C


3 Consider $f(x)=-2.5 x+5$ for $0 \leq x \leq 3$.
a Draw the graph of $f$ on a pair of axes. Use the same scale on both axes.
b Point A lies on the graph of $f$ and has coordinates $(3, b)$. Find the value of $b$.
c Complete the table:

| Function | Domain | Range |
| :---: | :---: | :---: |
| $f$ | $0 \leq x \leq 3$ |  |
| $f^{-1}$ |  |  |

d Draw the graph of $f^{-1}$ on the same set of axes used in part a.
e Find the coordinates of the point that lies on the graph of $f^{-1}$ and on the line $y=x$.

4 The one-to-one function $f$ maps $A=\{x: x \geq 0\}$ onto $B=\{y:-1 \leq y \leq 2\} ; f(1)=0.5$ and $f(0)=2$.
a Write down the domain and the range of $f^{-1}$, the inverse function of $f$.
b Write down the value of $f^{-1}(0.5)$.
c Determine whether $f$ is an increasing or a decreasing function.
d Solve the equation $f^{-1}(x)=0$.

## Developing inquiry skills

How will the taxi driver's profit change if he increases the profit per kilometre by $\$ 0.30$ after one month?
How can you model the daily profit for different values of profit per kilometre?


### 5.3 Arithmetic sequences

## Investigation 9

Pablo starts his first full-time job at the age of 24 . In his first year he earns $\$ 3250$ per month after taxes, or $\$ 39000$ per year. He is given a salary schedule that shows how his salary will increase over time:

| Years in job | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Annual salary (\$) | 39000 | 39900 | 40800 | 41700 | 42600 |

We can also write the salaries for each year in a list, like so:
$39000,39900,40800,41700,42600$
We call this a sequence, an ordered list of numbers. We call each number a term. It is helpful to have a way to write, for example, "the 13th number in this sequence". We use a letter and number combination for this, such as $a_{13}$. So , in the sequence above,
$a_{1}=39000$ (also called the first term)
$a_{2}=39000$ (the second term)
$a_{3}=40800$
and so on.
In general, $a_{n}$ is the $n$th term. The entire sequence is denoted by $\left\{a_{n}\right\}$.
1 Write down the fifth term of the sequence above as a number and in the notation.
Sequences often have patterns that help us to predict them.
2 Use technology to graph the sequence $\left\{a_{n}\right\}$ as a set of points. Use the number of the term $(1,2,3, \ldots)$ as the independent variable and the term in the sequence as the dependent variable. For example, the first point would be ( 1,39000 ).
What do you notice about the graph? Why do you think this is so?
3 A sequence that forms a straight-line graph is called arithmetic. How might we identify a sequence as arithmetic without graphing it? Make a conjecture.

4 Complete the table by calculating the difference between every pair of consecutive terms. The first is done as an example. What do you notice? Does this support your conjecture?

| Years in job <br> [term number] | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Annual salary (term] | 39000 | 39900 | 40800 | 41700 | 42600 |
| Difference between terms | - | $39900-39000$ <br> $=900$ |  |  |  |

5 a What is the gradient of the line formed by plotting the points of the sequence?
How can it be predicted from the terms of the sequence?
b What is the $y$-intercept of the line associated with the sequence? How can it be predicted from the terms of the sequence?
c Find the equation of the line associated with this sequence. Graph this line. Verify that it passes through the points that you plotted.
d Conceptual What advantages does representing Pablo's annual salary with a sequence have over representing it with a linear function?
6 Pablo decides to buy a car, but has to take out a bank loan to do so. Every month the bank charges him interest (a fee for the loan that he must pay). The table shows the amount that Pablo must pay each month.

| Month | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Payment (\$) | 55 | 52 | 49 | 46 | 43 |
| Difference |  |  |  |  |  |

a Find the difference between consecutive terms as you did before, completing the third row of the table.
b Call the sequence of Pablo's payments $\left\{b_{n}\right\}$. What is $b_{1}$ ? What do you predict $b_{7}$ will be?
c What is different here from the previous situation? What is the same?
7 Factual How do you determine whether a sequence is arithmetic without graphing it?

8 Factual If a sequence is arithmetic, how are the parameters of the corresponding linear function related to the first term and the difference between terms of the sequence?
9 Use the pattern in the sequence $\left\{a_{n}\right\}$ to predict Pablo's annual salary if he stays in the same job and retires at the age of 65 . You may find it helpful to complete the table:

| Age | 24 | 25 | 26 | 27 | $\ldots$ | 64 | 65 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of term $[n]$ | 1 | 2 | 3 | 4 | $\ldots$ |  |  |
| Number of differences <br> added $(d=900)$ | 0 | 1 | 2 |  | $\ldots$ |  |  |
| Term $\left(a_{n}\right)$ | 39000 | $39000+900$ <br> $=39900$ | $39000+2 \times 900$ <br> $=40800$ |  | $\ldots$ |  |  |

10 Conceptual How can we systematically calculate the $n$th term of an arithmetic sequence?

## General sequences

A sequence of numbers is a list of numbers (of finite or infinite length) arranged in an order that obeys a certain rule.
Each number in the sequence is called a term. The $\boldsymbol{n}$ th term, where $n$ is a positive integer, can be represented by the notation $u_{n}$.

For example, $3,6,9,12,15, \ldots$ is a sequence. What is the rule?

The first term is 3 , the second term is 6 and so on, so we can say $u_{1}=3, u_{2}=6$, and so on.
Note that a sequence can be thought of as a function from $\mathbb{Z}^{+}$to $\mathbb{R}$, where the notation $u_{n}$ is equivalent to $u(n)$.

## Example 11

For each of the sequences
i $2,4,6,8,10, \ldots$
ii $\quad 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$
a Write down a general formula for the $n$th term.
b Find the 12 th term.


## HINT

Sometimes other letters are used to represent the $n$th term of a sequence, like $a_{n}, b_{n}, \ldots$

## Exercise 51

1 Write down the next three terms in each sequence.
a $3,8,13,18, \ldots$
b $1,2,3,5,8, \ldots$
c $2,3,5,8,12, \ldots$
d $1,2,4,8, \ldots$
e $1,-1,1,-1,1, \ldots$
f $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
g $100,75,50,25, \ldots$

2 Write down the first three terms of these sequences. Remember that $n \in \mathbb{Z}^{+}$.
a $u_{n}=n+1$
b $a_{n}=3 n+1$
c $b_{n}=2^{n}$
d $t_{n}=4-0.5 n$

3 Write down a general formula for the $n$th term of each sequence.
a $1,4,9,16, \ldots$
b $1,8,27,64, \ldots$
c $1,2,3,4, \ldots$
d $2,1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \ldots$

4 Consider the sequence $u_{n}=3+4 \times(n-1)$.
a Find $u_{6}$.
b Find the position $n$ in which the number 207 appears in the sequence $\left\{u_{n}\right\}$.
c Find the value of $n$ for which $u_{n}=111$.
d Determine whether or not 400 is a term of this sequence.

## Arithmetic sequences and series

A sequence in which the difference between each term and its previous one
remains constant is called an arithmetic sequence.
This constant difference is called the common difference of the sequence.

## Investigation 10

1 Which of these sequences are arithmetic sequences? For those that are arithmetic, what is the common difference in each case?
a $2,4,6,8,10, \ldots$
b $10,12,14,16, \ldots$
c $1,10,100,1000, \ldots$
d $10,8,6,4, \ldots$
e $-1,-0.5,0,0.5, \ldots$
f $\frac{?}{4}, \frac{3}{2}, \frac{5}{4}, 1, \ldots$
g $1,3,4,7,11, \ldots$
i $a_{n}=3 n+1$
h $u_{n}=5 n+2$
k $\quad t_{n}=4-0.5 n$
j $\quad b_{n}=2^{n}$
I $r_{n}=n^{2}$

2 Write down:
a the first five terms of a sequence in which the difference between each term and the previous term is always equal to 4
b the first five terms of a sequence in which the common difference is equal to -3
c the first four terms of an arithmetic sequence in which the first term is 2 and the common difference is -0.5 .

3 The sum of the terms of an arithmetic sequence is called an arithmetic series.
a Which of these sums are arithmetic series? How can you tell?
i $\quad S_{4}=1+7+13+19$
ii $\quad S_{4}=5+2.5+1.25+0.0625$
iii $\quad S_{5}=9-7-5-3-1$
b Write down an arithmetic series with six terms in which the first term is equal to 4 and the common difference is equal to 0.20 .

4 Conceptual How do you distinguish an arithmetic sequence or series from other sequences or series?

An arithmetic sequence with first term $u_{1}$ and common difference $d$ can be generated as shown:
$u_{1}$
$u_{2}=u_{1}+d$
$u_{3}=u_{1}+d+d=u_{1}+2 d$
$u_{4}=u_{1}+d+d+d=u_{1}+3 d$
$u_{5}=u_{1}+d+d+d+d=u_{1}+4 d$
What would the expression be for $u_{6}$ ? What do you notice about the number that multiplies $d$ ?

Following the pattern, $u_{n}=u_{1}+(n-1) d$.
The general term (or $n$th term) of an arithmetic sequence with first term $u_{1}$ and common difference $d$ is $u_{n}=u_{1}+(n-1) d$, where $n \in \mathbb{Z}^{+}$.

## Example 12

The first term of an arithmetic sequence is 5 and its common difference is $\frac{2}{3}$.
a Find the second and third terms of this sequence.
b Write down an expression for the $n$th term.
c Determine whether or not $\frac{49}{3}$ is a term of this sequence.
d Find the first term of this sequence that is greater than 25 .
a $u_{2}=\frac{17}{3}, \quad u_{3}=\frac{19}{3}$
b $u_{n}=5+(n-1) \times \frac{2}{3}$
c $\frac{49}{3}$ is the 18 th term
d 205
$u_{1}=5$
Every new term is found by adding $\frac{2}{3}$ to the previous term.
Therefore, $u_{2}=5+\frac{2}{3}=\frac{17}{3}$ and $u_{3}=\frac{17}{3}+\frac{2}{3}=\frac{19}{3}$.
Substitute 5 for $u_{1}$ and $\frac{2}{3}$ for $d$ in the formula for the general term.
If $\frac{49}{3}$ is a term of this sequence then there will be an $n$ for which $u_{n}=\frac{49}{3}$ :
$5+(n-1) \times \frac{2}{3}=\frac{49}{3}$

$$
(n-1) \times \frac{2}{3}=\frac{34}{3}
$$

$$
n-1=17
$$

Since $n=18$ is a positive integer, $\frac{49}{3}$ is in the
sequence. sequence.

You can see this on your GDC.

## Example 13

Consider the finite arithmetic sequence $-3,5, \ldots, 1189$.
a Write down the common difference, $d$.
b Find the number of terms in the sequence.

| a $d=8$ | The common difference is the difference <br> between any two consecutive terms: <br> $d=5-(-3)=8$ |
| :--- | :--- |
| bethod 1: |  |
| The general term of this sequence is |  |
| $u_{n}=-3+(n-1) \times 8$. |  |
| We also know that $u_{n}=1189$. |  |
| So $-3+(n-1) \times 8=1189$ |  |
| Use your GDC to solve for $n$. |  |
| Method 2: |  |
| Use the GDC to create a table of all of |  |
| the terms and then scroll down until you |  |
| see the value 1189. However, this may |  |
| take you longer than Method 1 when the |  |
| value of $n$ is big (as in this example). |  |

## TOK

Is all knowledge concerned with identification and use of patterns?

## Exercise 5J

1 The first term of an arithmetic sequence is -10 and the seventh term is -1 .
a Find the value of the common difference.
b Find the 15 th term of this sequence.
2 The $n$th term of a sequence is defined by $b_{n}=n(2 n+1)$.
a Find the values of $b_{1}, b_{2}$ and $b_{3}$.
b Show that this sequence is not arithmetic.

3 The first terms of an arithmetic sequence are $5,9,13,17, \ldots$
a Write down the general term for this sequence.
b Determine whether or not 116 is a term in this sequence.

4 In an arithmetic sequence, $u_{3}=12$ and $u_{10}=40$. The common difference is $d$.
a Write down two equations in $u_{1}$ and $d$ to show this information.
b Find the values of $u_{1}$ and $d$.
c Find the 100 th term.
5 When a company first started it had 85 employees. It was decided to increase the number of employees by 10 at the beginning of each year.
a Find the number of employees during the second year and during the third year.
b How many employees will this company have during the 10 th year?
c After how many years will the company have 285 employees?

6 In Investigation 9, Pablo made monthly payments to his bank for a loan. The payments for the first five months were $\$ 55, \$ 52, \$ 49, \$ 46, \$ 43$.
a Write down an expression for the $n$th term of this sequence.
b Find the amount of Pablo's 12th payment.
c Determine when Pablo will make his last payment.
d Find the amount of the last payment.
7 A sequoia tree that was 2.6 m tall when it was planted in 1998 grows at a rate of 1.22 m per year.
a Write down a formula to represent the height of the tree each year, with $a_{1}$ representing the height in 1998.
b Find the height of the tree in 2025.
c The tallest living sequoia tree, the General Sherman tree in the US Sequoia National Park, has a height of 84 m . In what year would the tree planted in 1998 reach this height if it continues to grow at the same rate?

8 Consider the finite arithmetic sequence 1000, 975, 950, ..., -225.
a Write down the common difference.
b Find the 10th term.
c Find the number of terms in the sequence.
9 The diagram shows part of the graph of a sequence $u_{n}$.

a Explain why this sequence is arithmetic.
b Write down the common difference of this sequence.
c Write down the first term of the sequence.
d Write down the general term of the sequence.
e Determine whether or not the point $(20,36)$ lies on the graph of this sequence.

10 Pedro is bored and is making patterns with sticks. Pattern 1 was made with four sticks, pattern 2 with seven sticks and so on.


a Write down the number of sticks needed to make pattern 5 and pattern 6.
b Find the number of sticks needed to make pattern 20.
c Find the pattern number for the pattern made with 127 sticks.

11 Consider the arithmetic series $S=1+3+5+7+\ldots+61$.
a Find the number of terms in this series.
b Write the series using sigma notation.
c Find the value of $S$.

## Investigation 11

1 Consider the arithmetic sequence $1,2,3,4, \ldots, 100$. We are going to find the sum of these 100 terms, $S_{100^{\circ}}$
$S_{100}=1+2+3+4+\cdots+97+98+99+100$, and also
$S_{100}=100+99+98+97+\cdots+4+3+2+1$. Why?
Adding these two equations, we get
$2 S_{100}=\underbrace{101+101+101+101+\ldots+101+101+101+101}_{\text {This is } 100 \times 101}$
Therefore $2 S_{100}=100 \times 101$, so $S_{100}=\frac{100 \times 101}{2}=5050$.
In the calculation $S_{100}=\frac{100 \times 101}{2}$, where does the 100 come from? Where does the 101 come from?
Describe how you have found $S_{100}$.
2 Apply the method from part 1 to find $S_{100}$ for the arithmetic sequence $3,5,7,9, \ldots$
Show that $S_{100}=10200$.
3 Now consider the arithmetic sequence 1, 4, ᄀ, ..., 148.
a Show that the sequence has 50 terms.
b Find $S_{50}$.
4 Now consider the series
$S_{n}=u_{1}+\left(u_{1}+d\right)+\left(u_{1}+2 d\right)+\left(u_{1}+3 d\right)+\ldots+\left(u_{1}+(n-1) d\right)$
5 Factual Based on the previous examples, what formula do you suggest to find $S_{n}$ ? (Follow the same steps as in part 1.)
6 Conceptual How does the formula for the sum of the first $n$ terms of an arithmetic sequence use symmetry?

The sum, $S_{n}$, of the first $n$ terms of an arithmetic sequence $u_{1}, u_{2}, u_{3}, \ldots$ can be calculated using the formula $S_{n}=\frac{n}{2}\left(u_{1}+u_{n}\right)$.

Note that if we replace $u_{n}$ with $u_{1}+(n-1) d$, the formula can be written as

## TOK

$S_{n}=\frac{n}{2}\left(u_{1}+u_{1}+(n-1) d\right)=\frac{n}{2}\left(2 u_{1}+(n-1) d\right)$.

How is intuition used in mathematics?

## Example 14

1 Calculate the sum of the first 20 terms of the series $60+57+54+\ldots$
2 a Find the sum of the arithmetic series $(-10)+(-6)+(-2)+\ldots+90$.
b Find the least number of terms that must be added to the series $(-10)+(-6)+(-2)+\ldots$ to obtain a sum greater than 100.
$1 S_{20}=630$

2 a $S_{26}=1040$
b 11 terms

The common difference is $57-60=-3$, and the first term is 60 , so substitute $d=-3, n=20$ and $u_{1}=60$ into the formula:
$S_{20}=\frac{20}{2}(2 \times 60+(20-1)(-3))=630$
This is an arithmetic series with common difference 4 . The first and last terms are given, so we can use the formula $S_{n}=\frac{n}{2}\left(u_{1}+u_{n}\right)$ if we can find $n$.

The general term for the sequence is $u_{n}=-10+(n-1) \times 4$. Setting $u_{n}=90$ gives $n=26$.
So $S_{26}=\frac{26}{2}(-10+90)=1040$.
The sum of the terms (in terms of $n$ ) is $S_{n}=\frac{n}{2}(2 \times(-10)+(n-1) \times 4)=\frac{n}{2}(-20+(n-1) \times 4)$
and this must be greater than 100 :
$\frac{n}{2}(-20+(n-1) \times 4)>100$
Use the GDC to make a table of values with the formula for $S_{n}$.
Adding 10 terms gives a sum of 80 , and adding 11 terms gives 110 , so $n=11$.

## Sigma notation and series

The terms of a sequence can be added together. Adding the terms of a sequence gives a series.
If $u_{1}, u_{2}, u_{3}, \ldots, u_{n}$ is a sequence, then $u_{1}+u_{2}+u_{3}+\ldots+u_{n}$ is a series. $S_{n}$ denotes the sum of the first $n$ terms of a series: $S_{n}=u_{1}+u_{2}+u_{3}+\ldots+u_{n}$. The Greek letter $\Sigma$, called "sigma", is often used to represent a sum of values:
$S_{n}=\sum_{i=1}^{n} u_{i}=u_{1}+u_{2}+u_{3}+\ldots+u_{n}$.
$\sum_{i=1}^{n} u_{i}$ is read as "the sum of all $u_{i}$ from $i=1$ to $i=n$ ".

For example, for the sequence $2,4,6,8,10, \ldots, S_{1}=2, S_{2}=2+4$, $S_{3}=2+4+6, S_{4}=2+4+6+8$ and so on, and $S_{4}=2+4+6+8$ can be written as $\sum_{i=1}^{4} 2 i$.

## TOK

 language?
## Example 15

a Write the expression $\sum_{i=1}^{4}(2 i+3)$ as a sum of terms.
b Calculate the sum of these terms.

$$
\begin{array}{l|l}
\text { a } \begin{array}{l}
\sum_{i=1}^{4}(2 i+3) \\
=(2 \times 1+3)+(2 \times 2+3)+(2 \times 3+3) \\
\quad+(2 \times 4+3)
\end{array} & \begin{array}{l}
\text { It is important to remember that } \sum_{i=1}^{4}(2 i+3) \\
\text { is a sum. There will be four terms in } \\
\text { this sum. The first term is found by } \\
\text { substituting } i=1 \text { in the expression } 2 i+3, \\
\text { and the last term by substituting } i=4 .
\end{array} \\
\text { b } 5+7+9+11=32 & \text { The sum can be found with the GDC. }
\end{array}
$$

## Example 16

Write the series $1+3+5+7+9+11$ using sigma notation.

| $\sum_{n=1}^{6}(2 n-1)$ | There are six terms in this series, which <br> means that the index will vary from 1 to 6. <br> This is the sequence of odd numbers: <br> $u_{n}=2 n-1$, so that $u_{1}=1, u_{2}=3, u_{3}=5, \ldots$ |
| :--- | :--- |

## Exercise 5K

1 Find the sum of the first 17 terms of the arithmetic series $15+15.5+16+16.5+\ldots$

2 Find the sum of the first 20 terms of the arithmetic series $6+3+0-3-6-\ldots$

3 Find the sum of the first 30 multiples of 8 .
4 Consider the series $52+62+72+\ldots+462$.
a Find the number of terms.
b Find the sum of the terms.
5 Consider the arithmetic series $S_{n}=1+8+15+\ldots$
a Write down an expression for $S_{n}$ in terms of $n$.
b Find the value of $n$ for which $S_{n}=5500$.
6 An arithmetic series has $S_{1}=4$ and $d=-3$.
a Write down an expression for $S_{n}$ in terms of $n$.
b Find $S_{10}$.
c Find the smallest $n$ for which $S_{n}<-250$.
? Montserrat is training for her first race. In her first training week she runs 3 km , and in the second training week she runs 3.5 km . Every week she runs 0.5 km more than the previous one.
a Find how many kilometres Montserrat runs in her 10th training week.
b Calculate the total number of kilometres that Montserrat will have run by her 15th training week.

8 Consider the arithmetic sequence
$5, a, 14, b, \ldots$
a Find the common difference.
b Find the values of $a$ and $b$.
c Find $S_{10}$.
d Find the least value of $n$ for which $S_{n}>500$.

9 The first three terms of an arithmetic sequence are $2 k+1,-k+10$ and $k-1$.
a Show that $k=4$.
b Find the values of the first three terms of this sequence.
c Write down the common difference of the sequence.
d Find the sum of the first 20 terms of the sequence.

10 Consider the sum $S=\sum_{i=3}^{x}(3 i+1)$, where $x$ is a positive integer greater than 3 .
a Write down the first three terms of the series.
b Write down the number of terms in the series.
c Given that $S=520$, find the value of $x$.

## Simple interest

Capital is the money that you put in a savings institution or a bank. If you put money in a bank, the bank will pay you interest on this money.

If you borrow money from a bank or savings institution, then you have to pay them interest.

The interest is the extra amount you pay or earn. Its value depends on the capital.

Simple interest is the simplest way to calculate interest. Of course, there are other ways.

As an example, Wang wants to borrow $\$ 1000$ for three years at a rate of $4 \%$ per year.

The interest that Wang will have to pay for $\mathbf{1}$ year is $4 \%$ of 1000 :
$0.04 \times 1000$.
The interest that Wang will have to pay for $\mathbf{2}$ years is $(0.04 \times 1000) \times \mathbf{2}$.
The interest that Wang will have to pay for $\mathbf{3}$ years is $(0.04 \times 1000) \times \mathbf{3}$.
This is how the formula for simple interest is built.

```
The formula for simple interest is I=C\timesr\timesn, where
    C is the capital (or principal)
    ris the interest rate
    n}\mathrm{ is the number of interest periods
    I is the interest.
```

In the example above, how much money will Wang have to repay to the bank after three years?

## Investigation 12

1 Capital of $\$ 1000$ is invested in a bank account at a simple interest rate of $3 \%$ per annum.
a Complete the table for the amount of money in the bank account after $n$ years (assuming no withdrawals).

| Number of years | 0 | 1 | 2 | 3 | 4 | $n$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Amount in bank (\$] |  |  |  |  |  |  |

b What can you say about the difference in money between two consecutive years? What can you say about this sequence of numbers? What are the variables? What values does the independent variable take?
2 A capital amount $C$ is invested in a bank account at a simple interest rate of $r \%$ per annum.
a Complete the table for the amount of money in the bank account after $n$ years (assuming no withdrawals).

| Number of years | 0 | 1 | 2 | 3 | 4 | $n$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Amount in bank $[\$ A]$ |  |  |  |  |  |  |

b What can you say about the difference in money between two consecutive years? What is the relationship between this value and the type of account? What can you say about this sequence of numbers? What are the variables? What values does the independent variable take?
3 The amount of money in a particular bank account that pays $r \%$ simple interest per annum can be calculated with the following formula:
$A_{n}=1200+60 n$
What does $A_{n}$ mean? What is the capital in this bank account?
What is the value of $r$ ?
4 Conceptual What is a suitable model for simple interest?

## Example 17

An amount of UK£5000 is invested at a simple interest rate of $3 \%$ per annum for a period of 8 years.
a Calculate the interest received after 8 years.
b Find the total amount in the account after the 8 years.
a UK£1200
b UK£6200

In this example, $C=5000, r=0.03$ ( $3 \%$ is written as 0.03 ) and $n=8$.
Using the formula $I=C \times r \times n$,
$I=5000 \times 0.03 \times 8=1200$
The total amount is found by adding the interest to the capital:

$$
5000+1200=6200
$$

## Example 18

Riddhi invests $\$ 4000$ in a bank account at a simple interest rate of $7.3 \%$ per year.


Find the number of years it takes for Riddhi's money to double.

| 14 years | $C=4000$, and we are looking for it to double. <br> We therefore need the interest earned to be <br> $I=8000-4000=4000$. Using the simple interest <br> formula: <br> $4000=4000 \times 0.073 \times n$, so $n=13.69$. <br> Since interest is only applied after each whole <br> year, it will be 14 years before the capital doubles. |
| :--- | :--- |

## Exercise 5L

1 Calculate the simple interest on a loan of:
a $\$ 9000$ at a rate of $5 \%$ pa over three years (pa means per annum)
b $\$ 10000$ at a rate of $8.5 \%$ pa over 18 months (18 months is 1.5 years)
c $\$ 6500$ at a rate of $7 \%$ pa over 3 years and 5 months.

2 Find the amount of money borrowed if after seven years the simple interest charged is $\$ 9000$ at a rate of $7.5 \%$ per annum.

3 Find the rate of simple interest charged if UK $£ 1840$ is earned after 5 years on a deposit of UK $£ 8000$.

4 Stephen deposits $\$ 8600$ in a bank account that pays simple interest at a rate of $6.5 \%$ per annum. Find how long it will take for Stephen's money to double.

5 True or false: The formula $A_{n}=1000+30 n$ can be used to calculate the final amount in an investment account that pays $0.3 \%$ simple interest pa on a capital sum of $\$ 1000$.

## Developing inquiry skills

Will the taxi driver reach the average annual salary in one year's time? If not, how long will it take?
What assumptions do you need to make to answer this question?


## Developing your toolkit

Now do the Modelling and investigation activity on page 260.

### 5.4 Modelling

## Investigation 13

A water tank is leaking. The amount of water in the tank was measured three times after it started to leak. The table shows the measurements.

| Time $[\boldsymbol{t}$ minutes ] | 2 | 4 | 9 |
| :--- | :---: | :---: | :---: |
| Amount of water $[\boldsymbol{w}$ litres ) | 1900 | 1800 | 1500 |

1 Using technology, plot the information from the table on a set of axes.
2 What can you say about the positions of the points? What do you think would be a good model to fit this data? Why?
Explain how you decided to choose your model. It is important to justify why you have chosen a particular model.
Find an equation for your model using technology. Plot your model on the same set of axes as in part 1.
3 How could your model be useful? What results would it help to find?

