.

b State whether this a one-tailed test or a two-tailed test.

c Find the t-value and p-value for a t-test at the 10% significance level.

d Write down the conclusion to the test.

Developing inquiry skills

It has already been confirmed that the trees in the opening problem are normally distributed, so you can perform a t-test on the data.

State your null and alternative hypotheses.

Write down whether this a one-tailed test or a two-tailed test.

Perform a t-test at the 5% significance level.

Write down the conclusion to the test.

Chapter summary

- The product moment correlation coefficient of the ranks of a set of data is called Spearman's rank correlation coefficient. The IB notation is r_s.
- A value of 1 means the set of data is strictly increasing and a value of -1 means it is strictly decreasing.
  A value of 0 means the data shows no monotonic behaviour.
- The advantages of Spearman's over the PMCC are:
  - It can be used on data that is not linear.
  - It can be used on data that has been ranked even if the original data is unknown or cannot be quantified.
  - It is not greatly affected by outliers.
- A χ^2 test for independence can be performed to find out whether two data sets are independent of each other or not. It can be performed at various significance levels. In the examination it will only be tested at the 1%, 5% or 10% significance level.
- The number of degrees of freedom is $\nu = (\text{rows} - 1)(\text{columns} - 1)$.
- Expected values must be greater than 5. If there are expected values less than 5 then you will need to combine rows or columns.
- The formula for the χ^2 test is $\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$ where f_o are the observed values and f_e are the expected values.
- If the p-value is greater than the significance level (0.01, 0.05 or 0.10) then you accept the null hypothesis; if it is less than the significance level then you do not accept the null hypothesis.
- If the χ^2 test statistic is less than the critical value then you accept the null hypothesis; if it is greater than the critical value then you do not accept the null hypothesis.
- In a χ^2 goodness of fit test, $\nu = (n - 1)$.
- The t-test is used for two data sets that are measuring the same thing, and only applies to normally distributed data.

Developing inquiry skills

Thinking about the opening problem:

- Discuss how what you have learned in this chapter has helped you to answer the questions.
- Discuss how you decided whether the data was biased or not.
- Write down which statistical tests you were able to use from this chapter.
- State what claims you can make about the trees.
- Discuss what information you managed to find.

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