4 DIVIDING UP SPACE: COORDINATE GEOMETRY, LINES, VORONOI DIAGRAMS

Example 20

Brooklake town had only 50 social housing units when the new social programme started. Since then it has been adding new social housing units at a steady rate. Now, 17 years later, the town has a total of 322 social housing units.

a Find the rate of increase in social housing units per year.

b Write an equation for the total number of housing units, h, that the town had in relation to the number of years, x, that have passed since the social programme started.

c Draw the graph of the line representing this equation.

d Determine the gradient of the line. Interpret its meaning.

e Determine the y-intercept of the line. Interpret its meaning.

f Determine how many social housing units the town had 13 years after starting the programme.

g Predict the number of social housing units that the town will have 22 years after starting the programme. State your assumptions.

h Determine the year in which the town added its 200th social housing unit after the programme began.

a $m = \frac{322 - 50}{17 - 0} = 16$

b $h = mx + k$

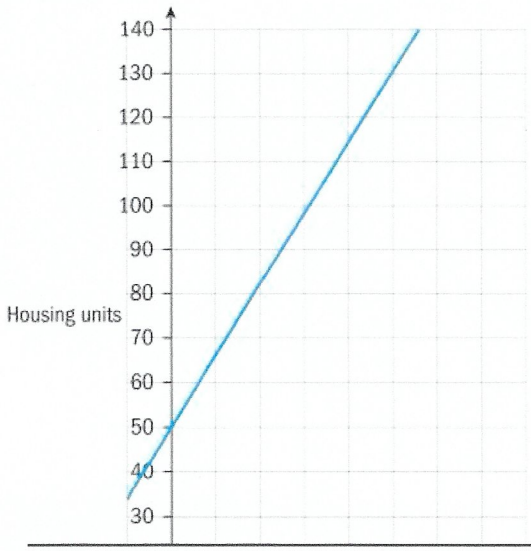
$h = 16x + k$

$50 = 16 \times 0 + k$

$k = 50$

$h = 16x + 50$

c



If we plot the given information, the y-coordinates of the points (0, 50) and (17, 322) represent the number of housing units at year 0 and at year 17.

The rate of increase in housing units is the gradient of the line connecting these two points.

TOK

"The object of mathematical rigour is to sanction and legitimize the conquests of intuition." Jacques Hadamard

Do you think that studying the graph of a function contains the same level of mathematical rigour as studying the function algebraically?

4.3

d Gradient is 16, which is the number of housing units added each year.

e (0, 50), which means that there were 50 housing units when the town started the social programme.

f $h = 16 \times 13 + 50 = 258$ housing units at the end of the 13th year.

g $h = 16 \times 22 + 50 = 402$ housing units at the end of the 22nd year.

We assume that they continue to add units at the same rate: 16 units a year.

h $16 \times x + 50 = 200$

$x = 9.375$

The 200th unit will be added in the 10th year after the programme began.

We need to round up as more than 9 years were needed to build the 200th housing unit.

Exercise 4F

1 Find, in gradient-intercept form, the equation of the line:

a with gradient 3 and y-intercept $(0, -2)$

b with gradient -4 and passing through $(-1, 5)$

c passing through $(-1, -1)$ and $(2, -3)$.

2 Determine the x- and y-intercepts of the following lines:

a $y = 5x$ **b** $y = -0.4x + 3$

c $-0.2x + y = 1$

3 An air traffic controller examines four flight routes represented by straight lines given in A–D below.

4 Albena lives in Belgium. She travels to the USA where the temperature is measured in degrees Fahrenheit. She wants to know how hot it is in °C if the thermometer shows 83°F.

Use the fact that water boils at 100°C a 212°F, and that water freezes at 0°C an 32°F.

a **i** Plot the two points representing t given information on a pair of ax labelled °F (x-axis) and °C (y-axis

ii Sketch the line passing through t two points.