

Using Euler Diagrams to Analyze Arguments

There are two types of reasoning: inductive and deductive. With inductive reasoning we observe patterns to solve problems. Now, in this section and the next, we will study how deductive reasoning may be used to determine whether logical arguments are valid or invalid. A logical argument is made up of **premises** (assumptions, laws, rules, widely held ideas, or observations) and a **conclusion**. Together, the premises and the conclusion make up the argument. Also recall that *deductive* reasoning involves drawing specific conclusions from given general premises. When reasoning from the premises of an argument to obtain a conclusion, we want the argument to be valid.

Valid and Invalid Arguments

An argument is **valid** if the fact that all the premises are true forces the conclusion to be true. An argument that is not valid is **invalid**, or a **fallacy**.

It is very important to note that “valid” and “true” are not the same—an argument can be valid even though the conclusion is false. (See Example 4 below.)

Several techniques can be used to check whether an argument is valid. One of these is the visual technique based on **Euler diagrams**, illustrated by the following examples. (Another is the method of truth tables, shown in the next section.)

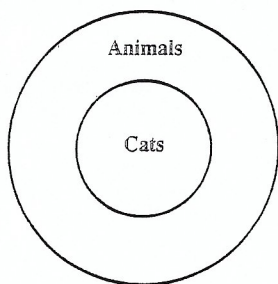
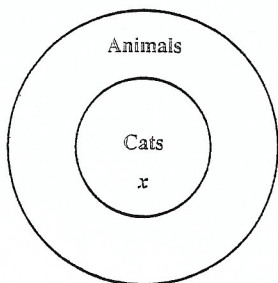


FIGURE 8



x represents Tom.

FIGURE 9

EXAMPLE 1 Is the following argument valid?

All cats are animals.
Tom is a cat.
 Tom is an animal.

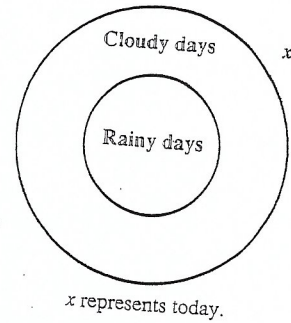
Here we use the common method of placing one premise over another, with the conclusion below a line. To begin, draw a region to represent the first premise. This is the region for “animals.” Since all cats are animals, the region for “cats” goes inside the region for “animals,” as in Figure 8.

The second premise, “Tom is a cat,” suggests that “Tom” would go inside the region representing “cats.” Let x represent “Tom.” Figure 9 shows that “Tom” is also inside the region for “animals.” Therefore, if both premises are true, the conclusion that Tom is an animal must be true also. The argument is valid, as checked by Euler diagrams.

The method of Euler diagrams is especially useful for arguments involving the quantifiers *all*, *some*, or *none*.

EXAMPLE 2 Is the following argument valid?

All rainy days are cloudy.
Today is not cloudy.
 Today is not rainy.

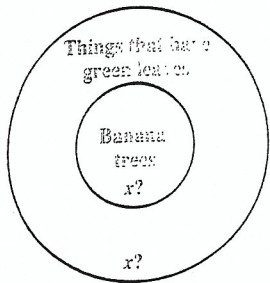


■ FIGURE 11

In Figure 10 the region for “rainy days” is drawn entirely inside the region for “cloudy days.” Since “Today is *not* cloudy,” place an x for “today” *outside* the region for “cloudy days.” (See Figure 11.) Placing the x outside the region for “cloudy days” forces it to be also outside the region for “rainy days.” Thus, if the first two premises are true, then it is also true that today is not rainy. The argument is valid. ■

EXAMPLE 3 Is the following argument valid?

All banana trees have green leaves.
That plant has green leaves.
 That plant is a banana tree.



■ FIGURE 12

The region for “banana trees” goes entirely inside the region for “things that have green leaves.” (See Figure 12.) There is a choice for locating the x that represents “that plant.” The x must go inside the region for “things that have green leaves,” but can go either inside or outside the region for “banana trees.” Even if the premises are true, we are not forced to accept the conclusion as true. This argument is invalid; it is a fallacy. ■

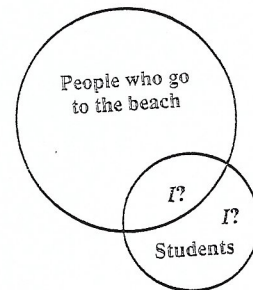
As mentioned earlier, the validity of an argument is not the same as the truth of its conclusion. The argument in Example 3 was invalid, but the conclusion “That plant is a banana tree” may or may not be true. We cannot be sure.

Arguments with the word “some” can be a little tricky. One is shown in the final example of this section.

EXAMPLE 5 Is the following argument valid?

Some students go to the beach.
I am a student.
 I go to the beach.

The first premise is sketched in Figure 14. As the sketch shows, some (but not necessarily *all*) students go to the beach. There are two possibilities for I , as shown in Figure 15.



■ FIGURE 15

One possibility is that I go to the beach; the other is that I don't. Since the truth of the premises does not force the conclusion to be true, the argument is invalid. ■

Decide whether each argument is valid or invalid.

1. All museums exhibit art.
The Louvre is a museum.
The Louvre exhibits art.
3. All homeowners have a plumber.
Vonalaine Crowe has a plumber.
Vonalaine Crowe is a homeowner.
5. All residents of St. Tammany parish live on farms.
Jay Beckenstein lives on a farm.
Jay Beckenstein is a resident of St. Tammany parish.
6. All dogs love to bury bones.
Archie is a dog.
Archie loves to bury bones.
7. All members of the credit union have savings accounts.
Kristyn Wasag does not have a savings account.
Kristyn Wasag is not a member of the credit union.
8. All engineers need mathematics.
Shane Stagg does not need mathematics.
Shane Stagg is not an engineer.
9. All residents of New Orleans have huge utility bills in July.
Erin Kelly has a huge utility bill in July.
Erin Kelly lives in New Orleans.
10. All people applying for a home loan must provide a down payment.
Cynthia Herring provided a down payment.
Cynthia Herring applied for a home loan.
11. Some mathematicians are absent-minded.
Diane Gray is a mathematician.
Diane Gray is absent-minded.
12. Some animals are nocturnal.
Oliver Owl is an animal.
Oliver Owl is nocturnal.
13. Some cars have automatic door locks.
Some cars are red.
Some red cars have automatic door locks.
14. Some doctors appreciate classical music.
Kevin Howell is a doctor.
Kevin Howell appreciates classical music.

As mentioned in the text, an argument can have a true conclusion yet be invalid. In these exercises, each argument has a true conclusion. Identify each argument as valid or invalid.

17. All birds fly.
All planes fly.
A bird is not a plane.
19. All chickens have a beak.
All hens are chickens.
All hens have a beak.
21. Quebec is northeast of Ottawa.
Quebec is northeast of Toronto.
Ottawa is northeast of Toronto.
18. All cars have tires.
All tires are rubber.
All cars have rubber.
20. All chickens have a beak.
All birds have a beak.
All chickens are birds.
22. Veracruz is south of Tampico.
Tampico is south of Monterrey.
Veracruz is south of Monterrey.

In Exercises 25–30, the premises marked A, B, and C are followed by several possible conclusions. Take each conclusion in turn, and check whether the resulting argument is valid or invalid.

- A. All people who drive contribute to air pollution.
- B. All people who contribute to air pollution make life a little worse.
- C. Some people who live in a suburb make life a little worse.
25. Some people who live in a suburb drive.
26. Some people who live in a suburb contribute to air pollution.
27. Some people who contribute to air pollution live in a suburb.
28. Suburban residents never drive.
29. All people who drive make life a little worse.
30. Some people who make life a little worse live in a suburb.
31. Find examples of arguments in magazine ads. Check them for validity.
32. Find examples of arguments on television commercials. Check them for validity.

Logic puzzles of the type found in Exercises 33–34 can be solved by using a “grid” where information is entered in each box. If the situation is impossible, write No in the grid. If it is possible write Yes. For example, consider the following puzzle.

Norma, Harriet, Betty, and Geneva are married to Don, Bill, John, and Nathan, but not necessarily in the order given. One couple has first names that

start with the same letter. Harriet is married to John. Don’s wife is neither Geneva nor Norma. Pair up each husband and wife.

The grid that follows from the puzzle is shown below.

	Don	Bill	John	Nathan
Norma	No	No	No	Yes
Harriet	No	No	Yes	No
Betty	Yes	No	No	No
Geneva	No	Yes	No	No

From the grid, we deduce that Harriet is married to John, Norma is married to Nathan, Betty is married to Don, and Geneva is married to Bill.

Solve each of the following puzzles.

33. Juanita, Evita, Li, Fred, and Butch arrived at a party at different times. Evita arrived after Juanita but before Butch. Butch was neither the first nor the last to arrive. Juanita and Evita arrived after Fred but all three of them were present when Li got there. In what order did the five arrive?
34. There are five girls whose first names are Cherie, Leann, Chris, Mary, and Monica. (The last two are twins.) The twins have the same last name, and the four last names are Parkerson, Bingle, Plunk, and Waters. Cherie’s last name begins with a “P.” The twins have never met the girl whose last name is Waters, and Leann has the longest last name. Determine the girls’ complete names.

Using Truth Tables to Analyze Arguments

The previous section showed how to use Euler diagrams to test the validity of arguments. While Euler diagrams often work well for simple arguments, difficulties can develop with more complex ones. These difficulties occur because Euler diagrams require a sketch showing every possible case. In complex arguments it is hard to be sure that all cases have been considered.

In deciding whether to use Euler diagrams to test the validity of an argument, look for quantifiers such as "all," "some," or "no." These words often indicate arguments best tested by Euler diagrams. If these words are absent, it may be better to use truth tables to test the validity of an argument.

As an example of this method, consider the following argument:

If the floor is dirty, then I must mop it.
The floor is dirty.
 I must mop it.

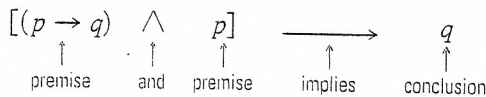
In order to test the validity of this argument, we begin by identifying the *component* statements found in the argument. They are "the floor is dirty" and "I must mop it." We shall assign the letters p and q to represent these statements:

p represents "the floor is dirty."
 q represents "I must mop it."

Now we write the two premises and the conclusion in symbols:

Premise 1: $p \rightarrow q$
Premise 2: p
 Conclusion: q

To decide if this argument is valid, we must determine whether the conjunction of both premises implies the conclusion for all possible cases of truth values for p and q . Therefore, write the conjunction of the premises as the antecedent of a conditional statement, and the conclusion as the consequent.



Finally, construct the truth table for this conditional statement, as shown below.

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$[(p \rightarrow q) \wedge p] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the final column, shown in color, indicates that the conditional statement that represents the argument is true for all possible truth values of p and q , the statement is a tautology. Thus, the argument is valid.

The pattern of the argument in the floor-mopping example,

$$\begin{array}{l}
 p \rightarrow q \\
 \underline{p} \\
 q
 \end{array}$$

is a common one, and is called **modus ponens**, or the *law of detachment*.

EXAMPLE 1 Determine whether the argument is *valid* or *invalid*.

If my check arrives in time, I'll register for the Fall semester.
 I've registered for the Fall semester.

My check arrived in time.


Let p represent "my check arrives (arrived) in time" and let q represent "I'll register (I've registered) for the Fall semester." Using these symbols, the argument can be written in the form

$$\begin{array}{r} p \rightarrow q \\ \underline{q} \\ p \end{array}$$

To test for validity, construct a truth table for the statement

$$[(p \rightarrow q) \wedge q] \rightarrow p.$$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The third row of the final column of the truth table shows F, and this is enough to conclude that the argument is invalid. 

If a conditional and its converse were logically equivalent, then an argument of the type found in Example 1 would be valid. Since a conditional and its converse are *not* equivalent, the argument is an example of what is sometimes called the **fallacy of the converse**.

EXAMPLE 2 Determine whether the argument is *valid* or *invalid*.

If a man could be in two places at one time, I'd be with you.
 I am not with you.

A man can't be in two places at one time.

$$\begin{array}{r} p \rightarrow q \\ \underline{\sim q} \\ \sim p \end{array}$$


If p represents "a man could be in two places at one time" and q represents "I'd be with you," the argument becomes

The symbolic statement of the entire argument is

$$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p.$$

The truth table for this argument, shown below, indicates a tautology, and the argument is valid.

p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \wedge \sim q$	$\sim p$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

The pattern of reasoning of this example is called **modus tollens**, or the *law of contraposition*, or *indirect reasoning*. 

With reasoning similar to that used to name the fallacy of the converse, the fallacy

$$\begin{array}{r} p \rightarrow q \\ \underline{\sim p} \\ \sim q \end{array}$$

is often called the **fallacy of the inverse**. An example of such a fallacy is "If it rains, I get wet. It doesn't rain. Therefore, I don't get wet."

EXAMPLE 3 Determine whether the argument is *valid* or *invalid*.

I'll buy a car or I'll take a vacation.
 I won't buy a car.

 I'll take a vacation.

If p represents "I'll buy a car" and q represents "I'll take a vacation," the argument becomes

$$\frac{p \vee q}{\frac{\sim p}{q}}$$

We must set up a truth table for

$$[(p \vee q) \wedge \sim p] \rightarrow q.$$

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

The statement is a tautology and the argument is valid. Any argument of this form is valid by the law of **disjunctive syllogism**.

EXAMPLE 4 Determine whether the argument is *valid* or *invalid*.

If it squeaks, then I use WD-40.
 If I use WD-40, then I must go to the hardware store.

 If it squeaks, then I must go to the hardware store.

Let p represent "it squeaks," let q represent "I use WD-40," and let r represent "I must go to the hardware store." The argument takes on the general form

$$\frac{p \rightarrow q}{\frac{q \rightarrow r}{p \rightarrow r}}$$

Make a truth table for the following statement:

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r).$$

It will require eight rows.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

This argument is valid since the final statement is a tautology. The pattern of argument shown in this example is called **reasoning by transitivity**, or the *law of hypothetical syllogism*.

Testing the Validity of an Argument with a Truth Table

1. Assign a letter to represent each component statement in the argument.
2. Express each premise and the conclusion symbolically.
3. Form the symbolic statement of the entire argument by writing the *conjunction* of *all* the premises as the antecedent of a conditional statement, and the conclusion of the argument as the consequent.
4. Complete the truth table for the conditional statement formed in part 3 above. If it is a tautology, then the argument is valid; otherwise, it is invalid.

Valid Argument Forms

Modus Ponens	Modus Tollens	Disjunctive Syllogism	Reasoning by Transitivity
$p \rightarrow q$ <hr/> p <hr/> q	$p \rightarrow q$ <hr/> $\sim q$ <hr/> $\sim p$	$p \vee q$ <hr/> $\sim p$ <hr/> q	$p \rightarrow q$ <hr/> $q \rightarrow r$ <hr/> $p \rightarrow r$

Invalid Argument Forms (Fallacies)

Fallacy of the Converse	Fallacy of the Inverse
$p \rightarrow q$ <hr/> q <hr/> p	$p \rightarrow q$ <hr/> $\sim p$ <hr/> $\sim q$

Each of the following arguments is either valid by one of the forms of valid arguments discussed in this section, or a fallacy by one of the forms of invalid arguments discussed. (See the summary boxes.) Decide whether the argument is valid or a fallacy, and give the form that applies.

1. If you build it, he will come.
If he comes, then you will see your father.
If you build it, then you will see your father.
2. If Harry Connick, Jr. comes to town, then I will go to the concert.
If I go to the concert, then I'll be broke until payday.
If Harry Connick, Jr. comes to town, then I'll be broke until payday.
3. If Doug Gilbert sells his quota, he'll get a bonus.
Doug Gilbert sold his quota.
He got a bonus.
4. If Cyndi Keen works hard enough, she will get a raise.
Cyndi Keen worked hard enough.
She got a raise.
5. If she buys a dress, then she will buy shoes.
She buys shoes.
She buys a dress.
6. If I didn't have to write a term paper, I'd be ecstatic.
I am ecstatic.
I don't have to write a term paper.
7. If beggars were choosers, then I could ask for it.
I cannot ask for it.
Beggars aren't choosers.
8. If Roger Clemens pitches, the Red Sox will win.
The Red Sox will not win.
Roger Clemens will not pitch.
9. "If I have seen farther than others, it is because I have stood on the shoulders of giants." (Sir Isaac Newton)
I have not seen farther than others.
I have not stood on the shoulders of giants.
10. "If we evolved a race of Isaac Newtons, that would not be progress." (Aldous Huxley)
We have not evolved a race of Isaac Newtons.
That is progress.
11. Alice Lavin sings or Barbara Lumer dances.
Barbara Lumer does not dance.
Alice Lavin sings.
12. She charges it on Visa or she orders it C.O.D.
She doesn't charge it on Visa.
She orders it C.O.D.

Use a truth table to determine whether the argument is valid or invalid.

- | | | | |
|--|--|---|---|
| 13. $p \wedge \sim q$
<u>p</u>
$\sim q$ | 14. $p \vee q$
<u>p</u>
$\sim q$ | 15. $p \vee \sim q$
<u>p</u>
$\sim q$ | 16. $\sim p \rightarrow \sim q$
<u>q</u>
p |
| 17. $\sim p \rightarrow q$
<u>p</u>
$\sim q$ | 18. $p \rightarrow q$
<u>q \rightarrow p</u>
p \wedge q | 19. $p \rightarrow \sim q$
<u>$\sim p$</u>
$\sim q$ | 20. $p \rightarrow \sim q$
<u>q</u>
$\sim p$ |
| 21. $(p \rightarrow q) \wedge (q \rightarrow p)$
<u>p</u>
p \vee q | 22. $(\sim p \vee q) \wedge (\sim p \rightarrow q)$
<u>p</u>
$\sim q$ | 23. $(r \wedge p) \rightarrow (r \vee q)$
<u>(q \wedge p)</u>
r \vee p | 24. $(\sim p \wedge r) \rightarrow (p \vee q)$
<u>($\sim r \rightarrow p$)</u>
q \rightarrow r |

25. Earlier we showed how to analyze arguments using Euler diagrams. Refer to Example 4 in this section, restate each premise and the conclusion using a quantifier, and then draw an Euler diagram to illustrate the relationship.
26. Explain in a few sentences how to determine the statement for which a truth table will be constructed so that the arguments in Exercises 27–36 can be analyzed for validity.

Determine whether the following arguments are valid or invalid.

27. Wally's hobby is amateur radio. If his wife likes to read, then Wally's hobby is not amateur radio. If his wife does not like to read, then Nikolas likes cartoons. Therefore, Nikolas likes cartoons.
28. If you are infected with a virus, then it can be transmitted. The consequences are serious and it cannot be transmitted. Therefore, if the consequences are not serious, then you are not infected with a virus.
29. Paula Abdul sings or Tom Cruise is not a hunk. If Tom Cruise is not a hunk, then Garth Brooks does not win a Grammy. Garth Brooks wins a Grammy. Therefore, Paula Abdul does not sing.
30. If Bill so desires, then Al will be the vice president. Magic is a spokesman or Al will be the vice president. Magic is not a spokesman. Therefore, Bill does not so desire.
31. The Falcons will be in the playoffs if and only if Morten is an all-pro. Janet loves the Falcons or Morten is an all-pro. Janet does not love the Falcons. Therefore, the Falcons will not be in the playoffs.
32. If you're a big girl, then you don't cry. If you don't cry, then your momma does not say, "Shame on you." You don't cry or your momma says, "Shame on you." Therefore, if you're a big girl, then your momma says, "Shame on you."
33. If I were your woman and you were my man, then I'd never stop loving you. I've stopped loving you. Therefore, I am not your woman or you are not my man.
34. If Charlie is a salesman, then he lives in Hattiesburg. Charlie lives in Hattiesburg and he loves to fish.

Therefore, if Charlie does not love to fish, he is not a salesman.

35. All men are mortal. Socrates is a man. Therefore, Socrates is mortal.
36. All men are created equal. All people who are created equal are women. Therefore, all men are women.
37. Mittie Arnold made the following observation: "If I want to determine whether an argument leading to the statement

$$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

is valid, I only need to consider the lines of the truth table which lead to T for the column headed $(p \rightarrow q) \wedge \sim q$." Mittie was very perceptive. Can you explain why her observation was correct?

38. Suppose that you ask someone for the time and you get the following response:

"If I tell you the time, then we'll start chatting. If we start chatting, then you'll want to meet me at a truck stop. If we meet at a truck stop, then we'll discuss my family. If we discuss my family, then you'll find out that my daughter is available for marriage. If you find out that she is available for marriage, then you'll want to marry her. If you want to marry her, then my life will be miserable since I don't want my daughter married to some fool who can't afford a \$10 watch."

Use reasoning by transitivity to draw a valid conclusion.

In the arguments used by Lewis Carroll, it is helpful to restate a premise in if . . . then form in order to more easily lead to a valid conclusion. The following premises come from Lewis Carroll. Write each premise in if . . . then form.

39. None of your sons can do logic.
40. All my poultry are ducks.
41. No teetotalers are pawnbrokers.
42. Guinea pigs are hopelessly ignorant of music.
43. Opium-eaters have no self-command.
44. No teachable kitten has green eyes.
45. All of them written on blue paper are filed.
46. I have not filed any of them that I can read.

The following exercises involve premises from Lewis Carroll. Write each premise in symbols, and then in the final part, give a valid conclusion.

47. Let p be "one is able to do logic," q be "one is fit to serve on a jury," r be "one is sane," and s be "he is your son."
 - (a) Everyone who is sane can do logic.
 - (b) No lunatics are fit to serve on a jury.
 - (c) None of your sons can do logic.
 - (d) Give a valid conclusion.
48. Let p be "it is a duck," q be "it is my poultry," r be "one is an officer," and s be "one is willing to waltz."
 - (a) No ducks are willing to waltz.
 - (b) No officers ever decline to waltz.
 - (c) All my poultry are ducks.
 - (d) Give a valid conclusion.
49. Let p be "it is a guinea pig," q be "it is hopelessly ignorant of music," r be "it keeps silent while the *Moonlight Sonata* is being played," and s be "it appreciates Beethoven."
 - (a) Nobody who really appreciates Beethoven fails to keep silent while the *Moonlight Sonata* is being played.
 - (b) Guinea pigs are hopelessly ignorant of music.
 - (c) No one who is hopelessly ignorant of music ever keeps silent while the *Moonlight Sonata* is being played.
 - (d) Give a valid conclusion.
50. Let p be "one is honest," q be "one is a pawnbroker," r be "one is a promise-breaker," s be "one is trust worthy," t be "one is very communicative," and u be "one is a wine-drinker."
 - (a) Promise-breakers are untrustworthy.
 - (b) Wine-drinkers are very communicative.
 - (c) A person who keeps a promise is honest.
 - (d) No teetotalers are pawnbrokers. (*Hint*: Assume "teetotaler" is the opposite of "wine-drinker.")
 - (e) One can always trust a very communicative person.
 - (f) Give a valid conclusion.
51. Let p be "he is going to a party," q be "he brushes his hair," r be "he has self-command," s be "he looks fascinating," t be "he is an opium-eater," u be "he is tidy," and v be "he wears white kid gloves."
 - (a) No one who is going to a party ever fails to brush his hair.
 - (b) No one looks fascinating if he is untidy.
 - (c) Opium-eaters have no self-command.
 - (d) Everyone who has brushed his hair looks fascinating.
 - (e) No one wears white kid gloves unless he is going to a party. (*Hint*: " a unless b " $\equiv \sim b \rightarrow a$.)
 - (f) A man is always untidy if he has no self-command.
 - (g) Give a valid conclusion.
52. Let p be "it begins with 'Dear Sir'," q be "it is crossed," r be "it is dated," s be "it is filed," t be "it is in black ink," u be "it is in the third person," v be "I can read it," w be "it is on blue paper," x be "it is on one sheet," and y be "it is written by Brown."
 - (a) All the dated letters in this room are written on blue paper.
 - (b) None of them are in black ink, except those that are written in the third person.
 - (c) I have not filed any of them that I can read.
 - (d) None of them that are written on one sheet are undated.
 - (e) All of them that are not crossed are in black ink.
 - (f) All of them written by Brown begin with "Dear Sir."
 - (g) All of them written on blue paper are filed.
 - (h) None of them written on more than one sheet are crossed.
 - (i) None of them that begin with "Dear Sir" are written in the third person.
 - (j) Give a valid conclusion.