Derivatives Practice test

Evaluate

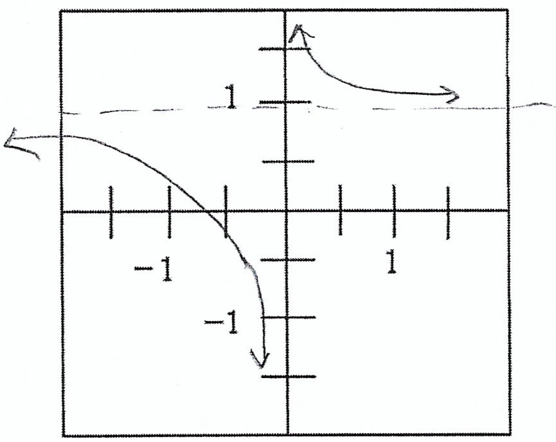
1. $\lim_{x \to 0} x \cdot \csc x = 1$

2. $\lim_{x \to -\infty} \frac{x+1}{x-1} = 1$

3. Sketch a graph that uses the following conditions. Pay attention to how the graph is labeled.

$\lim_{x \to 0^+} f(x) = \infty$, $\lim_{x \to 0^-} f(x) = -\infty$,

$\lim_{x \to \infty} f(x) = 1$, $\lim_{x \to -\infty} f(x) = 1$



4. Use the given table to find the following.

x	3	-1	2
$f(x)$	5	-2	3
$g(x)$	-1	2	7
$f'(x)$	-3	1	0
$g'(x)$	6	2	-3

a. Find $h'(2)$, if $h(x) = f(x)g(x)$ $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(2) = f'(2)g(2) + f(2)g'(2) = -9$

$0 \cdot 7 + 3(-3)$

b. Find $h'(3)$, if $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3)$

$= f'(-1)g'(3) = (1)(6) = 6$

5) $y' = 1e^{x^3} + xe^{x^3}(3x^2)$

6) $g'(x) = 1\sin x^2 + x\cos x^2 \cdot 2x$

7) $y' = \frac{1}{\sec x} \cdot \sec x \cdot \tan x$

8) $\ln y = (1-x)\ln x$

$\frac{y'}{y} = (-1)\ln x + (1-x)\frac{1}{x}$

$y' = \left[-\ln x + (1-x)\left(\frac{1}{x}\right)\right] x^{1-x}$

9) $y' = \dfrac{(4x^3e^x + x^4e^x)(x-1)^3 - x^4e^x\,3(x-1)^{-2}}{((x-1)^3)^2}$

10) $y' + 12 = 1y^5 + x5y^4y' + \sec^2 y \cdot y'$

$12 - y^5 = (5xy^4 + \sec^2 y - 1)y'$

$\dfrac{12-y^5}{5xy^4 + \sec^2 y - 1} = y'$

11) $f(\pi) = \sin\pi = 0$ $\quad(\pi, 0)$

$f'(x) = \cos x$ $\quad y - 0 = -1(x - \pi)$

$f'(\pi) = \cos\pi = -1$

12) $f(2) = 3(2)^2 - 5 = 7$ $\quad y - 7 = 12(x-2)$

$f'(x) = 6x \quad f'(2) = 12$

$y - 7 = 12(2.1 - 2)$

$y = 12(2.1-2) + 7$

13) $y=3x^5-20x^3$

a) $y'=15x^4-60x^2=15x^2(x^2-4)$

$x=0,-2,2$

$(x-2)$
$(x+2)$

inc +	max	dec −		dec −	min	inc +
	-2		0		2	

b) $y''=60x^3-120x=60x(x^2-2)$

$x=0, \pm\sqrt{2}$

$60x$
(x^2-2)

−		+		−		+
	$-\sqrt{2}$		0		$\sqrt{2}$	

up $(-\sqrt{2},0)\ (\sqrt{2},\infty)$

down $(-\infty,-\sqrt{2})\ (0,\sqrt{2})$

c) $y''=60x(x^2-2)$

$x=0 \quad y''=0$

$x=-2 \quad y''<0$ concave down $\Rightarrow$ max

$x=2 \quad y''>0$ concave up $\Rightarrow$ min

14) a) $y=\frac{1}{x} \quad y'=-x^{-2}$

$$\frac{\frac{1}{3}-\frac{1}{1}}{3-1}=\frac{-\frac{2}{3}}{\frac{2}{1}}=-\frac{1}{3}$$

$$-\frac{1}{x^2}=-\frac{1}{3} \qquad \boxed{x=\sqrt{3}}$$

$$\frac{1}{x^2}=\frac{1}{3}$$

$x=\pm\sqrt{3}$

b) $y=\frac{1}{x}$ is not continuous on $[-1,1]$

So MVT does not apply

15) $s(t) = t^3 - 9t^2 + 24t$

a) $s(2) = 8 - 32 + 48 = 56 - 32 = 24$ ft

b) $v(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$

$= 3(t-4)(t+2)$

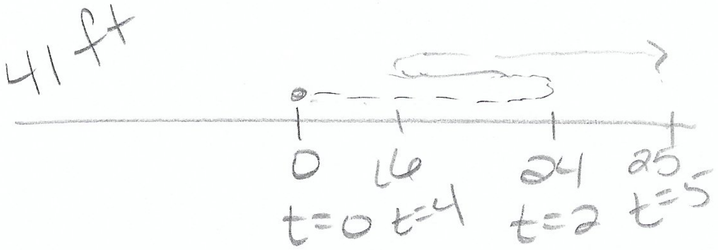
$t = 2, 4$ sec

c) $a(t) = 6t - 18$

$t = 3$ sec

d)

41 ft



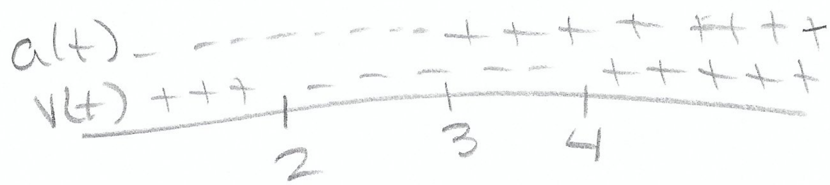
$s(0) = 0$

$s(2) = 24$

$s(4) = 4^3 - 9(4)^2 + 24(4) = 16$

$s(5) = 5^3 - 9(5)^2 + 24(5) = 25$

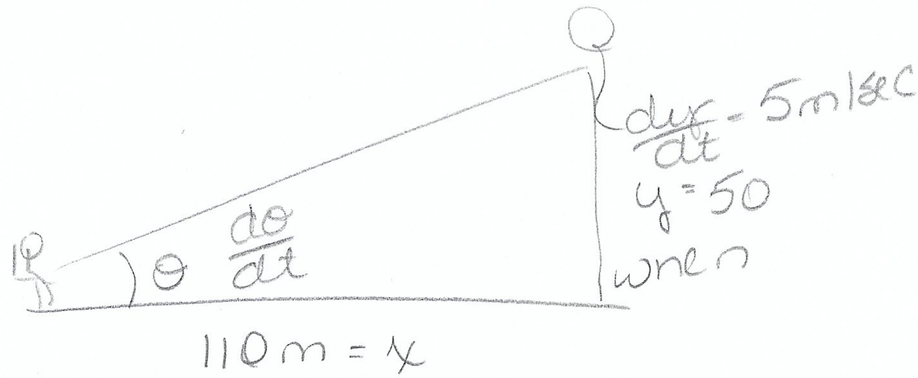
e)



Speeding up $(2,3), (4, \infty)$

16) a) hole b) VA c) jump

17)



$\tan\theta = \frac{50}{110}$

$\theta = \tan^{-1}(50/110)$

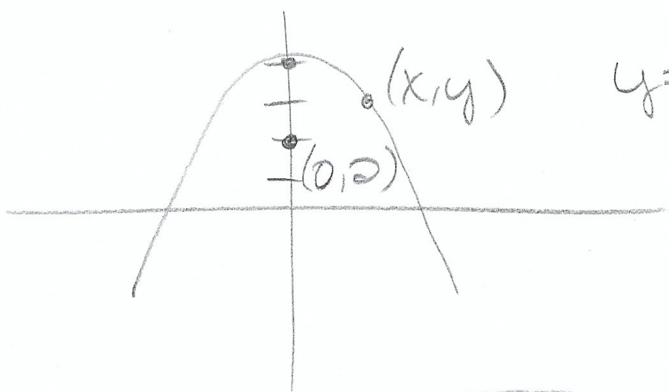
$\theta = .4266$

$\tan\theta = \frac{y}{x} \quad \tan\theta = \frac{y}{110}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{110}\frac{dy}{dt}$

$\frac{d\theta}{dt} = \frac{5}{110} \cdot \cos^2\theta = .0377$

18)



$y = 4 - x^2 \quad \frac{8}{2} \cdot \frac{3}{2}$

$d = \sqrt{(x-0)^2 + (y-2)^2} = \sqrt{x^2 + (4-x^2-2)^2}$

$d^2 = x^2 + (2-x^2)^2$

$(d^2)' = 2x + 2(2-x^2)(-2x) = 2x + -4x(2-x^2)$

$= 2x - 8x + 4x^3 = -6x + 4x^3 = 2x(2x^2 - 3)$

$x = 0 \quad x^2 = 3/2 \quad x = \pm\sqrt{3/2}$

$(0,4) \quad (\sqrt{3/2}, 5/2) \quad (-\sqrt{3/2}, 5/2)$