30. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$ (B) $\cos x$ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

45. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

(A) $0 < x < 2$

(B) $0 \le x \le 2$

(D) $-2 \le x < 0$

(E) $-2 \le x \le 0$

16. A series expansion of $\frac{\sin t}{t}$ is

(A) $1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$

(B) $\frac{1}{t} - \frac{t}{2!} + \frac{t^3}{4!} - \frac{t^5}{6!} + \cdots$

(C) $1 + \frac{t^2}{3!} + \frac{t^4}{5!} + \frac{t^6}{7!} + \cdots$

(D) $\frac{1}{t} + \frac{t}{2!} + \frac{t^3}{4!} + \frac{t^5}{6!} + \cdots$

(E) $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$

30. D Substitute $-x$ for x in $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ to get $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = e^{-x}$

45. E

The ratio test shows that the series is convergent for any value of x that makes $|x+1|<1$. The solutions to $|x+1|=1$ are the endpoints of the interval of convergence. Test $x=-2$ and $x=0$ in the series. The resulting series are $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$ and $\sum_{k=1}^{\infty} \frac{1}{k^2}$ which are both convergent. The interval is $-2 \le x \le 0$.

16. A $\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots \Rightarrow \frac{\sin t}{t} = 1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$

10. For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then $f'(x) =$

(A) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$

(B) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n-2}$

(C) $\sum_{n=1}^{\infty} (-1)^{2n} x^{2n}$

(D) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n}$

(E) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n}$

42. The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

13. $\sin(2x) =$

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!} + \ldots$

(B) $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1} (2x)^{2n-1}}{(2n-1)!} + \ldots$

(C) $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^{n} (2x)^{2n}}{(2n)!} + \ldots$

(D) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

(E) $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

10. A Take the derivative of the general term with respect to x: $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$

42. E Since $e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \cdots$, then $e^{3x} = 1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \cdots$

The coefficient we want is $\frac{3^3}{3!} = \frac{9}{2}$

13. B The Maclaurin series for $\sin t$ is $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \cdots$. Let $t = 2x$.

$$\sin(2x) = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \cdots + \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!} + \cdots$$

38. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges?

(A) $-1 \le x \le 1$ (B) $-1 < x \le 1$ (C) $-1 \le x < 1$

(D) $-1 < x < 1$ (E) All real x

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

(A) $1 - \frac{1}{2} + \frac{1}{24}$

(B) $1 - \frac{1}{2} + \frac{1}{4}$

(C) $1 - \frac{1}{3} + \frac{1}{5}$

(D) $1 - \frac{1}{4} + \frac{1}{8}$

(E) $1 - \frac{1}{6} + \frac{1}{120}$

27. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n{}^{n-1}$

38. C Check $x=-1$, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ which is convergent by alternating series test

Check $x=1$, $\sum_{n=1}^{\infty}\frac{1}{n}$ which is the harmonic series and known to diverge.

14. E $\sin x \approx x-\frac{x^3}{3!}+\frac{x^5}{5!}$; $\sin 1 \approx 1-\frac{1^3}{3!}+\frac{1^5}{5!}=1-\frac{1}{6}+\frac{1}{120}$

27. D If $f(x)=\sum_{n=0}^{\infty} a_n x^n$, then $f'(x)=\sum_{n=0}^{\infty} n a_n x^{n-1}=\sum_{n=1}^{\infty} n a_n x^{n-1}$.

$f'(1)=\sum_{n=1}^{\infty} n a_n 1^{n-1}=\sum_{n=1}^{\infty} n a_n$

27. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \le 3$ (B) $-3 \le x \le 3$ (C) $-2 < x < 4$

(D) $-2 \le x < 4$ (E) $0 \le x \le 2$

43. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin\left(x^2\right)$ is

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

45. If $f(x) = \sum_{k=1}^{\infty}\left(\sin^2 x\right)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

27. C This is a geometric series with $r = \frac{x-1}{3}$. Convergence for $-1 < r < 1$. Thus the series is convergent for $-2 < x < 4$.

43. A $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \Rightarrow \sin x^2 = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \cdots = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$

45. D This is an infinite geometric series with a first term of $\sin^2 x$ and a ratio of $\sin^2 x$. The series converges to $\frac{\sin^2 x}{1-\sin^2 x} = \tan^2 x$ for $x \neq (2k+1)\frac{\pi}{2}$, k an integer. The answer is therefore $\tan^2 1 = 2.426$.

17. Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

20. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 3^n}$ converges?

(A) $-3\le x\le 3$
(B) $-3<x<3$
(C) $-1<x\le 5$
(D) $-1\le x\le 5$
(E) $-1\le x<5$

24. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$. If f is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

17. B $f(x)=\ln(3-x);\ f'(x)=\frac{1}{x-3},\ f''(x)=-\frac{1}{(x-3)^2},\ f'''(x)=\frac{2}{(x-3)^3};$

$f(2)=0,\ f'(2)=-1,\ f''(2)=-1,\ f'''(2)=-2;\ \ a_0=0, a_1=-1, a_2=-\frac{1}{2}, a_3=-\frac{1}{3}$

$f(x)\approx-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-3)^3}{3}$

20. E $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 3^n}$. The endpoints of the interval of convergence are when $(x-2)=\pm 3;\ x=-1,5$.
Check endpoints: $x=-1$ gives the alternating harmonic series which converges. $x=5$ gives the harmonic series which diverges. Therefore the interval is $-1\le x<5$.

24. D $f'(x)=\sin(x^2)=x^2-\frac{(x^2)^3}{3!}+\cdots=x^2-\frac{1}{6}x^6\ +\cdots\Rightarrow f(x)=\frac{1}{3}x^3-\frac{1}{42}x^7+\cdots$ The coefficieint of x^7 is $-\frac{1}{42}$.

83. The Taylor series for $\ln x$, centered at $x=1$, is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

84. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

89. The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

32. For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) No values of x (B) $x < -1$ (C) $x \ge -1$ (D) $x > -1$ (E) All values of x

83. C Use a calculator. The maximum for $\left|\ln x-\left(\frac{(x-1)}{1}-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}\right)\right|$ on the interval $0.3 \le x \le 1.7$ occurs at $x = 0.3$.

84. B You may use the ratio test. However, the series will converge if the numerator is $(-1)^n$ and diverge if the numerator is 1^n. Any value of *x* for which $|x+2|>1$ in the numerator will make the series diverge. Hence the interval is $-3 \le x < -1$.

89. A The series is the Maclaurin expansion of e^{-x}. Use the calculator to solve $e^{-x} = x^3$.

32. B $1+2^x+3^x+4^x+\cdots+n^x+\cdots=\sum_{n=1}^{\infty}\frac{1}{n^p}$ where $p=-x$. This is a *p*-series and is convergent if $p>1 \Rightarrow -x>1 \Rightarrow x<-1$.