Distributions practice [136 marks]

1a. *[2 marks]*

The masses of Fuji apples are normally distributed with a mean of 163g and a standard deviation of 6.83g.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between 172g and 183g.

Determine the probability that a Fuji apple selected at random will be a large apple.

1b. *[3 marks]*

Approximately 68% of Fuji apples have a mass within the medium-sized category, which is between k and 172g.

Find the value of k.

2a. *[2 marks]*

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

Calculate the expected number of people who will pass this polygraph test.

2b. *[2 marks]*

Calculate the probability that exactly 4 people will fail this polygraph test.

2c. *[3 marks]*

Determine the probability that fewer than 7 people will pass this polygraph test.

3a. *[1 mark]*

Jae Hee plays a game involving a biased six-sided die.

The faces of the die are labelled −3, −1, 0, 1, 2 and 5.

The score for the game, X, is the number which lands face up after the die is rolled.

The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$P(X=x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

Find the exact value of p.

3b. *[2 marks]*

Jae Hee plays the game once.

Calculate the expected score.

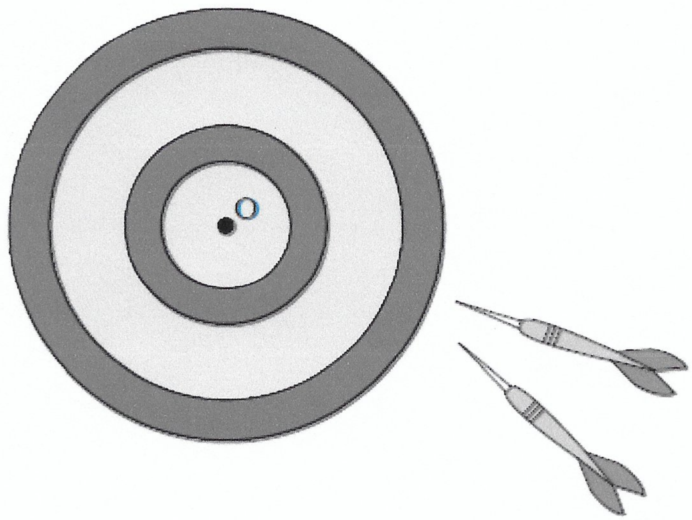
3c. *[3 marks]*

Jae Hee plays the game twice and adds the two scores together.

Find the probability Jae Hee has a **total** score of -3.

4a. *[2 marks]*

Arianne plays a game of darts.



The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean 10cm and standard deviation 3cm.

Find the probability that

a dart lands less than 13 cm from O.

4b. *[1 mark]*

a dart lands more than 15 cm from O.

4c. *[2 marks]*

Each of Arianne's throws is independent of her previous throws.

Find the probability that Arianne throws two consecutive darts that land more than 15 cm from O.

4d. *[2 marks]*

In a competition a player has three darts to throw on each turn. A point is scored if a player throws **all** three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

Find the probability that Arianne does **not** score a point on a turn of three darts.

4e. *[3 marks]*

In the competition Arianne has ten turns, each with three darts.

Find the probability that Arianne scores at least 5 points in the competition.

4f. *[2 marks]*

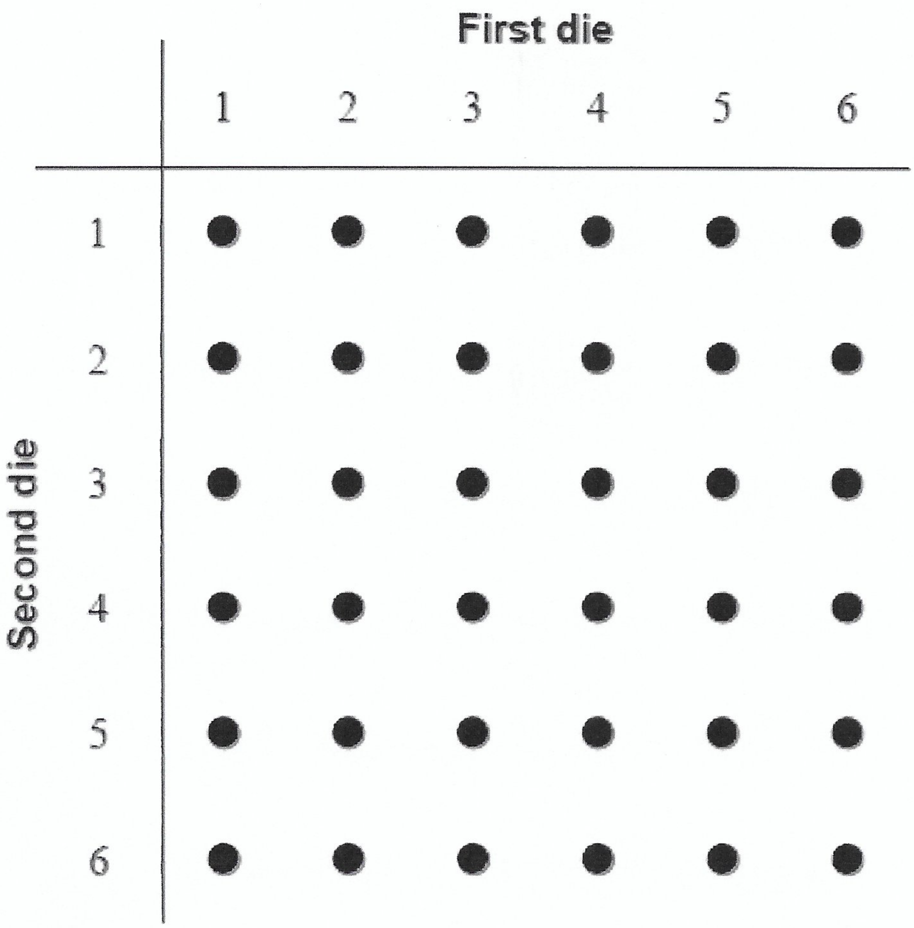
Find the probability that Arianne scores at least 5 points and less than 8 points.

4g. *[2 marks]*

Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points.

5a. *[2 marks]*

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable “the score in a game”.

Complete the table to show the probability distribution of T.

t	1	2	3	4	5	6
$P(T = t)$						

5b. *[1 mark]*

Find the probability that

a player scores at least 3 in a game.

5c. *[2 marks]*

a player scores 6, given that they scored at least 3.

5d. *[2 marks]*

Find the expected score of a game.

6a. *[2 marks]*

The stopping distances for bicycles travelling at 20 km h^{-1} are assumed to follow a normal distribution with mean 6.76 m and standard deviation 0.12 m.

Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at 20 km h^{-1} manages to stop

in less than 6.5 m.

6b. *[1 mark]*

in more than 7 m.

6c. *[2 marks]*

1000 randomly selected bicycles are tested and their stopping distances when travelling at 20 km h^{-1} are measured.

Find, correct to four significant figures, the expected number of bicycles tested that stop between

6.5 m and 6.75 m.

6d. *[1 mark]*

6.75 m and 7 m.

6e. *[2 marks]*

The measured stopping distances of the 1000 bicycles are given in the table.

Measured stopping distance	Number of bicycles
Less than 6.5 m	12
Between 6.5 m and 6.75 m	428
Between 6.75 m and 7 m	527
More than 7 m	33

It is decided to perform a χ^2 goodness of fit test at the 5% level of significance to decide whether the stopping distances of bicycles travelling at 20 km h^{-1} can be modelled by a normal distribution with mean 6.76 m and standard deviation 0.12 m.

State the null and alternative hypotheses.

6f. *[3 marks]*

Find the p-value for the test.

6g. *[2 marks]*

State the conclusion of the test. Give a reason for your answer.

7a. *[2 marks]*

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$P(X = x)$	q	$4p^2$	p	$0.7 - 4p^2$

Find an expression for q in terms of p.

7b. *[3 marks]*

Find the value of p which gives the largest value of $E(X)$.

7c. *[1 mark]*

Hence, find the largest value of $E(X)$.

8a. *[2 marks]*

Emlyn plays many games of basketball for his school team. The number of minutes he plays in each game follows a normal distribution with mean m minutes.

In any game there is a 30 % chance he will play less than 13.6 minutes.

Sketch a diagram to represent this information.

8b. *[2 marks]*

In any game there is a 70 % chance he will play less than 17.8 minutes.

Show that $m = 15.7$.

8c. *[2 marks]*

The standard deviation of the number of minutes Emlyn plays in any game is 4.

Find the probability that Emlyn plays between 13 minutes and 18 minutes in a game.

8d. *[2 marks]*

Find the probability that Emlyn plays more than 20 minutes in a game.

8e. *[2 marks]*

There is a 60 % chance Emlyn plays less than x minutes in a game.

Find the value of x.

8f. *[3 marks]*

Emlyn will play in two basketball games today.

Find the probability he plays between 13 minutes and 18 minutes in one game and more than 20 minutes in the other game.

8g. *[2 marks]*

Emlyn and his teammate Johan each practise shooting the basketball multiple times from a point X. A record of their performance over the weekend is shown in the table below.

	Emlyn	Johan
Saturday	42 successful shots from 70 attempts	16 successful shots from 30 attempts
Sunday	27 successful shots from 32 attempts	51 successful shots from 68 attempts

On Monday, Emlyn and Johan will practise and each will shoot 200 times from point X.

Find the expected number of successful shots Emlyn will make on Monday, based on the results from Saturday and Sunday.

8h. *[2 marks]*

Emlyn claims the results from Saturday and Sunday show that his expected number of successful shots will be more than Johan's.

Determine if Emlyn's claim is correct. Justify your reasoning.

9a. *[1 mark]*

The Malthouse Charity Run is a 5 kilometre race. The time taken for each runner to complete the race was recorded. The data was found to be normally distributed with a mean time of 28 minutes and a standard deviation of 5 minutes.

A runner who completed the race is chosen at random.

Write down the probability that the runner completed the race in more than 28 minutes.

9b. *[2 marks]*

Calculate the probability that the runner completed the race in less than 26 minutes.

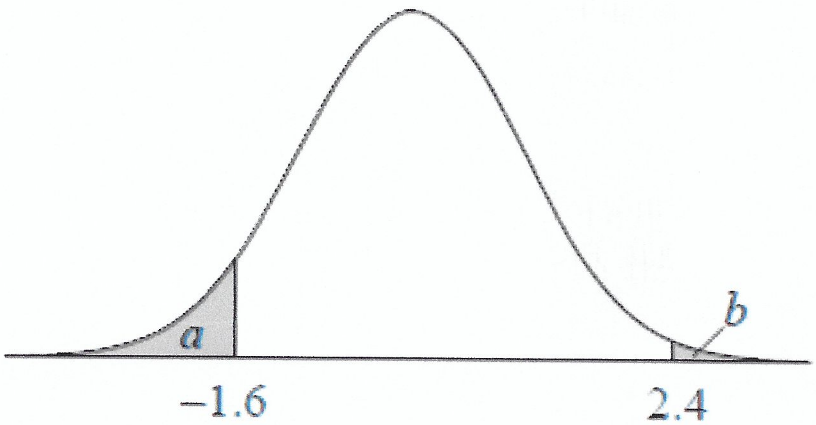
9c. *[3 marks]*

It is known that 20% of the runners took more than 28 minutes and less than k minutes to complete the race.

Find the value of k.

10a. *[2 marks]*

A random variable Z is normally distributed with mean 0 and standard deviation 1. It is known that $P(z < -1.6) = a$ and $P(z > 2.4) = b$. This is shown in the following diagram.



Find $P(-1.6 < z < 2.4)$. Write your answer in terms of a and b.

10b. *[4 marks]*

Given that $z > -1.6$, find the probability that $z < 2.4$. Write your answer in terms of a and b.

10c. *[1 mark]*

A second random variable X is normally distributed with mean m and standard deviation s.

It is known that $P(x < 1) = a$.

Write down the standardized value for $x = 1$.

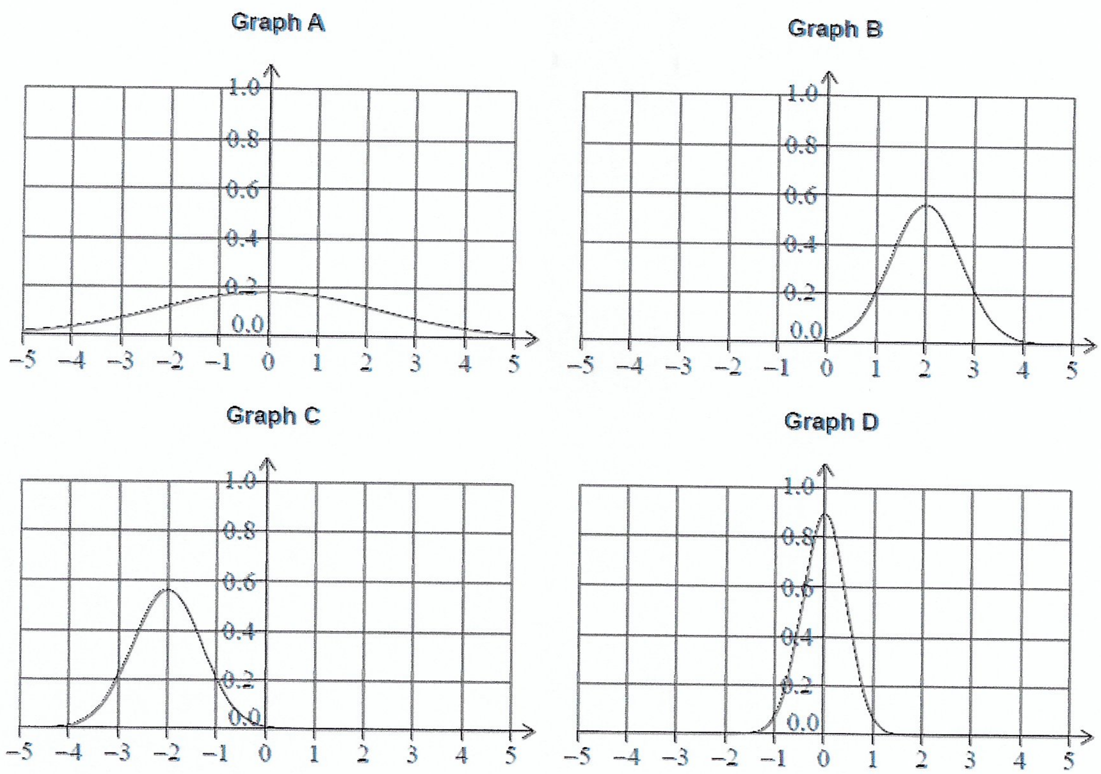
10d. *[6 marks]*

It is also known that $P(x > 2) = b$.

Find s.

11a. *[2 marks]*

Consider the following graphs of normal distributions.



In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation.

Mean and standard deviation	**Graph**
Mean $= -2$; standard deviation $= 0.707$	
Mean $= 0$; standard deviation $= 0.447$	

11b. *[2 marks]*

At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

Find the probability that a suitcase weighs less than 15 kg.

11c. *[2 marks]*

Any suitcase that weighs more than k kg is identified as excess baggage.
19.6 % of the suitcases at this airport are identified as excess baggage.

Find the value of k.

12a. *[4 marks]*

A discrete random variable X has the following probability distribution.

X	0	1	2	3
$P(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

Find the value of k.

12b. *[1 mark]*

Write down $P(X = 2)$.

12c. *[3 marks]*

Find $P(X = 2 | X > 0)$.

13a. *[2 marks]*

In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.

Find k.

13b. *[2 marks]*

Hence, find the probability that exactly k students are left handed;

13c. *[2 marks]*

Hence, find the probability that fewer than k students are left handed.

14a. *[1 mark]*

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

Find the probability that on any given day Mr Burke chooses a female student to answer a question.

14b. *[2 marks]*

In the first month, Mr Burke will teach his class 20 times.

Find the probability he will choose a female student 8 times.

14c. *[3 marks]*

Find the probability he will choose a male student at most 9 times.

15a. *[2 marks]*

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

Find the probability of rolling exactly one red face.

15b. *[3 marks]*

Find the probability of rolling two or more red faces.

15c. *[5 marks]*

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins \$10 for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

Show that, after a turn, the probability that Ted adds exactly \$10 to his winnings is $\frac{1}{3}$.

15d. *[1 mark]*

The random variable D (\$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where \$$w$ represents his winnings in the game so far.

D (\$)	$-w$	0	10	20	30
$P(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

Write down the value of x.

15e. *[2 marks]*

Hence, find the value of y.

15f. *[3 marks]*

Ted will always have another turn if he expects an increase to his winnings.