

- 10 **Conceptual** Where is the point that gives the solution to the toxic waste problem?
- 11 **Conceptual** What applications are there for the solutions of the toxic waste problem?
- 12 **Conceptual** Why are vertices of a Voronoi diagram the only points that give the solution to the toxic waste problem (assuming a location between the towns is sought)?

EXAM HINT

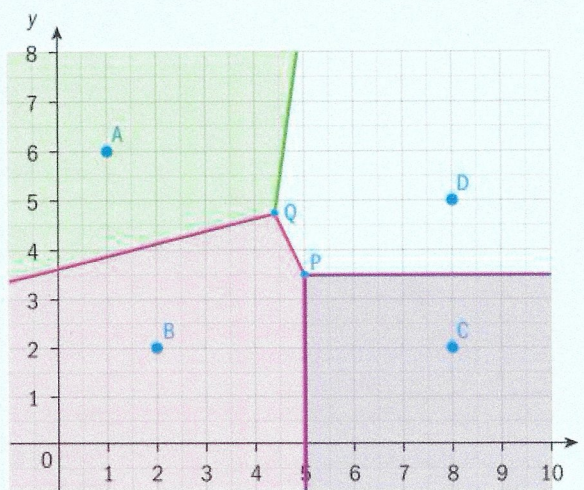
Within a Voronoi diagram the solution to the toxic waste problem will be at an intersection of cell boundaries or on the boundary of the diagram. In exams the solution will always be one of the internal vertices rather than a boundary edge.

Example 31

A town has four coffee shops. An entrepreneur wishes to open a new shop in the town but would like it to be as far as possible from all the other four coffee shops. Where should he put it?

Consider the Voronoi diagram below showing the positions of the four coffee shops on a set of coordinate axes: A(1, 6), B(2, 2), C(8, 2) and D(8, 5), where one unit represents 1 km.

- a Find the coordinates of the vertices P and Q in the Voronoi diagram.
- b Determine where a fifth shop should be sited so as to be as far as possible from any other shop, and how far this will be.



- a P is the point where $x = 5$ and $y = 3.5$ meet: (5, 3.5)

The perpendicular bisector of [AB] is $-x + 4y = 14.5$.

Three perpendicular bisectors meet at each of the vertices. Finding the intersection of any two perpendicular bisectors will be sufficient to find the vertex.

Any two of these equations need to be calculated by finding the midpoint and gradient.