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(b) Find $f(0)$.

(c) Find $\lim_{x \to -4} \dfrac{f(x)}{x^2 + 4x}$.

(d) Let h be the function defined by $h(x) = \dfrac{g(x)}{x^2 + 1}$. Find $h'(1)$.

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4. Consider the differential equation $\frac{dy}{dx} = (y - 2)(x^2 + 1)$.

(a) Find $y = g(x)$, the particular solution to the given differential equation with initial condition $g(0) = 5$.

(b) For the particular solution $y = g(x)$ found in part (a), find $\lim_{x \to -\infty} g(x)$.

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(c) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 3$.

Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 3)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(1, 3)$? Give a reason for your answer.

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5. The function f is defined by

$$f(x) = \begin{cases} 3x^2 + 2x & \text{for } x \leq 0 \\ e^{2x} + 2 & \text{for } x > 0. \end{cases}$$

(a) Is f continuous at $x = 0$? Justify your answer.

(b) Find $f'(-2)$ and $f'(3)$.

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(c) Explain why $f'(0)$ does not exist.

(d) Let g be the function given by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find $g(1)$.

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6. A hive contains 35 hundred bees at time $t = 0$. During the time interval $0 \le t \le 4$ hours, bees enter the hive at a rate modeled by $E(t) = 16t - 3t^2$, where $E(t)$ is measured in hundreds of bees per hour. During the same time interval, bees leave the hive at a rate modeled by $L(t) = -2t + 15$, where $L(t)$ is measured in hundreds of bees per hour.

(a) How many bees leave the hive during the time interval $0 \le t \le 2$?

(b) Write an expression involving one or more integrals for the total number of bees, in hundreds, in the hive at time t for $0 \le t \le 4$. Find the total number of bees in the hive at $t = 4$.

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(c) Find the minimum number of bees in the hive for $0 \le t \le 4$. Justify your answer.

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