2006 BC 5c; 6
2007 BC 6
2008 BC 6c
2009 BC 4b; 6
2010–2011 BC 6
2012 BC 4d; 6
2013–2014 BC 6

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used for the following questions.

1. Which of the following series is absolutely convergent?

   (A) $\sum_{k=0}^{\infty}(-1)^{k}\dfrac{k+3}{k+\sqrt{k}}$

   (B) $\sum_{k=0}^{\infty}(-1)^{k}\dfrac{3}{\sqrt{k}}$

   (C) $\sum_{k=0}^{\infty}(-1)^{k+1}\dfrac{\sqrt{k}}{k+3}$

   (D) $\sum_{k=0}^{\infty}(-1)^{k+1}\dfrac{\sqrt{3}}{k+k}$

2. The power series $\sum_{k=1}^{\infty}\dfrac{x^k}{k}$ converges for which values of x?

   (A) $-1 < x < 1$

   (B) $-1 < x < 1$

   (C) $-1 < x < 1$

   (D) x is any real number.

3. Which of the following series is the power series expansion for f (x) = x (cos x −1)?

   (A) $x-\dfrac{x^3}{2}+\dfrac{x^5}{24}-\cdots$

   (B) $-x^3+x^5-x^7-\cdots$

   (C) $\dfrac{x^3}{2}-\dfrac{x^5}{24}+\dfrac{x^7}{720}-\cdots$

   (D) $-\dfrac{x^3}{2}+\dfrac{x^5}{24}-\dfrac{x^7}{720}+\cdots$

4. What are all values of a for which the series $\sum_{k=0}^{\infty}\left(\dfrac{5}{9-a}\right)^{k}$ converges?

   (A) $a < 4$

   (B) $4 < a < 14$

   (C) $a < 9$

   (D) $a < 4$ or $a > 14$

5. The Maclaurin series for $f(x)=\frac{1}{1+x^2}$ is $\sum_{k=0}^{\infty}(-1)^k x^{2k}$. What is the Maclaurin series for $g(x) = \tan^{-1} x$?

(A) $C+\sum_{k=0}^{\infty}(-1)^k(2k)x^{2k-1}$

(B) $C+\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1}$

(C) $C+\sum_{k=0}^{\infty}(-1)^{k+1}\frac{x^{2k+1}}{2k}$

(D) $C+\sum_{k=0}^{\infty}(-1)^{k-1}\frac{x^{2k-1}}{2k}$

6. The Maclaurin series represents which function below?

(A) $\cos(3x^2)$

(B) $\sin(3x^2)$

(C) $\cos(9x^4)$

(D) e^{-3x2}

7. If the first five terms of the Taylor expansion for $f(x)$ about $x = 0$ are $3-7x+\frac{5}{2}x^2+\frac{3}{4}x^3-6x^4$, then $f'''(0) =$

(A) $\frac{1}{8}$

(B) $\frac{3}{4}$

(C) $\frac{9}{2}$

(D) 6

8. Which of the following series diverge?

I. $\sum_{k=0}^{\infty}\frac{k^{\frac{3}{2}}+1}{5k^2+7}$

II. $\sum_{k=2}^{\infty}(-1)^k\frac{1}{\ln(k)}$

III. $\sum_{k=0}^{\infty}(-1)^k\left(\frac{4}{3}\right)^k$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

A calculator may be used for the following questions.

9. The sixth degree term of the Taylor series expansion $f(x)=e^{\frac{1}{2}x^2}$ about $x=0$ has coefficient

(A) $-\frac{1}{48}$

(B) $-\frac{1}{6}$

(C) $\frac{1}{720}$

(D) $-\frac{1}{4608}$

10. Let $T_n(x)$ represent the Taylor Polynomial of degree n about $x=0$ for $f(x)=e^{-x}$. If $T_n(2)$ is used to approximate the value of $f(2)=e^{-2}$, then all of the following expressions are less than $\frac{2^7}{7!}$ EXCEPT

(A) $f(2)-T_6(2)$

(B) $T_6(2)-f(2)$

(C) $f(2)-T_5(2)$

(D) $T_5(2)-f(2)$

*11. For function $f(x)$, $f(0)=3$, $f'(0)=2$, $f''(0)=5$, and $f'''(0)=4$. Using the Taylor series expansion for $f(x)$ about $x=0$, the second degree estimate for $f'(0.1)$ is

(A) 2.500

(B) 2.520

(C) 3.120

(D) 3.225

A calculator may not be used for the following questions.

12. The Maclaurin series $\sum_{k=0}^{\infty}(-5)^k\frac{x^{3k+2}}{k!}$ represents which expression below?

(A) $x^2e^{-5x^3}$

(B) $-5e^{3x+2}$

(C) $x^2\sin(-5x^3)$

(D) $-5\cos(x^3)+2$

13. If $f(x) = \sin(x^2)$, the first three terms of the Taylor series expansion about $x = 0$ for $f'(x)$ are

(A) $2x - x^3 + \frac{1}{2}x^5$

(B) $1 - \frac{1}{2}x^4 + \frac{1}{16}x^8$

(C) $2x - x^5 + \frac{1}{12}x^9$

(D) $x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10}$

14. Which of the following series is conditionally convergent?

(A) $\sum_{k=1}^{\infty}(-1)^k \frac{k^2+1}{k+4}$

(B) $\sum_{k=1}^{\infty}(-1)^{k-1} \frac{2}{k+1}$

(C) $\sum_{k=1}^{\infty}(-1)^{k+1} \ln(k+1)$

(D) $\sum_{k=1}^{\infty}(-1)^k \frac{k+3}{k^3+7}$

15. The radius of convergence of the power series $\sum_{k=0}^{\infty}\frac{x^{2k}}{k \cdot 4^k}$ is

(A) 0

(B) 1

(C) 2

(D) 4

FREE-RESPONSE QUESTION

A calculator may be used for this question.

Let $f(x)$ be a function that is differentiable for all x. The Taylor expansion for $f(x)$ about $x = 0$ is given by $T(x) = \sum_{k=0}^{\infty}(-1)^k \frac{(k+1)x^{3k}}{k!}$. The first four nonzero terms of $T(x)$ are given by $T_4(x) = 1 - 2x^3 + \frac{3x^6}{2!} - \frac{4x^9}{3!}$.

(a) Show that $T(x)$ converges for all x.

(b) Let $W(x)$ be the Taylor expansion for $x^2 f'(x)$ about $x = 0$. Find the general term for $W(x)$ and find $W_3(x)$.

(c) $f(0.5) \approx T_4(0.5)$ and $f(1) \approx T_4(1)$. Find the values of $T_4(0.5)$ and $T_4(1)$.

(d) Which value is smaller, $|f(0.5) - T_4(0.5)|$ or $|f(1) - T_4(1)|$? Give a reason for your answer.