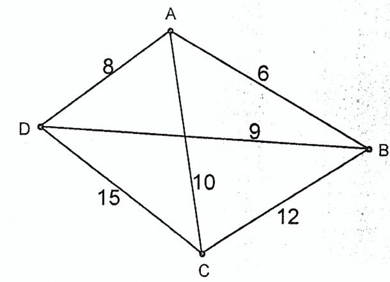
Brute Force

NAME ______________________

The Brute Force Algorithm

1. Beginning with any starting city, list all the possible round-trips. **Hint:** Draw a tree diagram.
2. Determine the distance of each round-trip.
3. Pick the shortest round-trip.

1. Refer to the map on the right to complete this question.



a) Starting with city A, list all the possible round-trips.

ABCDA
ABDCA
____
____
____
____

b) Determine the total distance of each round-trip.

ABCDA = AB + BC + CD + DA = 6 + 12 + 15 + 8 = ___
ABDCA = AB + BD + DC + CA = 6 + 9 + ___ + ___ = ___
ACBDA = ______________ = ______________
ACDBA = ______________ = ______________
ADBCA = ______________ = ______________
ADCBA = ______________ = ______________

c) Pick the shortest round trip.

2. For four cities, how many round trips were possible?

3. Did all the road trips result in different distances? If not, is there anything unique about the ones that gave the same distances?

4. Using your answer from Question 3 and the fact that 5 cities have 12 unique round-trips (URTs) and 6 cities have 60 URTs, can you find a mathematical formula that gives you the number of URTs for n cities?

The Number of Unique Round-Trips for n Cities

Given n cities, the number of unique round-trips is given by the following formula:

$$\text{Number of URTs} = \frac{(n-1)\cdot(n-2)\cdot(n-3)\cdots 3\cdot 2\cdot 1}{2} = \frac{(n-1)!}{2}$$

5. In Question 4 above, you are told that a tour of 5 cities has 12 URTs. If you are given cities S, T, U, V, and W, list all the URTs starting from city S.

6. You are a traveling salesperson for a local company. Determine the number of URTs if you need to visit:

a) 10 cities (including your starting city)

b) 25 cities (including your starting city)

c) The capitals of the lower 48 states (including your starting city)

Nearest Neighbor

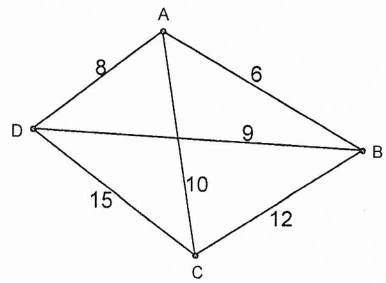
NAME ______________________

The Nearest Neighbor Algorithm

1. From your starting city, visit the *nearest* city.
2. From that city, visit the *nearest* city you have not already visited.
3. When you have visited all the cities, return to your starting city.

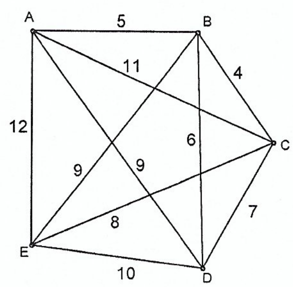
1. Given the table of distances between cities A, B, C, and D and the map, find the shortest round-trip starting at city A.

	A	B	C	D
A	—	6	10	8
B	6	—	12	9
C	10	12	—	15
D	8	9	15	—



A to ______ to ______ to ______ to **A**   Total Distance: __________

2. Given the map of cities A, B, C, D, and E, find the length of the round-trip starting at city B using the nearest neighbor algorithm.



B to ______ to ______ to ______ to ______ to **B**   Total Distance: __________

3. Given the table of distances between cities A, B, C, D, E, and F, find the length of the round-trip starting at city C using the nearest neighbor algorithm.

	A	B	C	D	E	F
A	—	10	12	4	6	20
B	10	—	2	15	9	18
C	12	2	—	8	13	5
D	4	15	8	—	17	21
E	6	9	13	17	—	3
F	20	18	5	21	3	—

C to ______ to ______ to ______ to ______ to ______ to **C**   Total Distance: __________

Cheapest Link

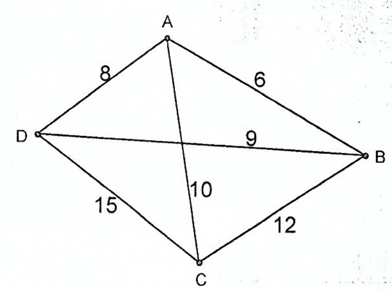
NAME ______________________

Note: For this activity, "route" refers to a path from one city to another, and "mini-tour" refers to a tour that does not include all cities.

The Cheapest Link Algorithm

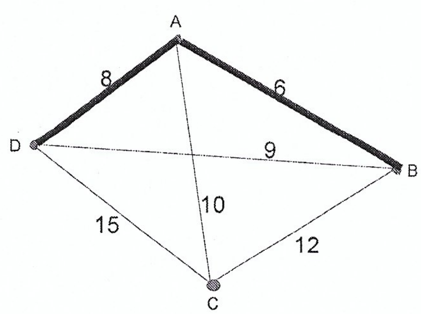
1. Sort the distances of all the routes between each pair of cities from shortest to longest.
2. Select the shortest route available on the list as long as:
   a) It does not cause three routes going to and from the same city.
   b) It does not form a "mini-tour."
3. Continue the process until there is a tour that includes all the cities.

1. Refer to the map on the right to complete this example.

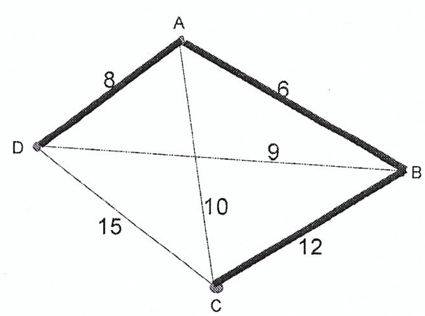


a) Complete the table of routes listed from shortest to longest.

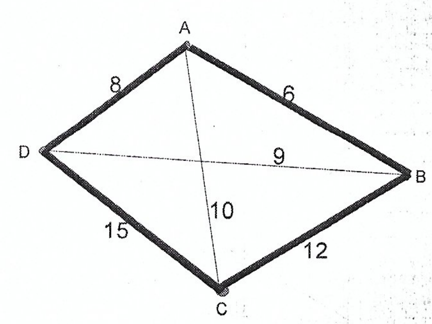
ROUTE	DISTANCE
AB	6
AD	8



Select route AB first since it is the shortest route. Next, select AD because it is the next shortest.



BD and AC are the next cities in the sorted list. However, BD would create a "mini-tour" of ABDA, and AC creates three routes at A. Therefore, select BC.

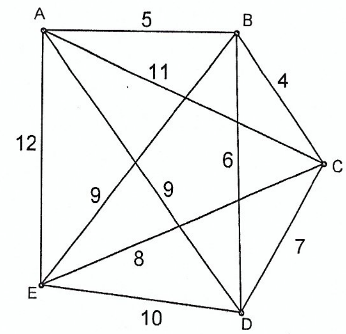


Finally, select CD to complete a tour that includes all the cities.

b) Fill in the distances between the routes used in the tour, then find the sum.

Total Distance = 6 + ___ + ___ + ___ = ____

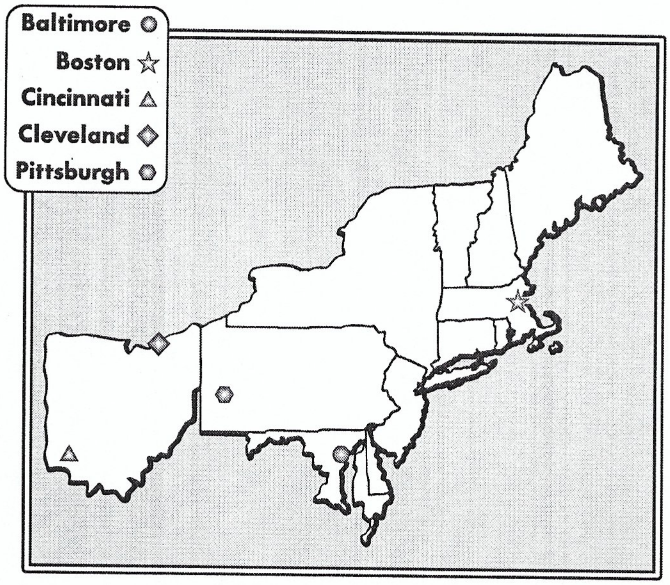
2. Use the Cheapest Link Algorithm to find the shortest round-trip using the map below.



3. Use the Cheapest Link Algorithm to find the shortest round-trip using the table below.

	A	B	C	D	E	F
A	—	10	12	4	6	20
B	10	—	2	15	9	18
C	12	2	—	8	13	5
D	4	15	8	—	17	21
E	6	9	13	17	—	3
F	20	18	5	21	3	—

Road Trip!



You live in Cleveland and you are planning a road trip to visit friends in Cincinnati, Pittsburgh, Baltimore, and Boston. However, with the price of gas over $3.00 a gallon, you want to calculate the order to visit the cities to make the round-trip the shortest and save money on gas.

Table of Distances (miles)

	CLEVELAND	BOSTON	BALTIMORE	CINCINNATI	PITTSBURGH
CLEVELAND	—	554	314	211	115
BOSTON	554	—	364	739	480
BALTIMORE	314	364	—	428	200
CINCINNATI	211	739	428	—	259
PITTSBURGH	115	480	200	259	—

Note: These distances are straight-line distances, not driving distances.

This type of problem is called the Traveling Salesperson Problem. It was first studied by Irish mathematician William Rowan Hamilton, who lived from 1805-1865.

Best Route

NAME ______________________

Use the information given on the Road Trip! overhead to solve these problems.

1. Using the Nearest Neighbor Algorithm, find the shortest round-trip starting in Cleveland.

   Cle to ______ to ______ to ______ to ______ to **Cle** Total Distance: ____________

2. Use the Nearest Neighbor steps using each of the other cities as the starting point.

   Cin to ______ to ______ to ______ to ______ to **Cin** Total Distance: ____________

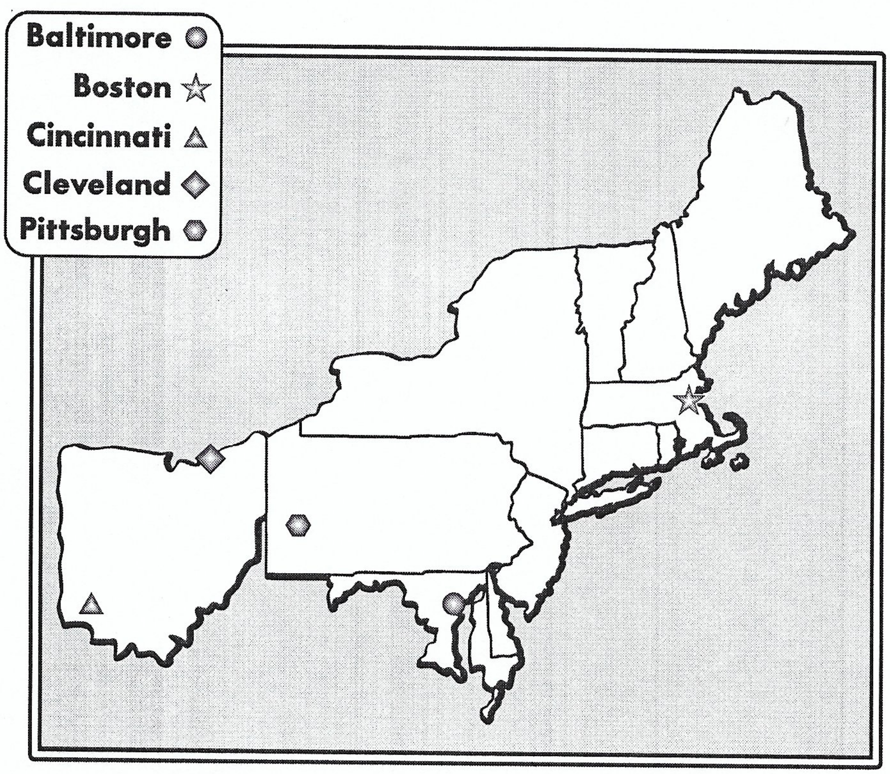
   Pitt to ______ to ______ to ______ to ______ to **Pitt** Total Distance: ____________

   Bos to ______ to ______ to ______ to ______ to **Bos** Total Distance: ____________

   Bal to ______ to ______ to ______ to ______ to **Bal** Total Distance: ____________

3. Were the total distances in Questions 1 and 2 the same or different? Is this what you expected? Why do you think the results turned out this way?

4. Using the Cheapest Link Algorithm, find the shortest round-trip. Draw the route on the map.



ROUTE	DISTANCE

5. What is the total distance of the route found using the Cheapest Link Algorithm?

6. Using the Brute Force Algorithm, how many unique round-trips are possible?

7. One of the possible round-trips results in a total distance of 1588 miles. Determine the tour that begins and ends at Cleveland for this round trip.

8. What is the best tour to follow on your road trip? Explain your reasoning.

9. Which algorithm was the easiest to implement? Explain your reasoning.

10. Which algorithm was the hardest to implement? Explain your reasoning.

11. Which algorithm will always find the shortest distance? Explain your reasoning.