

Consider the differential equation  $\frac{dy}{dx} = e^y(3x^2 - 6x)$ . Let  $y = f(x)$  be the particular solution to the differential equation that passes through  $(1, 0)$ .

- Write an equation for the line tangent to the graph of  $f$  at the point  $(1, 0)$ . Use the tangent line to approximate  $f(1.2)$ .
- Find  $y = f(x)$ , the particular solution to the differential equation that passes through  $(1, 0)$ .

Solutions to the differential equation  $\frac{dy}{dx} = xy^3$  also satisfy  $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$ . Let  $y = f(x)$  be a particular solution to the differential equation  $\frac{dy}{dx} = xy^3$  with  $f(1) = 2$ .

- Write an equation for the line tangent to the graph of  $y = f(x)$  at  $x = 1$ .
- Use the tangent line equation from part (a) to approximate  $f(1.1)$ . Given that  $f(x) > 0$  for  $1 < x < 1.1$ , is the approximation for  $f(1.1)$  greater than or less than  $f(1.1)$ ? Explain your reasoning.
- Find the particular solution  $y = f(x)$  with initial condition  $f(1) = 2$ .

The function  $f$  is differentiable for all real numbers. The point  $\left(3, \frac{1}{4}\right)$  is on the graph of  $y = f(x)$ , and the slope at each point  $(x, y)$  on the graph is given by  $\frac{dy}{dx} = y^2(6 - 2x)$ .

- Find  $\frac{d^2y}{dx^2}$  and evaluate it at the point  $\left(3, \frac{1}{4}\right)$ .
- Find  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = y^2(6 - 2x)$  with the initial condition  $f(3) = \frac{1}{4}$ .

Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .

- Find a solution  $y = f(x)$  to the differential equation satisfying  $f(0) = \frac{1}{2}$ .
- Find the domain and range of the function  $f$  found in part (a).