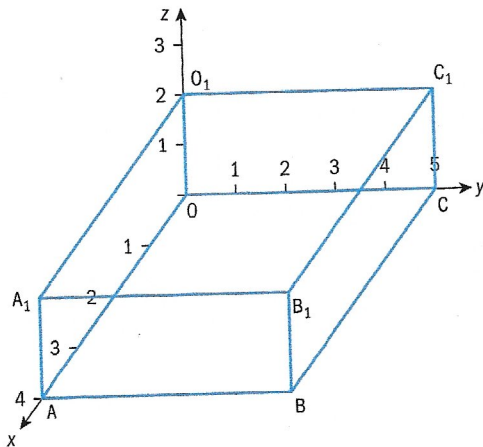


### Example 3

A cuboid  $ABCOA_1B_1C_1O_1$  has dimensions  $4 \times 5 \times 2$ . The vertices of the base have the following coordinates:  $A(4, 0, 0)$ ,  $B(4, 5, 0)$ ,  $C(0, 5, 0)$  and  $O(0, 0, 0)$ .

- Sketch a coordinate system in 3D and the cuboid  $ABCOA_1B_1C_1O_1$ . Determine the coordinates of the vertices  $A_1$ ,  $B_1$ ,  $C_1$  and  $O_1$ .
- Determine the coordinates of the midpoint  $M$  of diagonal  $[AC_1]$ .
- Explain whether or not you would expect  $[AC_1]$  and  $[A_1C]$  to have the same midpoint.

a



$A_1(4, 0, 2)$ ;  $B_1(4, 5, 2)$ ;  $C_1(0, 5, 2)$ ;  $O_1(0, 0, 2)$

b  $M\left(\frac{4+0}{2}, \frac{0+5}{2}, \frac{0+2}{2}\right) = M(2, 2.5, 1)$

- c Yes. This is justified by either verifying that the midpoint of  $[A_1C]$  is  $M(2, 2.5, 1)$  or noting that  $[AC_1]$  and  $[A_1C]$  are diagonals of the rectangle  $ACC_1A_1$ . Since the intersection point of the two diagonals of a rectangle divides each of them in half,  $[AC_1]$  and  $[A_1C]$  should have the same midpoint  $M$ .

Since the given coordinates are in 3D, you will need to draw three perpendicular coordinate axes,  $Ox$ ,  $Oy$  and  $Oz$ .

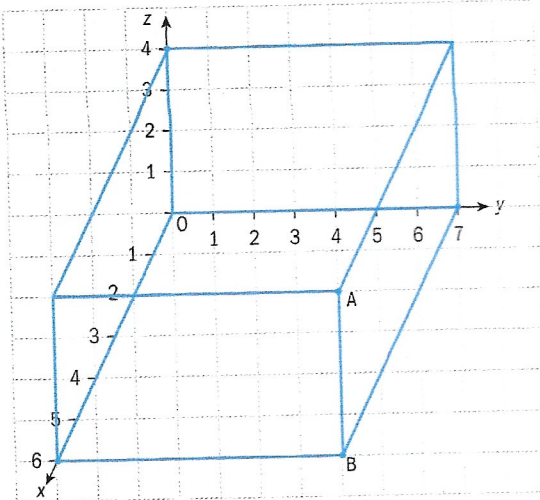
Keep in mind that each coordinate triple is ordered in the sequence  $(x, y, z)$ .

Use the 3D midpoint formula to determine the coordinates of point

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right).$$

4 For the cuboid below:

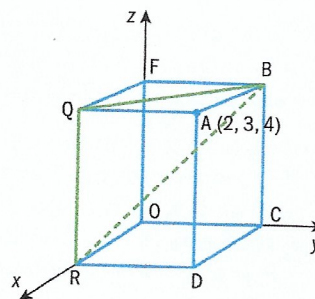
- write down the coordinates of point  $B$
- write down the coordinates of point  $A$
- find the coordinates of the midpoint,  $M$ , of the diagonal  $[AO]$  of the cuboid.



### Example 4

A storage box for garden tools in the shape of a cuboid, O C D R F B A Q, is shown in the diagram. The vertex A has coordinates  $(2, 3, 4)$ . All coordinates are given in metres. Determine:

- the coordinates of vertices R, C, B and Q
- the maximum length of a garden tool that can fit in the storage box
- the height, RQ, of the storage box
- the angle between the diagonals [BR] and [BQ].



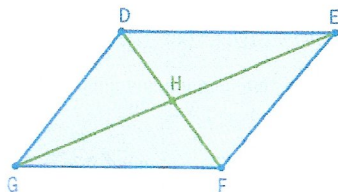
### Exercise 4B

Find the distance between each pair of points given below:

- $A(4, -3)$  and  $B(-1.5, 5)$
- $A(1, -2, 10)$  and  $B(-4, 0, 3)$ .

- The distance between two towns P and Q is 2 km. On a coordinate system, town P has coordinates  $(x, 7)$  and Q has coordinates  $(3, 5)$ . The values of the coordinates are given in kilometres. Find the value of  $x$ .

- The vertices of a sail are plotted on a pair of coordinate axes and have the following coordinates:  $A(-4, 6)$ ,  $B(6, 2)$  and  $C(-8, -4)$ . Show that the triangular sail,  $\triangle ABC$ , is shaped as a right-angled triangle.
- A fountain has a circular shape. The diameter of the circle has endpoints  $A(1, 7)$  and  $B(-2, 1)$ . Find the circumference of the fountain.
- Kari makes a kite in the form of a parallelogram, EDGF, as shown in the diagram. Its diagonals [DF] and [EG] intersect at point H. She plots the vertices on a coordinate system with the following coordinates:  $D(3, 6)$ ,  $G(1, 3)$ ,  $H(4, y)$  and  $E(x, 6)$ . Find the values of  $x$  and  $y$ .



- A tracking station lies at the origin of a coordinate system with the  $x$ -axis due east, the  $y$ -axis due north and the  $z$ -axis vertically upwards. Two aircraft have coordinates  $(20, 25, 11)$  and  $(26, 31, 12)$  relative to the tracking station.
  - Find the distance between the two aircraft at this time.
  - The radar at the tracking station has a range of 40 km. Determine whether it will be able to detect both aircraft.