1. If $y = x\sin x$, then $\frac{dy}{dx} =$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

3. If $f(x) = 4x^{-2} + \frac{1}{4}x^2 + 4$, then $f'(2) =$

(A) -62 (B) -58 (C) -3 (D) 0 (E) 1

4. If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

P

6. Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

P

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$

(A) $5\left(x^3 - \cos x\right)^4$

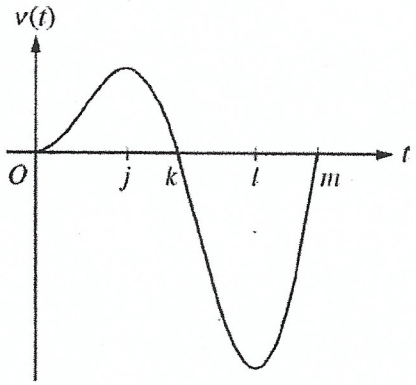
(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

12



13

8. A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 \le t \le m$, where j, k, l, and m are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of t is the speed of the particle decreasing?

(A) $j \le t \le l$

(B) $k \le t \le m$

(C) $j \le t \le k$ and $l \le t \le m$

(D) $0 \le t \le j$ and $k \le t \le l$

(E) $0 \le t \le j$ and $l \le t \le m$

9. Let f be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only

13

$$f(x) = \begin{cases} \dfrac{x^2 - 7x + 10}{b(x-2)} & \text{for } x \ne 2 \\ b & \text{for } x = 2 \end{cases}$$

14

10. Let f be the function defined above. For what value of b is f continuous at $x = 2$?

(A) -3 (B) $\sqrt{2}$ (C) 3 (D) 5 (E) There is no such value of b.

11. What is the slope of the line tangent to the graph of $y = \dfrac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\dfrac{1}{e}$ (B) $-\dfrac{3}{4e}$ (C) $-\dfrac{1}{4e}$ (D) $\dfrac{1}{4e}$ (E) $\dfrac{1}{e}$

P

12. If $f'(x) = \dfrac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) $\ln 25$ (C) $5 + \dfrac{2}{e} - \dfrac{2}{e^2}$ (D) 6 (E) 25

P

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \le 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

13

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\dfrac{7}{\sqrt{5}}$ (B) $\dfrac{14}{\sqrt{5}}$ (C) $\dfrac{18}{\sqrt{5}}$ (D) $\dfrac{15}{\sqrt{21}}$ (E) $\dfrac{30}{\sqrt{21}}$

12

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
-1	2	5	-2

15. The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

(A) $-\dfrac{5}{2}$ (B) -2 (C) 2 (D) 3 (E) 8

13

16. If $y = 5x\sqrt{x^2+1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$ (B) $\frac{15}{\sqrt{10}}$ (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

13

17. If $f(x) = ae^{-ax}$ for $a > 0$, then $f'(x) =$

(A) e^{-ax}

(B) ae^{-ax}

(C) a^2e^{-ax}

(D) $-ae^{-ax}$

(E) $-a^2e^{-ax}$

14

18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

9

19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

14

20. $\lim_{x \to 2} \dfrac{\ln(x+3) - \ln(5)}{x-2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

14

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x-5}$ (D) $y = \dfrac{5x}{1-x}$ (E) $y = \dfrac{20x^2 - x}{1+4x^2}$

12

22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

9

23. Let f be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all x and the point $(0, 1)$ is on the graph of f, what is the value of $g'(1)$?

(A) $\dfrac{1}{2+e}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3 (E) $2 + e$

13

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

12

25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

P

26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true?

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

P

27. A particle moves on the x-axis so that at any time t, $0 \le t \le 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

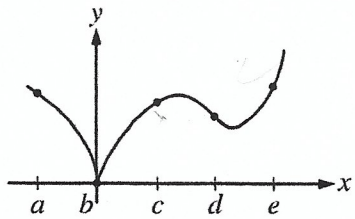
(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

14

28. The function f is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

13



Graph of f

12

76. The graph of the function f is shown in the figure above. For which of the following values of x is $f'(x)$ positive and increasing?

(A) a (B) b (C) c (D) d (E) e

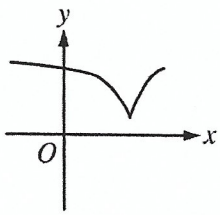
77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

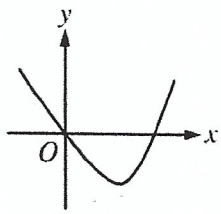
(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

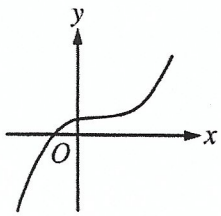
13

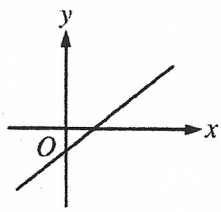
78. The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of f', the derivative of f?

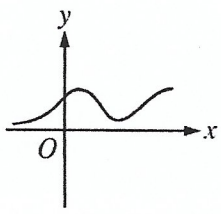
(A)


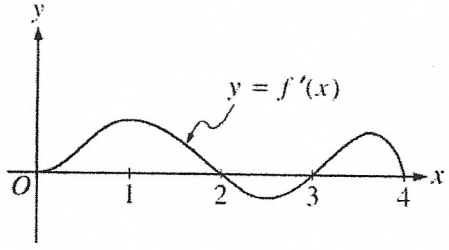
(B)


14

(C)


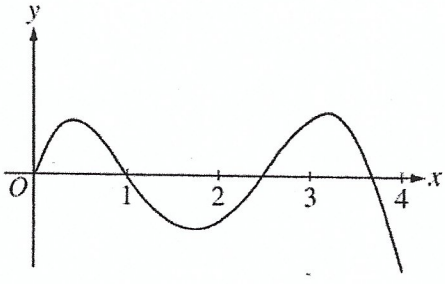
(D)


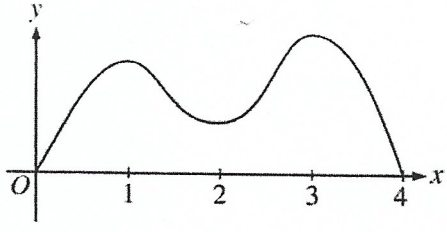
(E)


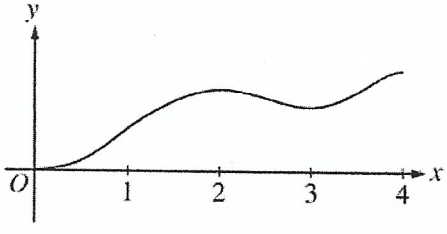


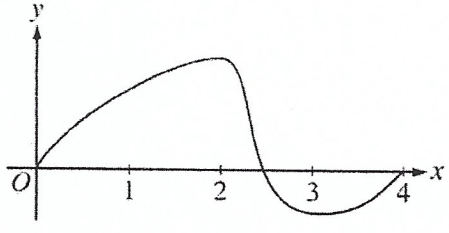
13

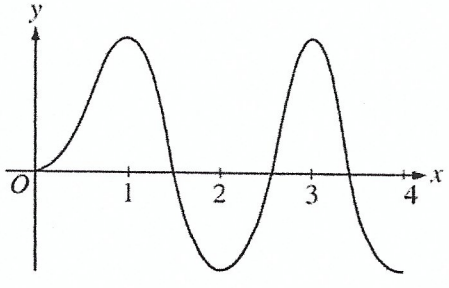
79. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

(A)


(B)


(C)


(D)


(E)


x	$f(x)$
1	2.4
3	3.6
5	5.4

14

80. The table above gives selected values of a function f. The function is twice differentiable with $f''(x) > 0$. Which of the following could be the value of $f'(3)$?

(A) 0.6 (B) 0.7 (C) 0.9 (D) 1.2 (E) 1.5

81. The first derivative of the function f is given by $f'(x) = x - 4e^{-\sin(2x)}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

(A) Three (B) Four (C) Five (D) Six (E) Seven

82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

85. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = \cos\sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) -1 (B) -0.624 (C) -0.318 (D) 0 (E) 0.065

13

86. Line ℓ is tangent to the graph of $y = e^x$ at the point $\left(k, e^k\right)$. What is the positive value of k for which the y-intercept of ℓ is $\frac{1}{2}$?

(A) 0.405

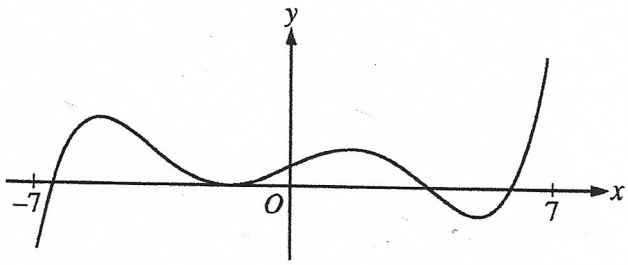
(B) 0.768

(C) 1.500

(D) 1.560

(E) There is no such value of k.

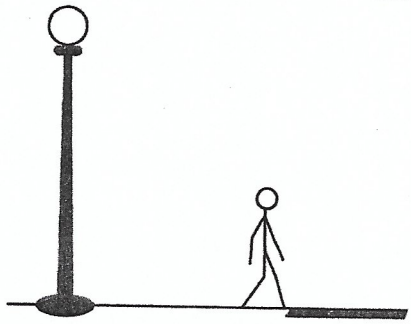
14



Graph of f'

87. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five



12

88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−2	−3	4

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) −28 (B) −16 (C) 40 (D) 44 (E) 47

P

90. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$

(B) $(0, 1.256)$ only

14

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

x	-2	0	3	5	6
$f'(x)$	3	1	4	7	5

91. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-2 < x < 6$?

(A) The graph of f is concave up.

(B) The graph of f has at least two points of inflection.

(C) f is increasing.

(D) f has no critical points.

(E) f has at least two relative extrema.

12

x	$f(x)$	$g(x)$	$f'(x)$
-4	0	-9	5
-2	4	-7	4
0	6	-4	2
2	7	-3	1
4	10	-2	3

14

92. The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$ (B $-\frac{1}{4}$ (C) $-\frac{3}{100}$ (D $\frac{1}{4}$ (E) $\frac{1}{3}$