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DIVIDING UP SPACE: COORDINATE GEOMETRY, LINES, VORONOI DIAGRAMS

Investigation 12

David is hiking. He is currently located at point A (25, 90). He wants to take the shortest path to reach highway L , which follows a straight line with equation $y = \frac{1}{3}x + 5$.

He takes a straight perpendicular path to the highway.

- 1 Draw the perpendicular from A to line L .
- 2 Find the equation of the perpendicular path that David takes to the highway.
- 3 Find the coordinates of the point, B, at which the perpendicular path meets the highway.
- 4 Point D(30, 15) is on the highway. Show that $AB < AD$.
- 5 Why is AB the shortest distance from A to the highway?

The shortest distance from a point A to a straight line (BC) is the segment [AD], where D lies on (BC) and $[AD] \perp (BC)$.

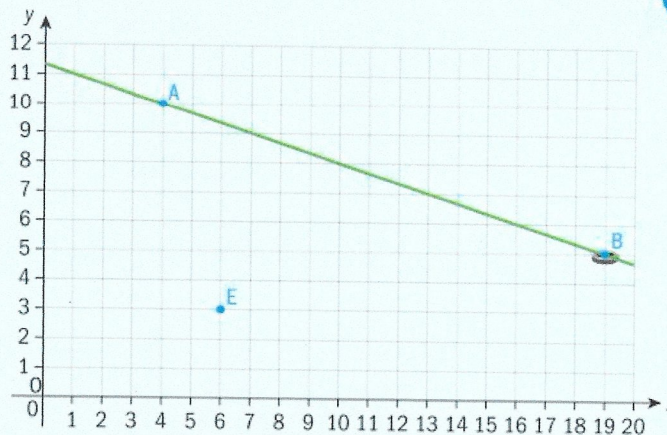
Reflect Why is the perpendicular passing through a given point to a given line the shortest distance from this point to the line?

TOK

Do you think that mathematics is just the manipulation of symbols under a set of rules?

Example 29

A robot vacuum cleaner is located at A(4, 10). It currently cleans by moving along a straight line from point A to point B(19, 5). The robot charging station is located at point E(6, 3). The vacuum cleaner will have to return to point E to recharge before it reaches point B. It is programmed so that it will turn towards E at a point D on [AB] such that [ED] is the shortest distance from E to [AB]. Find the coordinates of point D.



The shortest distance

$$\text{gradient}_{AB} = \frac{5 - 10}{19 - 4} = -\frac{5}{15} = -\frac{1}{3}$$

$$y = -\frac{1}{3}x + k$$

$$10 = -\frac{1}{3} \times 4 + k$$

To identify point D on [AB] such that [ED] is the shortest distance from E to [AB], use the fact that the shortest distance from a point to a line is the perpendicular drawn from the point to the line. Thus you have to find the equation of the line

4.4



The perpendicular to [AB] will have an equation of the form

$$y = 3x + k$$

$$3 = 3 \times 6 + k$$

$$k = -15$$

The equation of the line perpendicular to [AB] is $y = 3x - 15$.

$$3x - 15 = -\frac{1}{3}x + 11\frac{1}{3}$$

$$3\frac{1}{3}x = 26\frac{1}{3}$$

$$x = 7.9$$

$$y = 8.7$$

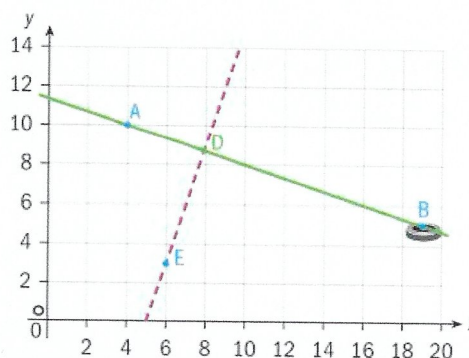
Therefore point D has coordinates (7.9, 8.7).

To find the equation of the line [AB], find the gradient and substitute it and the coordinates of A or B in the point-gradient or gradient-intercept form.

To determine k of the perpendicular to [AC] line, substitute the coordinates of point E.

Finally, determine the intersection point D of line [AB] and its perpendicular line passing through E. Equate the expressions for y from each of the two equations, and solve for the value x , which will be the x -coordinate of D. Use technology or an algebraic method. Substitute this x value in one of the two equations and find the corresponding y value.

As always, it is a good idea to draw a sketch that can help you reason and answer the question.



Reflect How do you find the shortest distance between a point and a line defined by two points?

Example 30

Two towns are located at point A(2, 20) and point B(14, 24), and a nearby road R is a straight