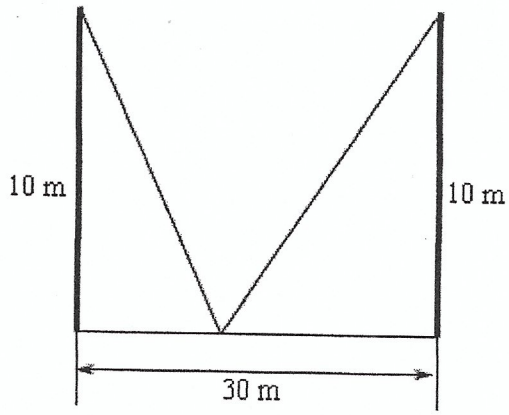
We have 45 m^2 of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.

We want to construct a cylindrical can with a bottom but no top that will have a volume of 30 cm^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

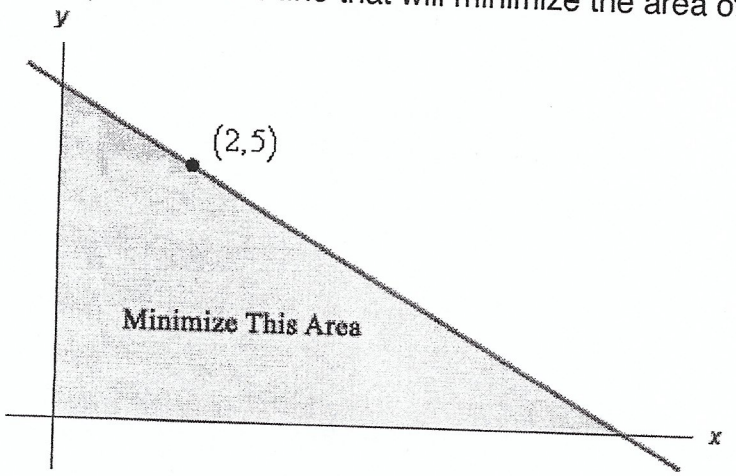
An 80 cm piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other will be bent into a rectangle with one side 4 times the length of the other side. Determine where, if anywhere, the wire should be cut to maximize the area enclosed by the two figures.

Find the point(s) on $x = 3 - 2y^2$ that are closest to $(-4, 0)$.

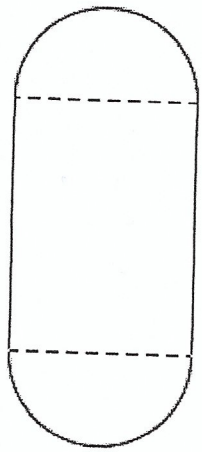
Two 10 meter tall poles are 30 meters apart. A length of wire is attached to the top of each pole and it is staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



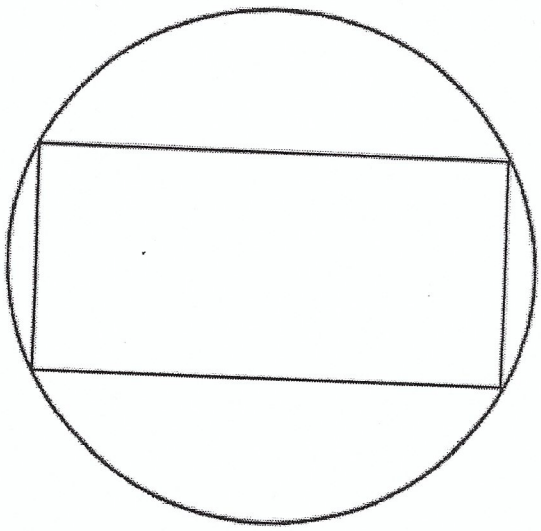
A line through the point $(2, 5)$ forms a right triangle with the x-axis and y-axis in the 1st quadrant. Determine the equation of the line that will minimize the area of this triangle.



We want to construct a window whose middle is a rectangle and the top and bottom of the window are semi-circles. If we have 50 meters of framing material what are the dimensions of the window that will let in the most light?

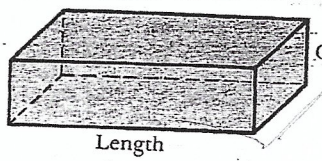


Determine the area of the largest rectangle that can be inscribed in a circle of radius 1.

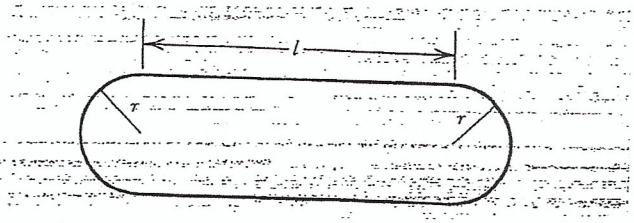


▶ EXERCISE SET 3.2

1. Find the positive number with the property that the sum of the number and its reciprocal is a minimum.
2. Find the positive number with the property that the sum of the square of the number and the square of its reciprocal is a minimum.
3. A rectangular plot of ground containing 432 ft² is fenced off in a large lot. Find the dimensions of the plot that requires the least amount of fence.
4. A rectangular plot of ground containing 432 ft² is fenced off in a large lot, and a fence is constructed down the middle of the lot to separate it into equal parts. Find the dimensions of the plot that requires the minimal amount of fencing.
5. Suppose the fence used to enclose the plot of ground described in Exercise 4 costs \$10 per foot and the fence used to divide the plot into parts costs \$5 per foot. Find the dimensions of the plot that requires the least expense for fencing.
6. A rectangular dog run is to contain 864 ft².
   a) If the dog's owner must pay for the fencing, what should be the dimensions of the run to minimize cost?
   b) Suppose a neighbor has agreed to let the owner use an already constructed fence for one side of the run. What should the dimensions of the run be in this situation if the owner's cost is to be a minimum?
7. A rectangular box with no top is to contain 2250 in³. Find the dimensions to minimize the amount of material used to construct the box if the length of the base is three times the width.
8. Suppose the box described in Exercise 7 is constructed with a top. What dimensions would minimize the amount of material required?
9. The United States Postal Service has recently decreed that no rectangular-shaped parcel can be mailed if the total of its length and girth (perimeter of a cross section) exceeds 108 in. (see the figure). Find the maximum volume in a rectangular parcel that can be mailed if a cross section of the parcel is a square.

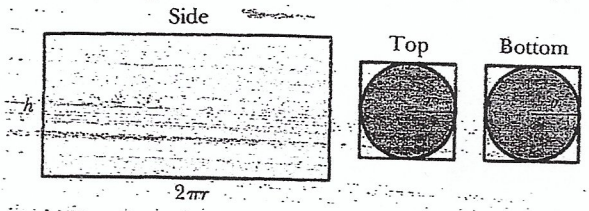


10. The speed of a point on the rim of a flywheel t sec after the flywheel has started to turn is given by the formula $v = 36t^2 - t^3$ ft/sec.
    a) Find its greatest speed.
    b) How long does it run before it reaches this speed?
11. The turning effect of a ship's rudder is found to be $T = k \cos\theta (\sin\theta)^2$, where k is a positive constant and θ is the angle that the direction of the rudder makes with the keel line of the ship ($0 \le \theta \le \pi/2$). For what value of θ is the rudder most effective?
12. A 1-mi race track is to be built with two straight sides and semicircles at the ends (see the figure). What is the maximum amount of area needed to construct the track?



13. A standard can contains a volume of 900 cm³. The can is in the shape of a right circular cylinder with a top and bottom. Find the dimensions of the can that minimize the amount of material needed for construction.
14. In constructing a can in the shape of a right circular cylinder, no waste is produced when the side of the can is cut, but the top and bottom are each stamped from a square sheet and the remainder is wasted (see

the figure). Find the relative dimensions of the can that uses the least amount of material with this construction method.



15. An open rectangular box is to be made from a piece of cardboard 8 in. wide and 8 in. long by cutting a square from each corner and bending up the sides. Find the dimensions of the box with the largest volume.

16. A rectangular plot that will contain a vineyard of one acre in area (43,560 ft²) is to be laid out. The vineyard must have a boundary of 8 ft on all sides in order for equipment to pass and an 8-ft pathway down the middle. What is the minimal acreage required for this plot?

17. A charter bus company charges \$10 per person for a round-trip to a ball game with a discount given for group fares. A group purchasing more than 10 tickets at one time receives a reduction per ticket of \$0.25 times the number of tickets purchased in excess of 10. Determine the maximum revenue that can be received by the bus company.

18. A hotel with 25 rooms normally charges \$40 for a room; however, special group rates are advertised: If the group requires more than 5 rooms, the price for each room is decreased by \$1 times the number of rooms exceeding 5. Find the maximum revenue that the hotel can receive from a group.

19. A rectangle is placed inside a circle of radius r with its corners on the boundary of the circle (see the figure). Of all such rectangles, find the dimensions of the one that encloses the maximum area.

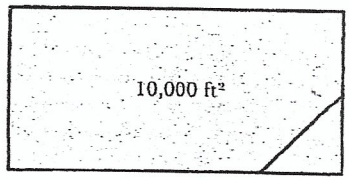
20. A rectangle is placed inside a circle of radius r with its corners on the boundary of the circle. What dimensions should be given to the rectangle to maximize the sum of its perimeter and the length of its two diagonals?

21. An isosceles triangle is placed inside a circle of radius r with its vertices on the boundary of the circle (see the figure). How should this be accomplished if the area of the triangle is to be maximized?

22. "The isosceles triangle with two fixed equal sides and maximum area is not an equilateral triangle." Show that this statement is true by finding the length of the base of an isosceles triangle that maximizes the area over all such triangles.

23. The area of the print on a book page is 42.5 in². The margins are 1 in. on the sides and bottom and 1/2 in. at the top. What should be the dimensions of a page of this book if the only object is to use the minimal amount of paper?

24. A field is fenced off in the form of a rectangle containing 10,000 ft². In addition to the fencing required for the perimeter of the field, an isosceles triangle is fenced off in one corner by running a fence from the midpoint on the shortest side to the adjacent side enclosing the corner of the rectangle (see the figure). Find the dimensions of the field that minimizes the amount of fencing required.



25. A wire 1 ft long is cut in two pieces: One piece is used to construct a square, the other to construct an equilateral triangle. Where should the cut be made in order to minimize the sum of the areas of the figures?

26. A wire 1 ft long is cut in two pieces: One piece is used to construct a square, the other to construct a circle. Where should the cut be made in order to minimize the sum of the areas of the figures?

27. Find the volume of the largest right circular cylinder that can be placed inside a sphere of radius 1.

28. Find the volume of the largest right circular cone that can be placed inside a sphere of radius 1.

29. The strength of a rectangular beam is directly proportional to the product of the width of the beam and the square of its depth. Find the dimensions of the strongest beam that can be cut from a log with radius r.

30. The stiffness of a rectangular beam is directly proportional to the product of the width of the beam and the cube of its depth. Find the dimensions of the stiffest beam that can be cut from a log with radius r.

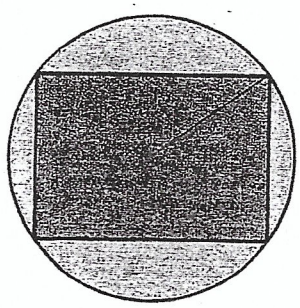


Figure for Exercise 19.

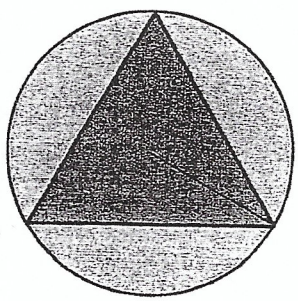


Figure for Exercise 21.

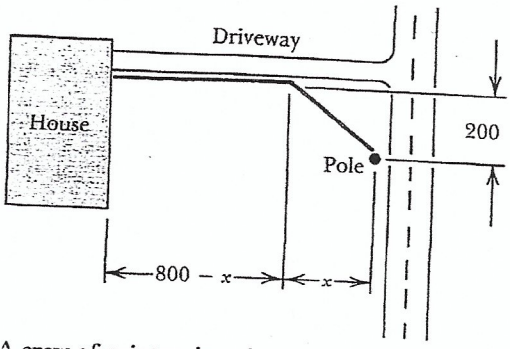
31. Suppose (x_0, y_0) is a point that does not lie on the circle $x^2 + y^2 = 1$. Show that the shortest distance from (x_0, y_0) to the circle is along a line that passes through the center of the circle.

32. Show that the shortest distance from the point (x_0, y_0) to the line with equation $Ax + By + C = 0$ is

$$\frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$

33. The maximum volume in the problem in Example 7 can be found without using trigonometry by expressing the area A totally as a function of x. Show that this method of solution gives the same result as that found in Example 7.

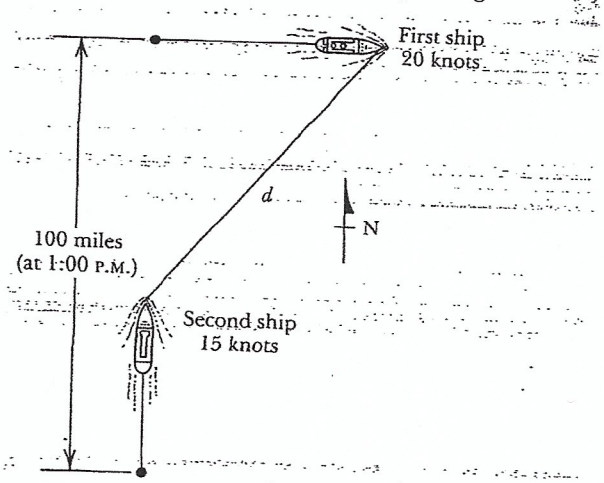
34. A house is built with a straight driveway 800 ft long, as shown in the figure. A utility pole on a line perpendicular to the driveway and 200 ft from the end of the driveway is the closest point from which electricity can be furnished. The utility company will furnish power with underground cable at \$2 per foot and with overhead lines at no charge. However, for overhead lines the company requires that a strip of 30 ft wide be cleared. The owner of the house estimates that to clear a strip this wide will cost \$3 for each foot of overhead wire used. At what point on the driveway should the switch from overhead to underground be made in order to minimize the cost?



35. A crew of painters is assigned to paint a second-floor wall on the outside of a building along a busy sidewalk. They must leave a corridor, for unsuperstitious pedestrians, between the wall and their ladders. The corridor is 6 ft wide and 8 ft high. What is the minimal length of ladder they can use to reach the wall, and how far from the base of the wall should it be placed?

36. A bully armed with a knowledge of calculus is planning an attack on his next victim. The attack must be made on a sidewalk between two lights that are 200 ft apart, one of which is twice as bright as the other. Before dropping out of high school the bully took a physics course and recalls that the intensity of illumination from a light varies inversely as the square of the distance from the light. Where will he attack if he always attacks at the darkest point between the lights?

37. Two tankers are traveling in the midst of the Atlantic Ocean. The first tanker is 100 nautical miles due north of the second at 1:00 P.M. GMT (Greenwich Mean Time) and traveling due east at the rate of 20 knots. The second tanker is traveling due north at 15 knots. At what time are the tankers closest together, and what is the minimal distance separating them?



38. A warehouse is to be built beside a long straight highway running north and south (see the figure at the top of page 203). This warehouse will house equipment produced in two factories and sent there by air for storage. The northern factory lies 80 mi east of the highway; the other lies on the highway. The point on the highway that is closest to the northern factory is 100 mi north of the second factory. Where should the

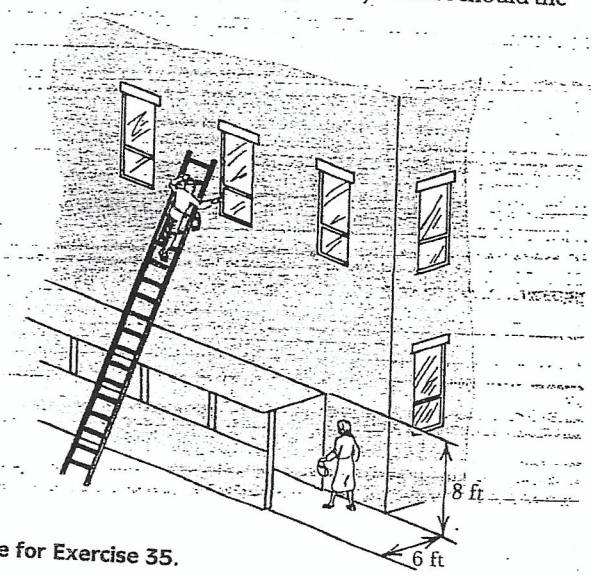


Figure for Exercise 35.