UNIT 7

Module

# 25 Proving Theorems about Parallelograms

COMMON CORE GPS

Contents



MATHEMATICAL PRACTICES

The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop. Opportunities to develop these practices are integrated throughout this program.

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

# Unpacking the Standards

my.hrw.com
Multilingual Glossary

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.

COMMON CORE GPS **MCC9-12.G.CO.11**

Prove theorems about parallelograms.

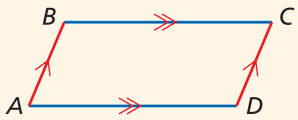
**Key Vocabulary**

**parallelogram** (paralelogramo)
A quadrilateral with two pairs of parallel sides.

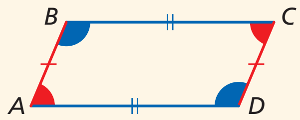
### What It Means For You

Parallelograms, including rectangles and squares, are everywhere around you. You can prove the many special relationships about their sides and angles that make them so important.

**EXAMPLE**




$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$




$\overline{AB} \cong \overline{CD}$ $\angle A \cong \angle C$

$\overline{BC} \cong \overline{DA}$ $\angle B \cong \angle D$

25-1 Geometry TASK

*Use with Properties of Parallelograms*

# Explore Properties of Parallelograms

In this task, you will investigate the relationships among the angles and sides of a special type of quadrilateral called a *parallelogram*. You will need to apply the Transitive Property of Congruence. That is, if figure $A \cong$ figure $B$ and figure $B \cong$ figure $C$, then figure $A \cong$ figure $C$.

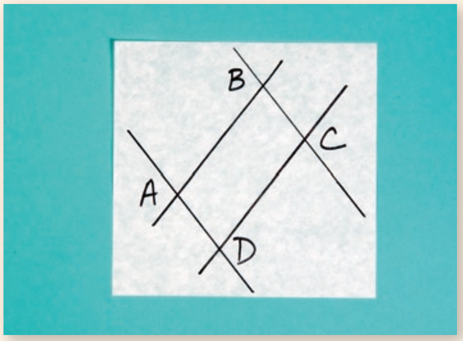
MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

**MCC9-12.G.CO.11** Prove theorems about parallelograms.

## Activity

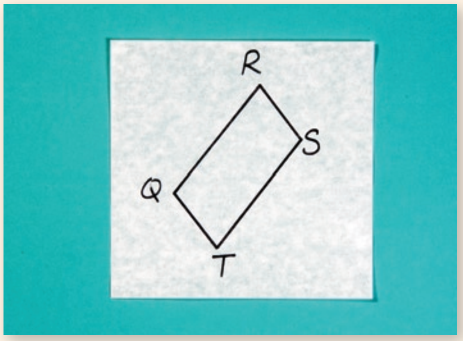
1. Use opposite sides of an index card to draw a set of parallel lines on a piece of patty paper. Then use opposite sides of a ruler to draw a second set of parallel lines that intersects the first. Label the points of intersection $A$, $B$, $C$, and $D$, in that order. Quadrilateral $ABCD$ has two pairs of parallel sides. It is a *parallelogram*.

2. Place a second piece of patty paper over the first and trace $ABCD$. Label the points that correspond to $A$, $B$, $C$, and $D$ as $Q$, $R$, $S$, and $T$, in that order. The parallelograms $ABCD$ and $QRST$ are congruent. Name all the pairs of congruent corresponding sides and angles.




3. Lay $ABCD$ over $QRST$ so that $\overline{AB}$ overlays $\overline{ST}$. What do you notice about their lengths? What does this tell you about $\overline{AB}$ and $\overline{CD}$? Now move $ABCD$ so that $\overline{DA}$ overlays $\overline{RS}$. What do you notice about their lengths? What does this tell you about $\overline{DA}$ and $\overline{BC}$?

4. Lay $ABCD$ over $QRST$ so that $\angle A$ overlays $\angle S$. What do you notice about their measures? What does this tell you about $\angle A$ and $\angle C$? Now move $ABCD$ so that $\angle B$ overlays $\angle T$. What do you notice about their measures? What does this tell you about $\angle B$ and $\angle D$?




5. Arrange the pieces of patty paper so that $\overline{RS}$ overlays $\overline{AD}$. What do you notice about $\overline{QR}$ and $\overline{AB}$? What does this tell you about $\angle A$ and $\angle R$? What can you conclude about $\angle A$ and $\angle B$?

6. Draw diagonals $\overline{AC}$ and $\overline{BD}$. Fold $ABCD$ so that $A$ matches $C$, making a crease. Unfold the paper and fold it again so that $B$ matches $D$, making another crease. What do you notice about the creases? What can you conclude about the diagonals?

## Try This

1. Repeat the above steps with a different parallelogram. Do you get the same results?

2. **Make a Conjecture** How do you think the sides of a parallelogram are related to each other? the angles? the diagonals? Write your conjectures as conditional statements.

# 25-1 Properties of Parallelograms

***Essential Question:*** If a quadrilateral is a parallelogram, what are some conclusions you can make about its angles, sides, and diagonals?

***Objectives***
Prove and apply properties of parallelograms.

Use properties of parallelograms to solve problems.

***Vocabulary***
parallelogram

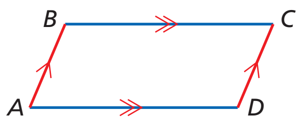
**Who uses this?**
Race car designers can use a parallelogram-shaped linkage to keep the wheels of the car vertical on uneven surfaces.
(See Example 1.)

Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names.

**Helpful Hint**
Opposite sides of a quadrilateral do not share a vertex. Opposite angles do not share a side.

A quadrilateral with two pairs of parallel sides is a **parallelogram**. To write the name of a parallelogram, you use the symbol ▱.

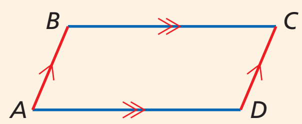
Parallelogram *ABCD*
▱*ABCD*




$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

**Know it! Note**

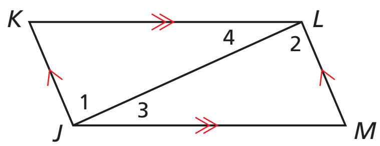
**Theorem 25-1-1** Properties of Parallelograms

| THEOREM | HYPOTHESIS | CONCLUSION |
|---|---|---|
| If a quadrilateral is a parallelogram, then its opposite sides are congruent. (▱ → opp. sides ≅) |   | $\overline{AB} \cong \overline{CD}$ <br> $\overline{BC} \cong \overline{DA}$ |

**PROOF**

**Theorem 25-1-1**

**Given:** *JKLM* is a parallelogram.
**Prove:** $\overline{JK} \cong \overline{LM}$, $\overline{KL} \cong \overline{MJ}$

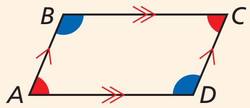
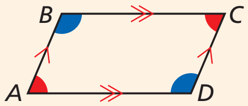
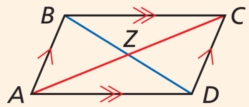



**Proof:**

| Statements | Reasons |
|---|---|
| 1. *JKLM* is a parallelogram. | 1. Given |
| 2. $\overline{JK} \parallel \overline{LM}$, $\overline{KL} \parallel \overline{MJ}$ | 2. Def. of ▱ |
| 3. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ | 3. Alt. Int. ∠s Thm. |
| 4. $\overline{JL} \cong \overline{JL}$ | 4. Reflex. Prop. of ≅ |
| 5. $\triangle JKL \cong \triangle LMJ$ | 5. ASA *Steps 3, 4* |
| 6. $\overline{JK} \cong \overline{LM}$, $\overline{KL} \cong \overline{MJ}$ | 6. CPCTC |

Know it! Note

## Theorems: Properties of Parallelograms

| THEOREM | HYPOTHESIS | CONCLUSION |
|---|---|---|
| **25-1-2** If a quadrilateral is a parallelogram, then its opposite angles are congruent. (▱ → opp. ∠s ≅) |  | $\angle A \cong \angle C$ <br> $\angle B \cong \angle D$ |
| **25-1-3** If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. (▱ → cons. ∠s supp.) |  | $m\angle A + m\angle B = 180°$ <br> $m\angle B + m\angle C = 180°$ <br> $m\angle C + m\angle D = 180°$ <br> $m\angle D + m\angle A = 180°$ |
| **25-1-4** If a quadrilateral is a parallelogram, then its diagonals bisect each other. (▱ → diags. bisect each other) |  | $\overline{AZ} \cong \overline{CZ}$ <br> $\overline{BZ} \cong \overline{DZ}$ |

*You will prove Theorems 25-1-3 and 25-1-4 in Exercises 45 and 44.*

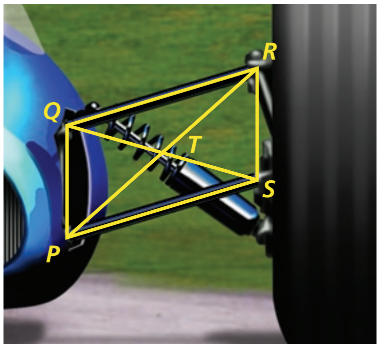
COMMON CORE GPS **EXAMPLE 1** MCC9-12.G.MG.1

my.hrw.com

Online Video Tutor

### *Racing Application*

**The diagram shows the parallelogram-shaped linkage that joins the frame of a race car to one wheel of the car. In ▱*PQRS*, *QR* = 48 cm, *RT* = 30 cm, and m∠*QPS* = 73°. Find each measure.**




**A** *PS*

| | |
|---|---|
| $\overline{PS} \cong \overline{QR}$ | *▱ → opp. sides ≅* |
| $PS = QR$ | *Def. of ≅ segs.* |
| $PS = 48$ cm | *Substitute 48 for QR.* |

**B** $m\angle PQR$

| | |
|---|---|
| $m\angle PQR + m\angle QPS = 180°$ | *▱ → cons. ∠s supp.* |
| $m\angle PQR + 73 = 180$ | *Substitute 73 for m∠QPS.* |
| $m\angle PQR = 107°$ | *Subtract 73 from both sides.* |

**C** *PT*

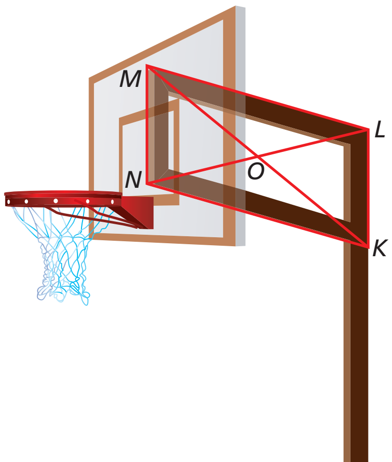
| | |
|---|---|
| $\overline{PT} \cong \overline{RT}$ | *▱ → diags. bisect each other* |
| $PT = RT$ | *Def. of ≅ segs.* |
| $PT = 30$ cm | *Substitute 30 for RT.* |

**CHECK IT OUT!** **In ▱*KLMN*, *LM* = 28 in., *LN* = 26 in., and m∠*LKN* = 74°. Find each measure.**

**1a.** *KN*

**1b.** $m\angle NML$

**1c.** *LO*



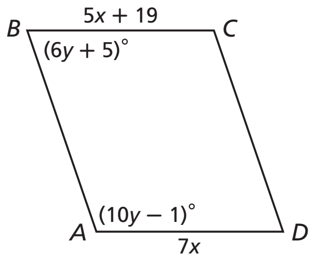

Art Reference: BasketballHoopsUnlimited

COMMON CORE GPS **EXAMPLE 2** MCC9-12.A.CED.1

my.hrw.com
Online Video Tutor

## Using Properties of Parallelograms to Find Measures

***ABCD* is a parallelogram. Find each measure.**




**A** $AD$

$\overline{AD} \cong \overline{BC}$ — *▱ → opp. sides ≅*

$AD = BC$ — *Def. of ≅ segs.*

$7x = 5x + 19$ — *Substitute the given values.*

$2x = 19$ — *Subtract 5x from both sides.*

$x = 9.5$ — *Divide both sides by 2.*

$AD = 7x = 7(9.5) = 66.5$

**B** $m\angle B$

$m\angle A + m\angle B = 180°$ — *▱ → cons. ∡ supp.*

$(10y - 1) + (6y + 5) = 180$ — *Substitute the given values.*

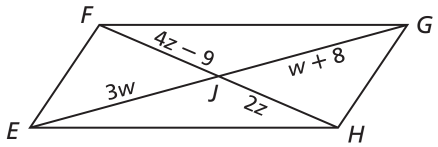
$16y + 4 = 180$ — *Combine like terms.*

$16y = 176$ — *Subtract 4 from both sides.*

$y = 11$ — *Divide both sides by 16.*

$m\angle B = (6y + 5)° = [6(11) + 5]° = 71°$

**CHECK IT OUT!** ***EFGH* is a parallelogram. Find each measure.**




**2a.** $JG$

**2b.** $FH$

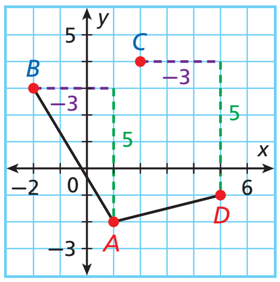
COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.GPE.4

my.hrw.com
Online Video Tutor

## Parallelograms in the Coordinate Plane

**Three vertices of ▱*ABCD* are $A(1, -2)$, $B(-2, 3)$, and $D(5, -1)$. Find the coordinates of vertex *C*.**

Since *ABCD* is a parallelogram, both pairs of opposite sides must be parallel.




**Step 1** Graph the given points.

**Step 2** Find the slope of $\overline{AB}$ by counting the units from *A* to *B*.
The rise from −2 to 3 is 5.
The run from 1 to −2 is −3.

**Step 3** Start at *D* and count the same number of units.
A rise of 5 from −1 is 4.
A run of −3 from 5 is 2. Label (2, 4) as vertex *C*.

**Step 4** Use the slope formula to verify that $\overline{BC} \parallel \overline{AD}$.

$$\text{slope of } \overline{BC} = \frac{4 - 3}{2 - (-2)} = \frac{1}{4}$$

$$\text{slope of } \overline{AD} = \frac{-1 - (-2)}{5 - 1} = \frac{1}{4}$$

The coordinates of vertex *C* are (2, 4).

**Remember!**
When you are drawing a figure in the coordinate plane, the name *ABCD* gives the order of the vertices.

**CHECK IT OUT!** **3.** Three vertices of ▱*PQRS* are $P(-3, -2)$, $Q(-1, 4)$, and $S(5, 0)$. Find the coordinates of vertex *R*.

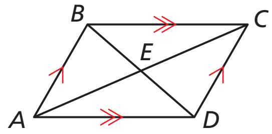
COMMON CORE GPS **EXAMPLE 4** MCC9-12.G.CO.11

## Using Properties of Parallelograms in a Proof

my.hrw.com

Online Video Tutor

Write a two-column proof.




**A** Theorem 25-1-2

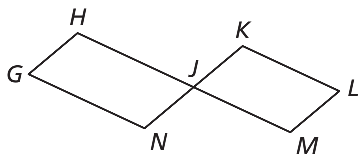
Given: *ABCD* is a parallelogram.

Prove: $\angle BAD \cong \angle DCB$, $\angle ABC \cong \angle CDA$

Proof:

| Statements | Reasons |
|---|---|
| 1. *ABCD* is a parallelogram. | 1. Given |
| 2. $\overline{AB} \cong \overline{CD}$, $\overline{DA} \cong \overline{BC}$ | 2. ▱ → opp. sides ≅ |
| 3. $\overline{BD} \cong \overline{BD}$ | 3. Reflex. Prop. of ≅ |
| 4. $\triangle BAD \cong \triangle DCB$ | 4. SSS *Steps 2, 3* |
| 5. $\angle BAD \cong \angle DCB$ | 5. CPCTC |
| 6. $\overline{AC} \cong \overline{AC}$ | 6. Reflex. Prop. of ≅ |
| 7. $\triangle ABC \cong \triangle CDA$ | 7. SSS *Steps 2, 6* |
| 8. $\angle ABC \cong \angle CDA$ | 8. CPCTC |

**B** Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.




Prove: $\angle G \cong \angle L$

Proof:

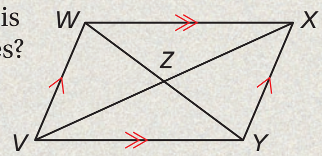
| Statements | Reasons |
|---|---|
| 1. *GHJN* and *JKLM* are parallelograms. | 1. Given |
| 2. $\angle HJN \cong \angle G$, $\angle MJK \cong \angle L$ | 2. ▱ → opp. ∠s ≅ |
| 3. $\angle HJN \cong \angle MJK$ | 3. Vert. ∠s Thm. |
| 4. $\angle G \cong \angle L$ | 4. Trans. Prop. of ≅ |

CHECK IT OUT!

4. Use the figure in Example 4B to write a two-column proof.
Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.
Prove: $\angle N \cong \angle K$

MCC.MP.3 MATHEMATICAL PRACTICES

## THINK AND DISCUSS




1. The measure of one angle of a parallelogram is 71°. What are the measures of the other angles?
2. In ▱*VWXY*, $VW = 21$, and $WY = 36$. Find as many other measures as you can. Justify your answers.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each cell, draw a figure with markings that represents the given property.

| Properties of Parallelograms | | | | |
|---|---|---|---|---|
| Opp. sides ∥ | Opp. sides ≅ | Opp. ∠s ≅ | Cons. ∠s supp. | Diags. bisect each other. |

# 25-1 Exercises

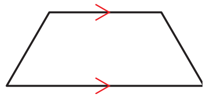
my.hrw.com
Homework Help

## GUIDED PRACTICE

**Vocabulary** **Apply the vocabulary from this lesson to answer each question.**

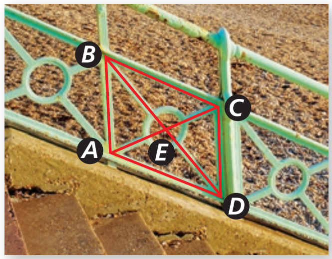
**1.** Explain why the figure at right is NOT a *parallelogram.*

**2.** Draw ▱*PQRS*. Name the opposite sides and opposite angles.



SEE EXAMPLE 1

**Safety** **The handrail is made from congruent parallelograms. In ▱*ABCD*, $AB = 17.5$, $DE = 18$, and $m\angle BCD = 110°$. Find each measure.**



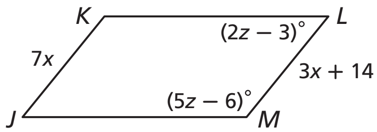

**3.** *BD* **4.** *CD*

**5.** *BE* **6.** $m\angle ABC$

**7.** $m\angle ADC$ **8.** $m\angle DAB$

SEE EXAMPLE 2

***JKLM*** **is a parallelogram. Find each measure.**




**9.** *JK* **10.** *LM*

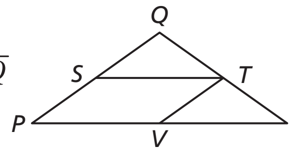
**11.** $m\angle L$ **12.** $m\angle M$

SEE EXAMPLE 3

**13. Multi-Step** Three vertices of ▱*DFGH* are $D(-9, 4)$, $F(-1, 5)$, and $G(2, 0)$. Find the coordinates of vertex *H*.

SEE EXAMPLE 4

**14.** Write a two-column proof.
**Given:** *PSTV* is a parallelogram. $\overline{PQ} \cong \overline{RQ}$
**Prove:** $\angle STV \cong \angle R$

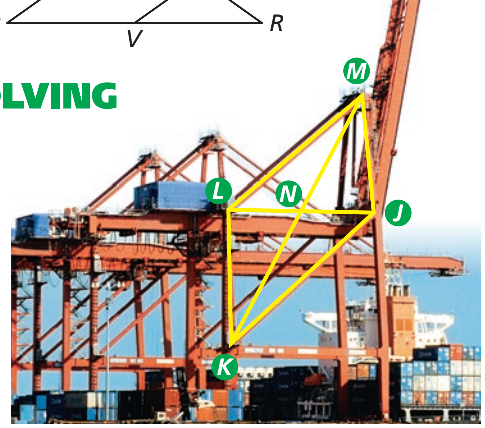



## PRACTICE AND PROBLEM SOLVING

| Independent Practice | |
|---|---|
| For Exercises | See Example |
| 15–20 | 1 |
| 21–24 | 2 |
| 25 | 3 |
| 26 | 4 |

my.hrw.com
Online Extra Practice

**Shipping** **Cranes can be used to load cargo onto ships. In ▱*JKLM*, $JL = 165.8$, $JK = 110$, and $m\angle JML = 50°$. Find the measure of each part of the crane.**

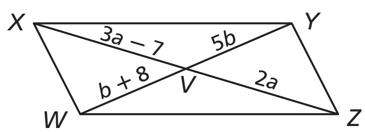



**15.** *JN* **16.** *LM*

**17.** *LN* **18.** $m\angle JKL$

**19.** $m\angle KLM$ **20.** $m\angle MJK$

***WXYZ*** **is a parallelogram. Find each measure.**

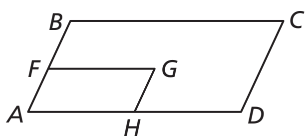



**21.** *WV* **22.** *YW*

**23.** *XZ* **24.** *ZV*

**25. Multi-Step** Three vertices of ▱*PRTV* are $P(-4, -4)$, $R(-10, 0)$, and $V(5, -1)$. Find the coordinates of vertex *T*.

**26.** Write a two-column proof.
**Given:** *ABCD* and *AFGH* are parallelograms.
**Prove:** $\angle C \cong \angle G$



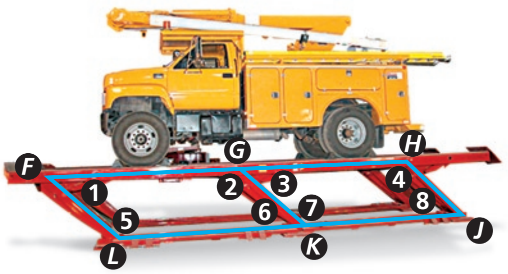
**Algebra** **The perimeter of $\square PQRS$ is 84. Find the length of each side of $\square PQRS$ under the given conditions.**

**27.** $PQ = QR$  **28.** $QR = 3(RS)$  **29.** $RS = SP - 7$  **30.** $SP = RS^2$

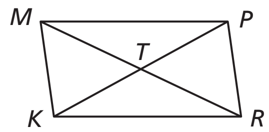
**31. Cars** To repair a large truck, a mechanic might use a *parallelogram lift*. In the lift, $\overline{FG} \cong \overline{GH} \cong \overline{LK} \cong \overline{KJ}$, and $\overline{FL} \cong \overline{GK} \cong \overline{HJ}$.

**a.** Which angles are congruent to $\angle 1$? Justify your answer.

**b.** What is the relationship between $\angle 1$ and each of the remaining labeled angles? Justify your answer.




**Complete each statement about $\square KMPR$. Justify your answer.**



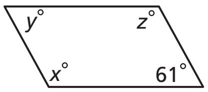

**32.** $\angle MPR \cong$ ___?___  **33.** $\angle PRK \cong$ ___?___  **34.** $\overline{MT} \cong$ ___?___

**35.** $\overline{PR} \cong$ ___?___  **36.** $\overline{MP} \parallel$ ___?___  **37.** $\overline{MK} \parallel$ ___?___

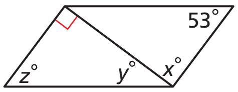
**38.** $\angle MPK \cong$ ___?___  **39.** $\angle MTK \cong$ ___?___  **40.** $m\angle MKR + m\angle PRK =$ ___?___

**Find the values of *x*, *y*, and *z* in each parallelogram.**

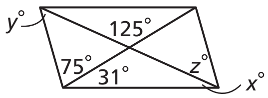
**41.**




**42.**

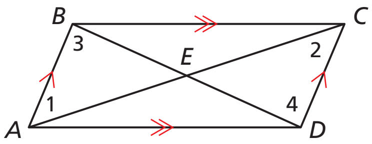



**43.**




**44.** Complete the paragraph proof of Theorem 25-1-4 by filling in the blanks.

**Given:** *ABCD* is a parallelogram.
**Prove:** $\overline{AC}$ and $\overline{BD}$ bisect each other at *E*.

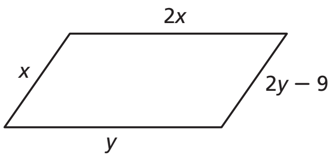



**Proof:** It is given that *ABCD* is a parallelogram. By the definition of a parallelogram, $\overline{AB} \parallel$ **a.** ___?___ . By the Alternate Interior Angles Theorem, $\angle 1 \cong$ **b.** ___?___ , and $\angle 3 \cong$ **c.** ___?___ . $\overline{AB} \cong \overline{CD}$ because **d.** ___?___ . This means that $\triangle ABE \cong \triangle CDE$ by **e.** ___?___ . So by **f.** ___?___ , $\overline{AE} \cong \overline{CE}$, and $\overline{BE} \cong \overline{DE}$. Therefore $\overline{AC}$ and $\overline{BD}$ bisect each other at *E* by the definition of **g.** ___?___ .

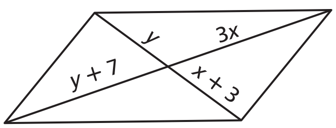
H.O.T. **45.** Write a two-column proof of Theorem 25-1-3: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

**Algebra** **Find the values of *x* and *y* in each parallelogram.**

**46.**




**47.**

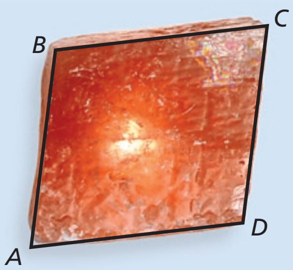



**Real-World Connections**

**48.** In this calcite crystal, the face *ABCD* is a parallelogram.

**a.** In $\square ABCD$, $m\angle B = (6x + 12)^\circ$, and $m\angle D = (9x - 33)^\circ$. Find $m\angle B$.

**b.** Find $m\angle A$ and $m\angle C$. Which theorem or theorems did you use to find these angle measures?




(tr), Photo Courtesy Mohawk Lifts; (bl), HMH Photo; (br), HMH Photo by Peter Van Steen

H.O.T. **49. Critical Thinking** Draw any parallelogram. Draw a second parallelogram whose corresponding sides are congruent to the sides of the first parallelogram but whose corresponding angles are not congruent to the angles of the first.

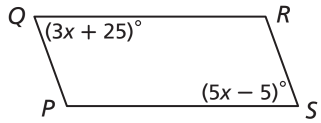
**a.** Is there an SSSS congruence postulate for parallelograms? Explain.

**b.** Remember the meaning of triangle rigidity. Is a parallelogram rigid? Explain.

**50. Write About It** Explain why every parallelogram is a quadrilateral but every quadrilateral is not necessarily a parallelogram.

## TEST PREP

**51.** What is the value of $x$ in $\square PQRS$?




Ⓐ 15 Ⓒ 30

Ⓑ 20 Ⓓ 70

**52.** The diagonals of $\square JKLM$ intersect at $Z$. Which statement is true?

Ⓕ $JL = KM$ Ⓖ $JL = \frac{1}{2}KM$ Ⓗ $JL = \frac{1}{2}JZ$ Ⓙ $JL = 2JZ$

**53. Gridded Response** In $\square ABCD$, $BC = 8.2$, and $CD = 5$. What is the perimeter of $\square ABCD$?

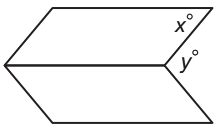
## CHALLENGE AND EXTEND

H.O.T. **The coordinates of three vertices of a parallelogram are given. Give the coordinates for all possible locations of the fourth vertex.**

**54.** $(0, 5), (4, 0), (8, 5)$

**55.** $(-2, 1), (3, -1), (-1, -4)$

H.O.T. **56.** The feathers on an arrow form two congruent parallelograms that share a common side. Each parallelogram is the reflection of the other across the line they share. Show that $y = 2x$.

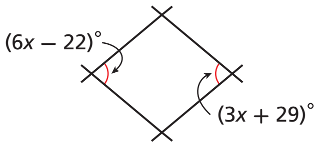



H.O.T. **57.** Prove that the bisectors of two consecutive angles of a parallelogram are perpendicular.

MATHEMATICAL PRACTICES

## FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **58. Problem Solving** A fence uses a pattern of quadrilaterals that are parallelograms like the one shown at the right. Find the value of $x$ and the angle measures of the parallelogram.




H.O.T. **59. Modeling** A parallelogram has one right angle. What is a more specific name for the parallelogram? Justify your answer.

H.O.T. **60. Error Analysis** Tony said the diagonals of a parallelogram are always congruent. Do you agree with Tony? If you disagree, correct his statement.

# 25-2 Conditions for Parallelograms

***Essential Question:*** What information about the angles, sides, or diagonals of a quadrilateral allows you to conclude it is a parallelogram?

***Objective***
Prove that a given quadrilateral is a parallelogram.

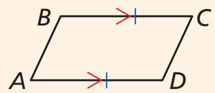
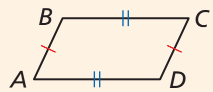
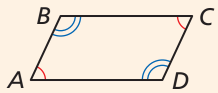
**Who uses this?**
A bird watcher can use a *parallelogram mount* to adjust the height of a pair of binoculars without changing the viewing angle. (See Example 4.)

You have learned to identify the properties of a parallelogram. Now you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. To do this, you can use the definition of a parallelogram or the conditions below.

**Know it! Note**

**Remember!**
In the converse of a theorem, the hypothesis and conclusion are exchanged.

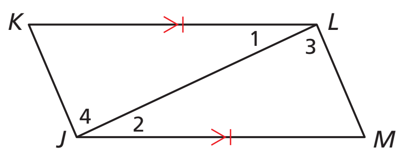
**Theorems** | **Conditions for Parallelograms**

| THEOREM | EXAMPLE |
|---|---|
| **25-2-1** If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides ∥ and ≅ → ▱) |  |
| **25-2-2** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides ≅ → ▱) |  |
| **25-2-3** If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. ∠s ≅ → ▱) |  |

*You will prove Theorems 25-2-2 and 25-2-3 in Exercises 26 and 29.*

**PROOF**

### Theorem 25-2-1



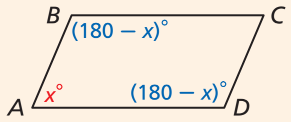
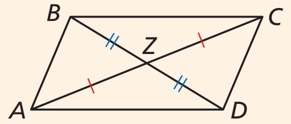

**Given:** $\overline{KL} \parallel \overline{MJ}$, $\overline{KL} \cong \overline{MJ}$
**Prove:** *JKLM* is a parallelogram.

**Proof:**
It is given that $\overline{KL} \cong \overline{MJ}$. Since $\overline{KL} \parallel \overline{MJ}$, $\angle 1 \cong \angle 2$ by the Alternate Interior Angles Theorem. By the Reflexive Property of Congruence, $\overline{JL} \cong \overline{JL}$. So $\triangle JKL \cong \triangle LMJ$ by SAS. By CPCTC, $\angle 3 \cong \angle 4$, and $\overline{JK} \parallel \overline{LM}$ by the Converse of the Alternate Interior Angles Theorem. Since the opposite sides of *JKLM* are parallel, *JKLM* is a parallelogram by definition.

The two theorems below can also be used to show that a given quadrilateral is a parallelogram.

Know it! Note

## Theorems: Conditions for Parallelograms

| THEOREM | EXAMPLE |
|---|---|
| **25-2-4** If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with ∠ supp. to cons. ∠s → ▱) |   |
| **25-2-5** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other → ▱) |   |

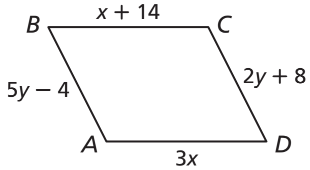
*You will prove Theorems 25-2-4 and 25-2-5 in Exercises 27 and 30.*

COMMON CORE GPS **EXAMPLE 1** MCC9-12.A.CED.1

my.hrw.com

Online Video Tutor

### Verifying Figures are Parallelograms




**A** **Show that *ABCD* is a parallelogram for $x = 7$ and $y = 4$.**

**Step 1** Find *BC* and *DA*.

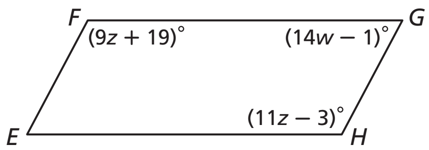
| | | |
|---|---|---|
| $BC = x + 14$ | *Given* | $DA = 3x$ |
| $BC = 7 + 14 = 21$ | *Substitute and simplify.* | $DA = 3x = 3(7) = 21$ |

**Step 2** Find *AB* and *CD*.

| | | |
|---|---|---|
| $AB = 5y - 4$ | *Given* | $CD = 2y + 8$ |
| $AB = 5(4) - 4 = 16$ | *Substitute and simplify.* | $CD = 2(4) + 8 = 16$ |

Since $BC = DA$ and $AB = CD$, *ABCD* is a parallelogram by Theorem 25-2-2.

**B** **Show that *EFGH* is a parallelogram for $z = 11$ and $w = 4.5$.**

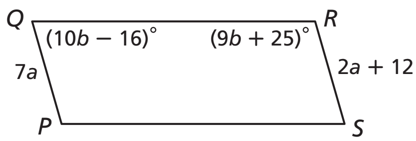



| | |
|---|---|
| $m\angle F = (9z + 19)°$ | *Given* |
| $m\angle F = [9(11) + 19]° = 118°$ | *Substitute 11 for z and simplify.* |
| $m\angle H = (11z - 3)°$ | *Given* |
| $m\angle H = [11(11) - 3]° = 118°$ | *Substitute 11 for z and simplify.* |
| $m\angle G = (14w - 1)°$ | *Given* |
| $m\angle G = [14(4.5) - 1]° = 62°$ | *Substitute 4.5 for w and simplify.* |

Since $118° + 62° = 180°$, $\angle G$ is supplementary to both $\angle F$ and $\angle H$. *EFGH* is a parallelogram by Theorem 25-2-4.

**CHECK IT OUT!**

**1.** Show that *PQRS* is a parallelogram for $a = 2.4$ and $b = 9$.



COMMON CORE GPS **EXAMPLE 2** MCC9-12.G.CO.11

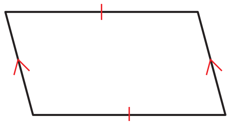
my.hrw.com

Online Video Tutor

## Applying Conditions for Parallelograms

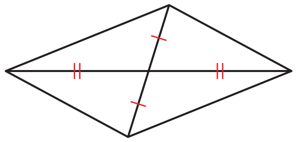
**Determine if each quadrilateral must be a parallelogram. Justify your answer.**

**A**



No. One pair of opposite sides are parallel. A different pair of opposite sides are congruent. The conditions for a parallelogram are not met.

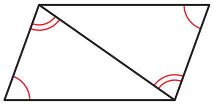
**B**



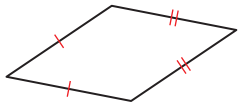
Yes. The diagonals bisect each other. By Theorem 25-2-5, the quadrilateral is a parallelogram.

**CHECK IT OUT!** **Determine if each quadrilateral must be a parallelogram. Justify your answer.**

**2a.**



**2b.**



COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.GPE.4

my.hrw.com

Online Video Tutor

## Proving Parallelograms in the Coordinate Plane

**Show that quadrilateral *ABCD* is a parallelogram by using the given definition or theorem.**

**A** $A(-3, 2), B(-2, 7), C(2, 4), D(1, -1)$; **definition of parallelogram**

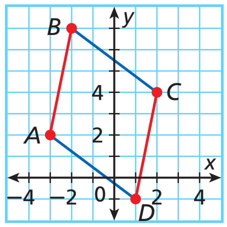
Find the slopes of both pairs of opposite sides.

$$\text{slope of } \overline{AB} = \frac{7-2}{-2-(-3)} = \frac{5}{1} = 5$$

$$\text{slope of } \overline{CD} = \frac{-1-4}{1-2} = \frac{-5}{-1} = 5$$

$$\text{slope of } \overline{BC} = \frac{4-7}{2-(-2)} = \frac{-3}{4} = -\frac{3}{4}$$

$$\text{slope of } \overline{DA} = \frac{2-(-1)}{-3-1} = \frac{3}{-4} = -\frac{3}{4}$$




Since both pairs of opposite sides are parallel, *ABCD* is a parallelogram by definition.

**Helpful Hint**

To say that a quadrilateral is a parallelogram *by definition*, you must show that both pairs of opposite sides are parallel.

**B** $F(-4, -2), G(-2, 2), H(4, 3), J(2, -1)$; **Theorem 25-2-1**

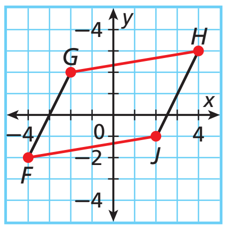
Find the slopes and lengths of one pair of opposite sides.

$$\text{slope of } \overline{GH} = \frac{3-2}{4-(-2)} = \frac{1}{6}$$

$$\text{slope of } \overline{JF} = \frac{-2-(-1)}{-4-2} = \frac{-1}{-6} = \frac{1}{6}$$

$$GH = \sqrt{[4-(-2)]^2 + (3-2)^2} = \sqrt{37}$$

$$JF = \sqrt{(-4-2)^2 + [-2-(-1)]^2} = \sqrt{37}$$




$\overline{GH}$ and $\overline{JF}$ have the same slope, so $\overline{GH} \parallel \overline{JF}$. Since $GH = JF$, $\overline{GH} \cong \overline{JF}$. So by Theorem 25-2-1, *FGHJ* is a parallelogram.

**CHECK IT OUT!** **3.** Use the definition of a parallelogram to show that the quadrilateral with vertices $K(-3, 0)$, $L(-5, 7)$, $M(3, 5)$, and $N(5, -2)$ is a parallelogram.

You have learned several ways to determine whether a quadrilateral is a parallelogram. You can use the given information about a figure to decide which condition is best to apply.

**Helpful Hint**

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

| Conditions for Parallelograms |
|---|
| Both pairs of opposite sides are parallel. (definition) |
| One pair of opposite sides are parallel and congruent. (Theorem 25-2-1) |
| Both pairs of opposite sides are congruent. (Theorem 25-2-2) |
| Both pairs of opposite angles are congruent. (Theorem 25-2-3) |
| One angle is supplementary to both of its consecutive angles. (Theorem 25-2-4) |
| The diagonals bisect each other. (Theorem 25-2-5) |

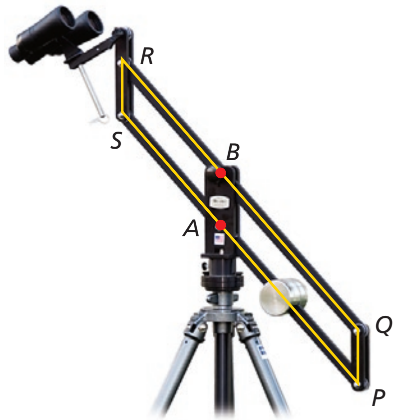
COMMON CORE GPS **EXAMPLE 4** MCC9-12.G.MG.1

my.hrw.com

Online Video Tutor

## *Bird-Watching Application*

**In the parallelogram mount, there are bolts at *P*, *Q*, *R*, and *S* such that *PQ* = *RS* and *QR* = *SP*. The frame *PQRS* moves when you raise or lower the binoculars. Why is *PQRS* always a parallelogram?**




When you move the binoculars, the angle measures change, but *PQ*, *QR*, *RS*, and *SP* stay the same. So it is always true that *PQ* = *RS* and *QR* = *SP*. Since both pairs of opposite sides of the quadrilateral are congruent, *PQRS* is always a parallelogram.

**CHECK IT OUT!** **4.** The frame is attached to the tripod at points *A* and *B* such that *AB* = *RS* and *BR* = *SA*. So *ABRS* is also a parallelogram. How does this ensure that the angle of the binoculars stays the same?

MCC.MP.2 MATHEMATICAL PRACTICES

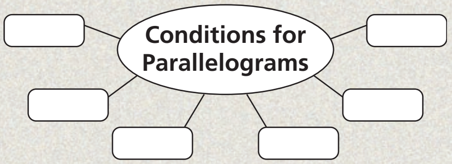
## THINK AND DISCUSS

**1.** What do all the theorems in this lesson have in common?

**2.** How are the theorems in this lesson different from the theorems in the lesson *Properties of Parallelograms*?

Know it! Note

**3. GET ORGANIZED** Copy and complete the graphic organizer. In each box, write one of the six conditions for a parallelogram. Then sketch a parallelogram and label it to show how it meets the condition.



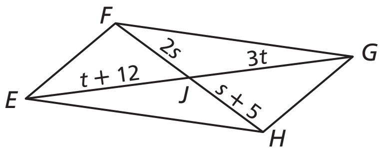
# 25-2 Exercises

my.hrw.com
Homework Help

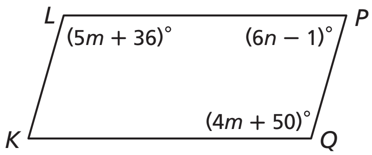
## GUIDED PRACTICE

SEE EXAMPLE 1

1. Show that *EFGH* is a parallelogram for $s = 5$ and $t = 6$.



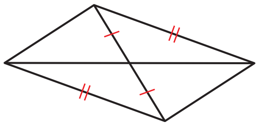

2. Show that *KLPQ* is a parallelogram for $m = 14$ and $n = 12.5$.

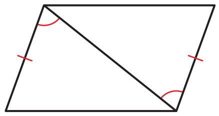



SEE EXAMPLE 2

**Determine if each quadrilateral must be a parallelogram. Justify your answer.**

3. 

4. 

5. 

SEE EXAMPLE 3

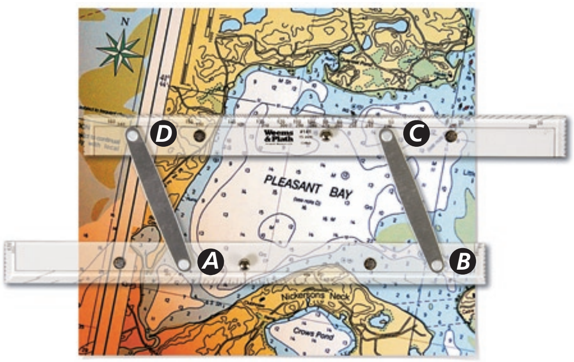
**Show that the quadrilateral with the given vertices is a parallelogram.**

6. $W(-5, -2)$, $X(-3, 3)$, $Y(3, 5)$, $Z(1, 0)$

7. $R(-1, -5)$, $S(-2, -1)$, $T(4, -1)$, $U(5, -5)$

SEE EXAMPLE 4

8. **Navigation** A parallel rule can be used to plot a course on a navigation chart. The tool is made of two rulers connected at hinges to two congruent crossbars $\overline{AD}$ and $\overline{BC}$. You place the edge of one ruler on your desired course and then move the second ruler over the compass rose on the chart to read the bearing for your course. If $\overline{AD} \parallel \overline{BC}$, why is $\overline{AB}$ always parallel to $\overline{CD}$?



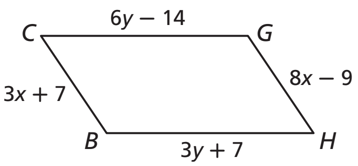

## PRACTICE AND PROBLEM SOLVING

**Independent Practice**

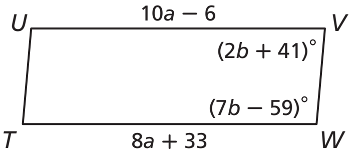
| For Exercises | See Example |
|---|---|
| 9–10 | 1 |
| 11–13 | 2 |
| 14–15 | 3 |
| 16 | 4 |

my.hrw.com
Online Extra Practice

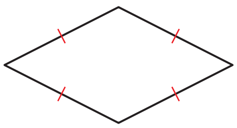
9. Show that *BCGH* is a parallelogram for $x = 3.2$ and $y = 7$.

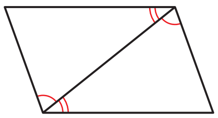


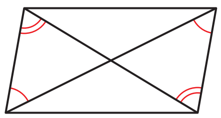

10. Show that *TUVW* is a parallelogram for for $a = 19.5$ and $b = 22$.




**Determine if each quadrilateral must be a parallelogram. Justify your answer.**

11. 

12. 

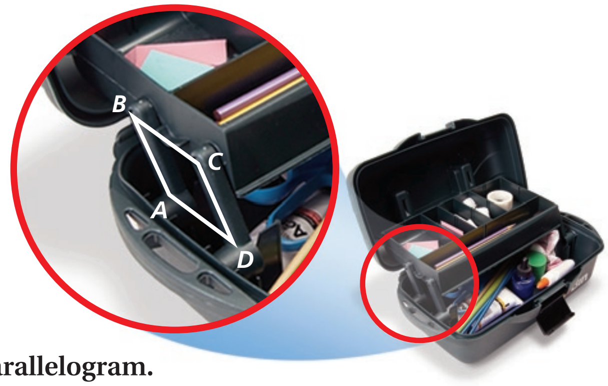
13. 

**Show that the quadrilateral with the given vertices is a parallelogram.**

14. $J(-1, 0)$, $K(-3, 7)$, $L(2, 6)$, $M(4, -1)$

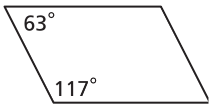
15. $P(-8, -4)$, $Q(-5, 1)$, $R(1, -5)$, $S(-2, -10)$

16. **Design** The toolbox has cantilever trays that pull away from the box so that you can reach the items beneath them. Two congruent brackets connect each tray to the box. Given that $AD = BC$, how do the brackets $\overline{AB}$ and $\overline{CD}$ keep the tray horizontal?

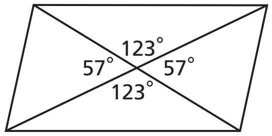



**Determine if each quadrilateral must be a parallelogram. Justify your answer.**

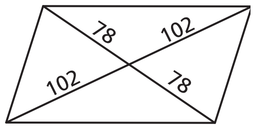
17.




18.

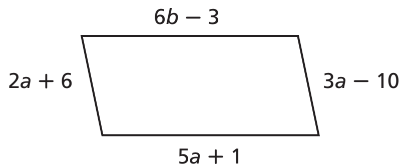



19.

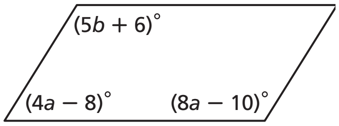



H.O.T. **Algebra** **Find the values of *a* and *b* that would make the quadrilateral a parallelogram.**

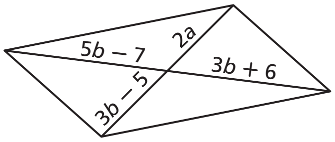
20.



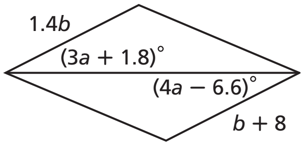

21.




22.




23.



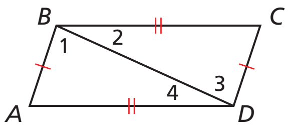

24. **Critical Thinking** Draw a quadrilateral that has congruent diagonals but is not a parallelogram. What can you conclude about using congruent diagonals as a condition for a parallelogram?

25. **Social Studies** The angles at the corners of the flag of the Republic of the Congo are right angles. The red and green triangles are congruent isosceles right triangles. Why is the shape of the yellow stripe a parallelogram?

26. Complete the two-column proof of Theorem 25-2-2 by filling in the blanks.

**Given:** $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$

**Prove:** *ABCD* is a parallelogram.



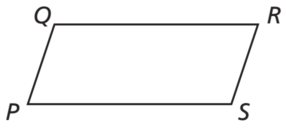

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$ | 1. Given |
| 2. $\overline{BD} \cong \overline{BD}$ | 2. **a.** ___?___ |
| 3. $\triangle DAB \cong$ **b.** ___?___ | 3. **c.** ___?___ |
| 4. $\angle 1 \cong$ **d.** ___?___ , $\angle 4 \cong$ **e.** ___?___ | 4. CPCTC |
| 5. $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$ | 5. **f.** ___?___ |
| 6. *ABCD* is a parallelogram. | 6. **g.** ___?___ |

**27.** Complete the paragraph proof of Theorem 25-2-4 by filling in the blanks.

**Given:** $\angle P$ is supplementary to $\angle Q$.
$\angle P$ is supplementary to $\angle S$.

**Prove:** $PQRS$ is a parallelogram.




**Proof:**

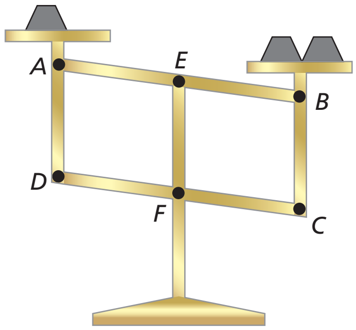
It is given that $\angle P$ is supplementary to **a.** ___?___ and **b.** ___?___ . By the Converse of the Same-Side Interior Angles Theorem, $\overline{QR} \parallel$ **c.** ___?___ and $\overline{PQ} \parallel$ **d.** ___?___ . So $PQRS$ is a parallelogram by the definition of **e.** ___?___ .

**LINK**

**Measurement**

Ancient balance scales had one beam that moved on a single hinge. The stress on the hinge often made the scale imprecise.

**28.** **Measurement** In the eighteenth century, Gilles Personne de Roberval designed a scale with two beams and two hinges. In $\square ABCD$, $E$ is the midpoint of $\overline{AB}$, and $F$ is the midpoint of $\overline{CD}$. Write a paragraph proof that $AEFD$ and $EBCF$ are parallelograms.

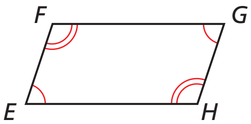



**Prove each theorem.**

**29.** Theorem 25-2-3

**Given:** $\angle E \cong \angle G$, $\angle F \cong \angle H$

**Prove:** $EFGH$ is a parallelogram.

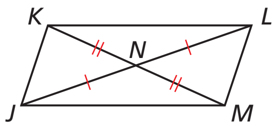



**Plan:** Show that the sum of the interior angles of $EFGH$ is 360°. Then apply properties of equality to show that $m\angle E + m\angle F = 180°$ and $m\angle E + m\angle H = 180°$. Then you can conclude that $\overline{EF} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{HE}$.

**30.** Theorem 25-2-5

**Given:** $\overline{JL}$ and $\overline{KM}$ bisect each other.

**Prove:** $JKLM$ is a parallelogram.




**Plan:** Show that $\triangle JNK \cong \triangle LNM$ and $\triangle KNL \cong \triangle MNJ$. Then use the fact that the corresponding angles are congruent to show $\overline{JK} \parallel \overline{LM}$ and $\overline{KL} \parallel \overline{MJ}$.

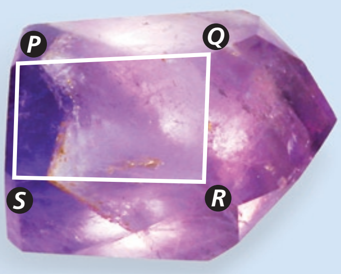
**H.O.T.** **31.** Prove that the figure formed by two midsegments of a triangle and their corresponding bases is a parallelogram.

**32.** **Write About It** Use the theorems about properties of parallelograms to write three biconditional statements about parallelograms.

**H.O.T.** **33.** **Construction** Explain how you can construct a parallelogram based on the conditions of Theorem 25-2-1. Use your method to construct a parallelogram.

**Real-World Connections**

**34.** A geologist made the following observations while examining this amethyst crystal. Tell whether each set of observations allows the geologist to conclude that $PQRS$ is a parallelogram. If so, explain why.



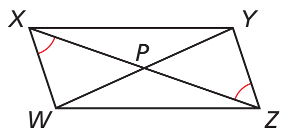

**a.** $\overline{PQ} \cong \overline{SR}$, and $\overline{PS} \parallel \overline{QR}$.

**b.** $\angle S$ and $\angle R$ are supplementary, and $\overline{PS} \cong \overline{QR}$.

**c.** $\angle S \cong \angle Q$, and $\overline{PQ} \parallel \overline{SR}$.

## TEST PREP

**35.** What additional information would allow you to conclude that $WXYZ$ is a parallelogram?




Ⓐ $\overline{XY} \cong \overline{ZW}$ Ⓒ $\overline{WY} \cong \overline{WZ}$

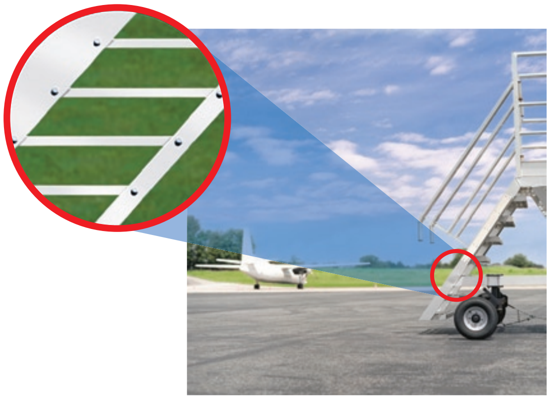
Ⓑ $\overline{WX} \cong \overline{YZ}$ Ⓓ $\angle XWY \cong \angle ZYW$

**36.** Which could be the coordinates of the fourth vertex of $\square ABCD$ with $A(-1, -1)$, $B(1, 3)$, and $C(6, 1)$?

Ⓕ $D(8, 5)$ Ⓖ $D(4, -3)$ Ⓗ $D(13, 3)$ Ⓙ $D(3, 7)$

**37.** **Short Response** The vertices of quadrilateral $RSTV$ are $R(-5, 0)$, $S(-1, 3)$, $T(5, 1)$, and $V(2, -2)$. Is $RSTV$ a parallelogram? Justify your answer.

## CHALLENGE AND EXTEND



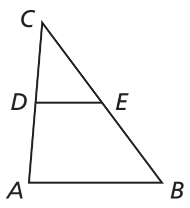
**38.** **Write About It** As the upper platform of the movable staircase is raised and lowered, the height of each step changes. How does the upper platform remain parallel to the ground?

H.O.T. **39.** **Multi-Step** The diagonals of a parallelogram intersect at $(-2, 1.5)$. Two vertices are located at $(-7, 2)$ and $(2, 6.5)$. Find the coordinates of the other two vertices.

H.O.T. **40.** **Given:** $D$ is the midpoint of $\overline{AC}$, and $E$ is the midpoint of $\overline{BC}$.

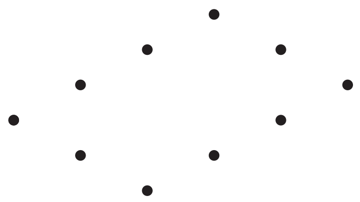
**Prove:** $\overline{DE} \parallel \overline{AB}$, $DE = \frac{1}{2}AB$

(*Hint:* Extend $\overline{DE}$ to form $\overline{DF}$ so that $\overline{EF} \cong \overline{DE}$. Then show that $DFBA$ is a parallelogram.)




## MATHEMATICAL PRACTICES — FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **41.** **Proof** A precision ice skating team with 10 members formed the figure shown. The skaters positioned themselves along four lines, and the space between each pair of adjacent skaters was 3 feet. Prove that the skaters formed a parallelogram.

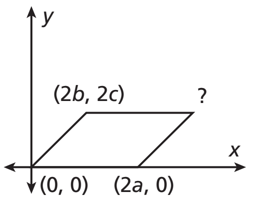


H.O.T. **42.** **Problem Solving** The figure shows a parallelogram.

**a.** Find the coordinates of the fourth vertex.

**b.** Find the midpoints of the sides of the parallelogram.

**c.** Show that the quadrilateral formed by connecting the midpoints of adjacent sides is also a parallelogram.



# 25-3 Properties of Special Parallelograms

***Essential Question:*** What are the geometric properties of rectangles, rhombuses, and squares?

**Objectives**

Prove and apply properties of rectangles, rhombuses, and squares.

Use properties of rectangles, rhombuses, and squares to solve problems.

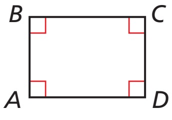
**Vocabulary**

rectangle
rhombus
square

**Who uses this?**

Artists who work with stained glass can use properties of rectangles to cut materials to the correct sizes.

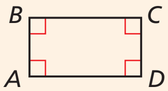
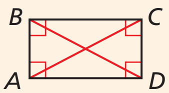
A second type of special quadrilateral is a *rectangle*. A **rectangle** is a quadrilateral with four right angles.



Rectangle *ABCD*

Know it! Note

**Theorems** Properties of Rectangles

| THEOREM | | HYPOTHESIS | CONCLUSION |
|---|---|---|---|
| **25-3-1** | If a quadrilateral is a rectangle, then it is a parallelogram. (rect. → ▱) |  | *ABCD* is a parallelogram. |
| **25-3-2** | If a parallelogram is a rectangle, then its diagonals are congruent. (rect. → diags. ≅) |  | $\overline{AC} \cong \overline{BD}$ |

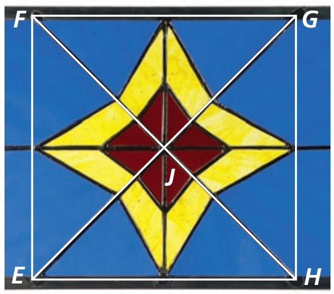
*You will prove Theorems 25-3-1 and 25-3-2 in Exercises 38 and 35.*

Since a rectangle is a parallelogram by Theorem 25-3-1, a rectangle "inherits" all the properties of parallelograms.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.G.MG.1

### *Craft Application*

**An artist connects stained glass pieces with lead strips. In this rectangular window, the strips are cut so that $FG = 24$ in. and $FH = 34$ in. Find $JG$.**



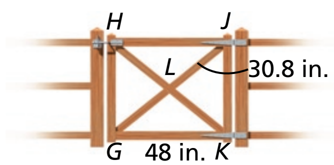

| | |
|---|---|
| $\overline{EG} \cong \overline{FH}$ | *Rect. → diags. ≅* |
| $EG = FH = 34$ | *Def. of ≅ segs.* |
| $JG = \frac{1}{2}EG$ | *▱ → diags. bisect each other* |
| $JG = \frac{1}{2}(34) = 17$ in. | *Substitute and simplify.* |

my.hrw.com

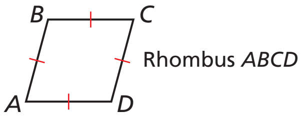
Online Video Tutor

CHECK IT OUT! **Carpentry** The rectangular gate has diagonal braces. Find each length.

**1a.** $HJ$ **1b.** $HK$

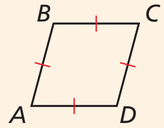
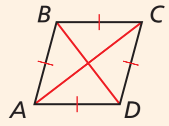
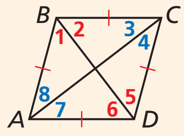


A *rhombus* is another special quadrilateral. A **rhombus** is a quadrilateral with four congruent sides.




**Know it! Note**

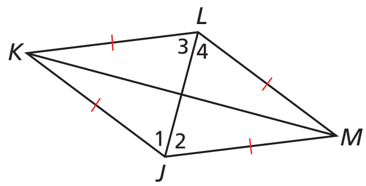
## Theorems — Properties of Rhombuses

| THEOREM | HYPOTHESIS | CONCLUSION |
|---|---|---|
| **25-3-3** If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus → ▱) |  | *ABCD* is a parallelogram. |
| **25-3-4** If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus → diags. ⊥) |  | $\overline{AC} \perp \overline{BD}$ |
| **25-3-5** If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus → each diag. bisects opp. ∠s) |  | $\angle 1 \cong \angle 2$<br>$\angle 3 \cong \angle 4$<br>$\angle 5 \cong \angle 6$<br>$\angle 7 \cong \angle 8$ |

*You will prove Theorems 25-3-3 and 25-3-4 in Exercises 34 and 37.*

**PROOF**

### Theorem 25-3-5




**Given:** *JKLM* is a rhombus.
**Prove:** $\overline{JL}$ bisects $\angle KJM$ and $\angle KLM$.
$\overline{KM}$ bisects $\angle JKL$ and $\angle JML$.

**Proof:**

Since *JKLM* is a rhombus, $\overline{JK} \cong \overline{JM}$, and $\overline{KL} \cong \overline{ML}$ by the definition of a rhombus. By the Reflexive Property of Congruence, $\overline{JL} \cong \overline{JL}$. Thus $\triangle JKL \cong \triangle JML$ by SSS. Then $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$ by CPCTC. So $\overline{JL}$ bisects $\angle KJM$ and $\angle KLM$ by the definition of an angle bisector. By similar reasoning, $\overline{KM}$ bisects $\angle JKL$ and $\angle JML$.

Like a rectangle, a rhombus is a parallelogram. So you can apply the properties of parallelograms to rhombuses.

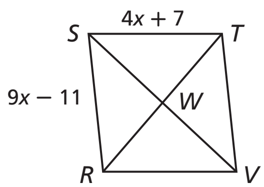
COMMON CORE GPS **EXAMPLE 2** MCC9-12.A.CED.1

my.hrw.com

Online Video Tutor

### Using Properties of Rhombuses to Find Measures

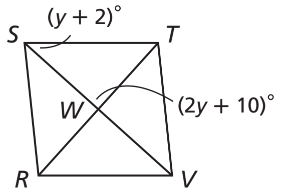
**RSTV is a rhombus. Find each measure.**




**A** *VT*

| | |
|---|---|
| $ST = SR$ | *Def. of rhombus* |
| $4x + 7 = 9x - 11$ | *Substitute the given values.* |
| $18 = 5x$ | *Subtract 4x from both sides and add 11 to both sides.* |
| $3.6 = x$ | *Divide both sides by 5.* |
| $VT = ST$ | *Def. of rhombus* |
| $VT = 4x + 7$ | *Substitute 4x + 7 for ST.* |
| $VT = 4(3.6) + 7 = 21.4$ | *Substitute 3.6 for x and simplify.* |

***RSTV* is a rhombus. Find each measure.**

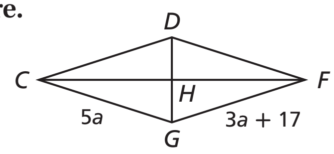



**B** **m∠*WSR***

| | |
|---|---|
| $m\angle SWT = 90°$ | *Rhombus → diags. ⊥* |
| $2y + 10 = 90$ | *Substitute 2y + 10 for m∠SWT.* |
| $y = 40$ | *Subtract 10 from both sides and divide both sides by 2.* |
| $m\angle WSR = m\angle TSW$ | *Rhombus → each diag. bisects opp. ∠s* |
| $m\angle WSR = (y + 2)°$ | *Substitute y + 2 for m∠TSW.* |
| $m\angle WSR = (40 + 2)° = 42°$ | *Substitute 40 for y and simplify.* |

CHECK IT OUT!

***CDFG* is a rhombus. Find each measure.**



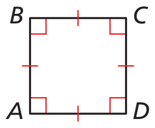

**2a.** *CD*

**2b.** $m\angle GCH$ if $m\angle GCD = (b + 3)°$ and $m\angle CDF = (6b - 40)°$

**Helpful Hint**

Rectangles, rhombuses, and squares are sometimes referred to as *special parallelograms*.

A **square** is a quadrilateral with four right angles and four congruent sides. In the exercises, you will show that a square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.




Square *ABCD*

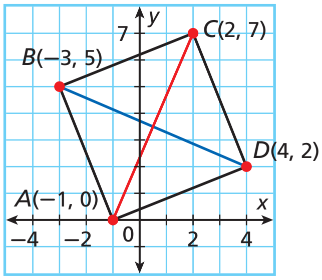
COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.GPE.4

## Verifying Properties of Squares

my.hrw.com

Online Video Tutor

**Show that the diagonals of square *ABCD* are congruent perpendicular bisectors of each other.**




**Step 1** Show that $\overline{AC}$ and $\overline{BD}$ are congruent.

$$AC = \sqrt{[2 - (-1)]^2 + (7 - 0)^2} = \sqrt{58}$$

$$BD = \sqrt{[4 - (-3)]^2 + (2 - 5)^2} = \sqrt{58}$$

Since $AC = BD$, $\overline{AC} \cong \overline{BD}$.

**Step 2** Show that $\overline{AC}$ and $\overline{BD}$ are perpendicular.

$$\text{slope of } \overline{AC} = \frac{7 - 0}{2 - (-1)} = \frac{7}{3}$$

$$\text{slope of } \overline{BD} = \frac{2 - 5}{4 - (-3)} = \frac{-3}{7} = -\frac{3}{7}$$

Since $\left(\frac{7}{3}\right)\left(-\frac{3}{7}\right) = -1$, $\overline{AC} \perp \overline{BD}$.

**Step 3** Show that $\overline{AC}$ and $\overline{BD}$ bisect each other.

$$\text{mdpt. of } \overline{AC}: \left(\frac{-1 + 2}{2}, \frac{0 + 7}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

$$\text{mdpt. of } \overline{BD}: \left(\frac{-3 + 4}{2}, \frac{5 + 2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

Since $\overline{AC}$ and $\overline{BD}$ have the same midpoint, they bisect each other. The diagonals are congruent perpendicular bisectors of each other.

CHECK IT OUT!

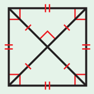
**3.** The vertices of square *STVW* are $S(-5, -4)$, $T(0, 2)$, $V(6, -3)$, and $W(1, -9)$. Show that the diagonals of square *STVW* are congruent perpendicular bisectors of each other.

## Student to Student

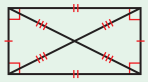
### *Special Parallelograms*

**Taylor Gallinghouse**
Central High School

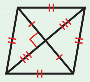
*To remember the properties of rectangles, rhombuses, and squares, I start with a* ***square****, which has all the properties of the others.*



*To get a* ***rectangle*** *that is not a square, I stretch the square in one direction. Its diagonals are still congruent, but they are no longer perpendicular.*



*To get a* ***rhombus*** *that is not a square, I go back to the square and slide the top in one direction. Its diagonals are still perpendicular and bisect the opposite angles, but they aren't congruent.*



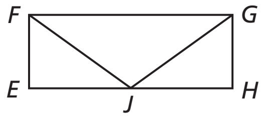
COMMON CORE GPS **EXAMPLE 4** MCC9-12.G.CO.11

### Using Properties of Special Parallelograms in Proofs

my.hrw.com

Online Video Tutor

**Given:** *EFGH* **is a rectangle.** *J* **is the midpoint of** $\overline{EH}$.
**Prove:** △*FJG* **is isosceles.**

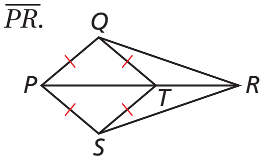



**Proof:**

| Statements | Reasons |
|---|---|
| **1.** *EFGH* is a rectangle. *J* is the midpoint of $\overline{EH}$. | **1.** Given |
| **2.** ∠*E* and ∠*H* are right angles. | **2.** Def. of rect. |
| **3.** ∠*E* ≅ ∠*H* | **3.** Rt. ∠ ≅ Thm. |
| **4.** *EFGH* is a parallelogram. | **4.** Rect. → ▱ |
| **5.** $\overline{EF} \cong \overline{HG}$ | **5.** ▱ → opp. sides ≅ |
| **6.** $\overline{EJ} \cong \overline{HJ}$ | **6.** Def. of mdpt. |
| **7.** △*FJE* ≅ △*GJH* | **7.** SAS *Steps 3, 5, 6* |
| **8.** $\overline{FJ} \cong \overline{GJ}$ | **8.** CPCTC |
| **9.** △*FJG* is isosceles. | **9.** Def. of isosc. △ |

CHECK IT OUT!

**4.** **Given:** *PQTS* is a rhombus with diagonal $\overline{PR}$.
**Prove:** $\overline{RQ} \cong \overline{RS}$




MCC.MP.7 MATHEMATICAL PRACTICES

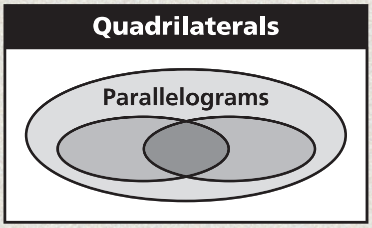
## THINK AND DISCUSS

**1.** Which theorem means "The diagonals of a rectangle are congruent"? Why do you think the theorem is written as a conditional?

**2.** What properties of a rhombus are the same as the properties of all parallelograms? What special properties does a rhombus have?

Know it! Note

**3. GET ORGANIZED** Copy and complete the graphic organizer. Write the missing terms in the three unlabeled sections. Then write a definition of each term.



# 25-3 Exercises

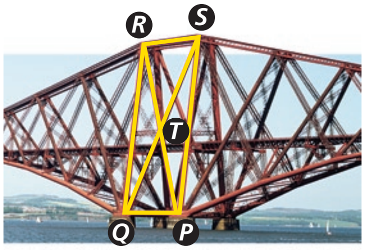
my.hrw.com
Homework Help

## GUIDED PRACTICE

1. **Vocabulary** What is another name for an *equilateral quadrilateral*? an *equiangular quadrilateral*? a *regular quadrilateral*?

SEE EXAMPLE 1

**Engineering** The braces of the bridge support lie along the diagonals of rectangle *PQRS*. *RS* = 160 ft, and *QS* = 380 ft. Find each length.

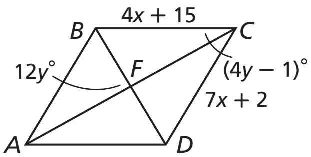



2. *TQ*
3. *PQ*
4. *ST*
5. *PR*

SEE EXAMPLE 2

***ABCD* is a rhombus. Find each measure.**

6. *AB*
7. m∠*ABC*

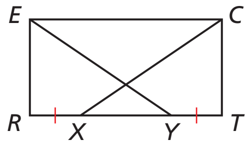



SEE EXAMPLE 3

8. **Multi-Step** The vertices of square *JKLM* are $J(-3, -5)$, $K(-4, 1)$, $L(2, 2)$, and $M(3, -4)$. Show that the diagonals of square *JKLM* are congruent perpendicular bisectors of each other.

SEE EXAMPLE 4

9. **Given:** *RECT* is a rectangle. $\overline{RX} \cong \overline{TY}$
**Prove:** △*REY* ≅ △*TCX*

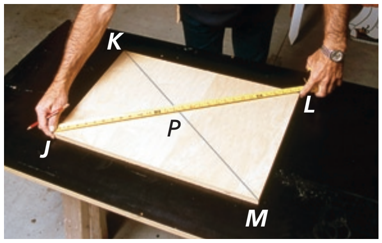



## PRACTICE AND PROBLEM SOLVING

**Independent Practice**

| For Exercises | See Example |
|---|---|
| 10–13 | 1 |
| 14–15 | 2 |
| 16 | 3 |
| 17 | 4 |

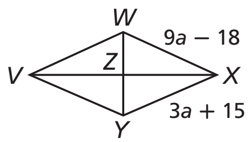
**Carpentry** A carpenter measures the diagonals of a piece of wood. In rectangle *JKLM*, *JM* = 25 in., and $JP = 14\frac{1}{2}$ in. Find each length.




10. *JL*
11. *KL*
12. *KM*
13. *MP*

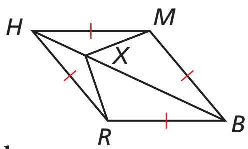
***VWXY* is a rhombus. Find each measure.**

14. *VW*
15. m∠*VWX* and m∠*WYX* if m∠*WVY* = $(4b + 10)°$ and m∠*XZW* = $(10b - 5)°$



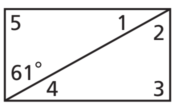

H.O.T. 16. **Multi-Step** The vertices of square *PQRS* are $P(-4, 0)$, $Q(4, 3)$, $R(7, -5)$, and $S(-1, -8)$. Show that the diagonals of square *PQRS* are congruent perpendicular bisectors of each other.

17. **Given:** *RHMB* is a rhombus with diagonal $\overline{HB}$.
**Prove:** ∠*HMX* ≅ ∠*HRX*

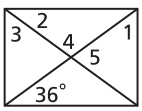



**Find the measures of the numbered angles in each rectangle.**

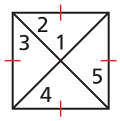
18.




19.




20.

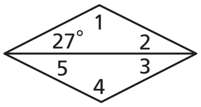


my.hrw.com

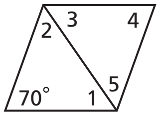
Online Extra Practice

**Find the measures of the numbered angles in each rhombus.**

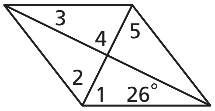
**21.**




**22.**




**23.**




**Tell whether each statement is sometimes, always, or never true. (*Hint:* Refer to your graphic organizer for this lesson.)**

**24.** A rectangle is a parallelogram.

**25.** A rhombus is a square.

**26.** A parallelogram is a rhombus.

**27.** A rhombus is a rectangle.

**28.** A square is a rhombus.

**29.** A rectangle is a quadrilateral.

**30.** A square is a rectangle.

**31.** A rectangle is a square.

H.O.T. **32.** **Critical Thinking** A triangle is equilateral if and only if the triangle is equiangular. Can you make a similar statement about a quadrilateral? Explain your answer.

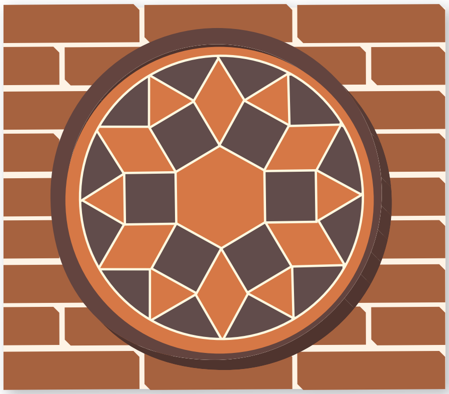
**33.** **History** There are five shapes of clay tiles in this tile mosaic from the ruins of Pompeii.

**a.** Make a sketch of each shape of tile and tell whether the shape is a polygon.

**b.** Name each polygon by its number of sides. Does each shape appear to be regular or irregular?

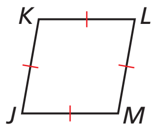
**c.** Do any of the shapes appear to be special parallelograms? If so, identify them by name.

**d.** Find the measure of each interior angle of the center polygon.



H.O.T. **34.** **ERROR ANALYSIS** Find and correct the error in this proof of Theorem 25-3-3.

**Given:** *JKLM* is a rhombus.
**Prove:** *JKLM* is a parallelogram.

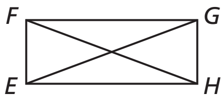



**Proof:**

It is given that *JKLM* is a rhombus. So by the definition of a rhombus, $\overline{JK} \cong \overline{LM}$, and $\overline{KL} \cong \overline{MJ}$. If a quadrilateral is a parallelogram, then its opposite sides are congruent. So *JKLM* is a parallelogram.

**35.** Complete the two-column proof of Theorem 25-3-2 by filling in the blanks.

**Given:** *EFGH* is a rectangle.
**Prove:** $\overline{FH} \cong \overline{GE}$




**Proof:**

| Statements | Reasons |
|---|---|
| **1.** *EFGH* is a rectangle. | **1.** Given |
| **2.** *EFGH* is a parallelogram. | **2.** **a.** ___?___ |
| **3.** $\overline{EF} \cong$ **b.** ___?___ | **3.** ▱ → opp. sides ≅ |
| **4.** $\overline{EH} \cong \overline{EH}$ | **4.** **c.** ___?___ |
| **5.** $\angle FEH$ and $\angle GHE$ are right angles. | **5.** **d.** ___?___ |
| **6.** $\angle FEH \cong$ **e.** ___?___ | **6.** Rt. ∠ ≅ Thm. |
| **7.** $\triangle FEH \cong \triangle GHE$ | **7.** **f.** ___?___ |
| **8.** $\overline{FH} \cong \overline{GE}$ | **8.** **g.** ___?___ |

**Real-World Connections**

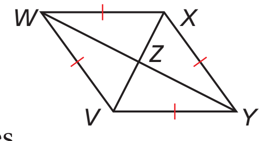

**36.** The organizers of a fair plan to fence off a plot of land given by the coordinates $A(2, 4)$, $B(4, 2)$, $C(-1, -3)$, and $D(-3, -1)$.

**a.** Find the slope of each side of quadrilateral *ABCD*.

**b.** What type of quadrilateral is formed by the fences? Justify your answer.

**c.** The organizers plan to build a straight path connecting *A* and *C* and another path connecting *B* and *D*. Explain why these two paths will have the same length.

**37.** Use this plan to write a proof of Theorem 25-3-4.

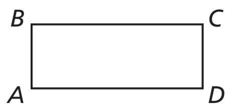



**Given:** *VWXY* is a rhombus.

**Prove:** $\overline{VX} \perp \overline{WY}$

**Plan:** Use the definition of a rhombus and the properties of parallelograms to show that $\triangle WZX \cong \triangle YZX$. Then use CPCTC to show that $\angle WZX$ and $\angle YZX$ are right angles.

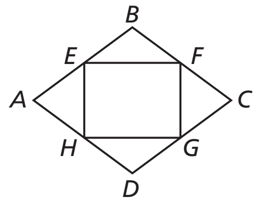
**38.** Write a paragraph proof of Theorem 25-3-1.




**Given:** *ABCD* is a rectangle.

**Prove:** *ABCD* is a parallelogram.

**39.** Write a two-column proof.

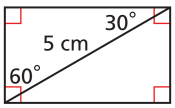



**Given:** *ABCD* is a rhombus. *E*, *F*, *G*, and *H* are the midpoints of the sides.

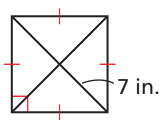
**Prove:** *EFGH* is a parallelogram.

**H.O.T. Multi-Step** **Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.**

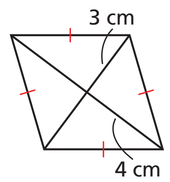
**40.**




**41.**




**42.**




**H.O.T. 43. Write About It** Explain why each of these conditional statements is true.

**a.** If a quadrilateral is a square, then it is a parallelogram.

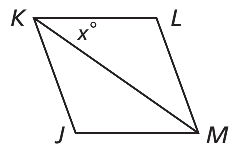
**b.** If a quadrilateral is a square, then it is a rectangle.

**c.** If a quadrilateral is a square, then it is a rhombus.

**44. Write About It** List the properties that a square "inherits" because it is (1) a parallelogram, (2) a rectangle, and (3) a rhombus.

## TEST PREP

**45.** Which expression represents the measure of $\angle J$ in rhombus *JKLM*?




Ⓐ $x°$ Ⓒ $(180 - x)°$

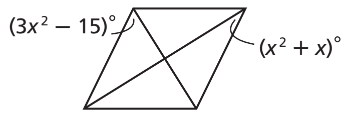
Ⓑ $2x°$ Ⓓ $(180 - 2x)°$

**46. Short Response** The diagonals of rectangle *QRST* intersect at point *P*. If $QR = 1.8$ cm, $QP = 1.5$ cm, and $QT = 2.4$ cm, find the perimeter of $\triangle RST$. Explain how you found your answer.

47. Which statement is NOT true of a rectangle?
- (F) Both pairs of opposite sides are congruent and parallel.
- (G) Both pairs of opposite angles are congruent and supplementary.
- (H) All pairs of consecutive sides are congruent and perpendicular.
- (J) All pairs of consecutive angles are congruent and supplementary.

## CHALLENGE AND EXTEND

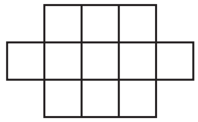
48. **Algebra** Find the value of $x$ in the rhombus.




H.O.T. 49. Prove that the segment joining the midpoints of two consecutive sides of a rhombus is perpendicular to one diagonal and parallel to the other.

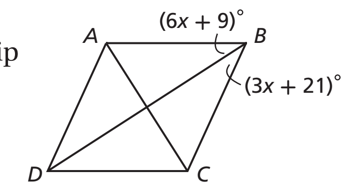
50. Extend the definition of a triangle midsegment to write a definition for the midsegment of a rectangle. Prove that a midsegment of a rectangle divides the rectangle into two congruent rectangles.

51. The figure is formed by joining eleven congruent squares. How many rectangles are in the figure?

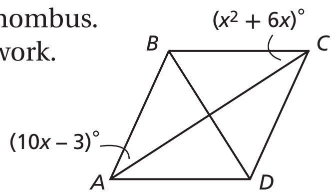


## FOCUS ON MATHEMATICAL PRACTICES

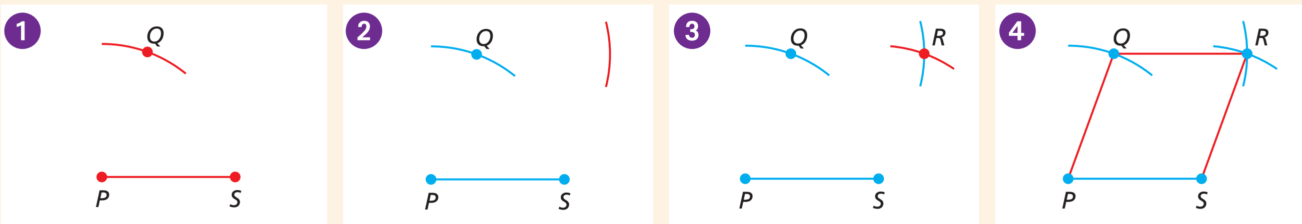
H.O.T. 52. **Reasoning** Explain the relationship between the two labeled angles in the rhombus shown and their relationship to $\angle BAD$, then find the value of $x$ and $m\angle BAD$.




H.O.T. 53. **Problem Solving** $ABCD$ is a rhombus. Find $x$ and $m\angle DAC$. Show your work.




### Construction Rhombus




1. Draw $\overline{PS}$. Set the compass to the length of $\overline{PS}$. Place the compass point at $P$ and draw an arc above $\overline{PS}$. Label a point $Q$ on the arc.
2. Place the compass point at $Q$ and draw an arc to the right of $Q$.
3. Place the compass point at $S$ and draw an arc that intersects the arc drawn from $Q$. Label the point of intersection $R$.
4. Draw $\overline{PQ}$, $\overline{QR}$, and $\overline{RS}$.

25-4 Technology TASK

# Predict Conditions for Special Parallelograms

In this task, you will use geometry software to predict the conditions that are sufficient to prove that a parallelogram is a rectangle, rhombus, or square.

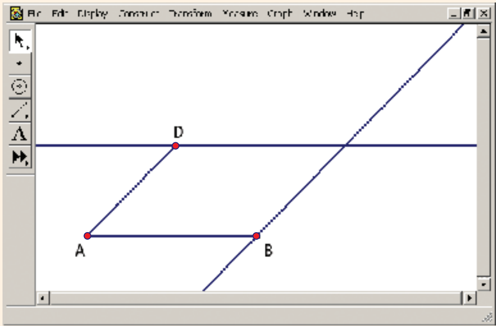
Use with *Conditions for Special Parallelograms*

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

**MCC9-12.G.CO.11** Prove theorems about parallelograms.

## Activity 1

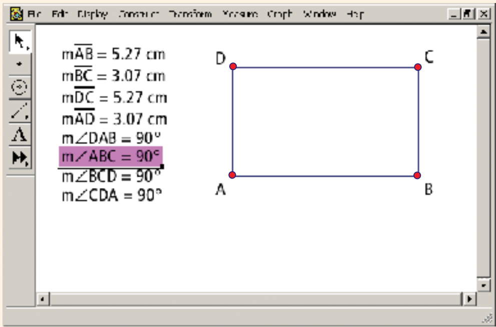
1. Construct $\overline{AB}$ and $\overline{AD}$ with a common endpoint $A$. Construct a line through $D$ parallel to $\overline{AB}$. Construct a line through $B$ parallel to $\overline{AD}$.




2. Construct point $C$ at the intersection of the two lines. Hide the lines and construct $\overline{BC}$ and $\overline{CD}$ to complete the parallelogram.

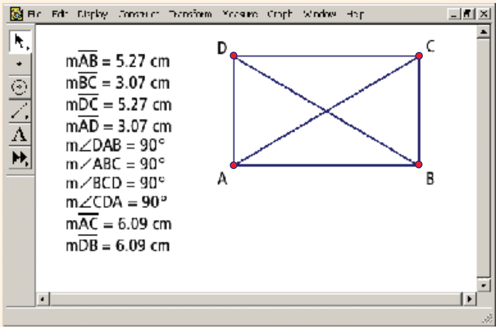
3. Measure the four sides and angles of the parallelogram.

4. Move $A$ so that $m\angle ABC = 90°$. What type of special parallelogram results?




5. Move $A$ so that $m\angle ABC \neq 90°$.

6. Construct $\overline{AC}$ and $\overline{BD}$ and measure their lengths. Move $A$ so that $AC = BD$. What type of special parallelogram results?



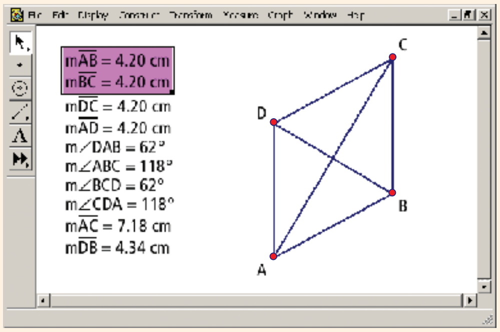

## Try This

1. How does the method of constructing $ABCD$ in Steps 1 and 2 guarantee that the quadrilateral is a parallelogram?

2. **Make a Conjecture** What are two conditions for a rectangle? Write your conjectures as conditional statements.

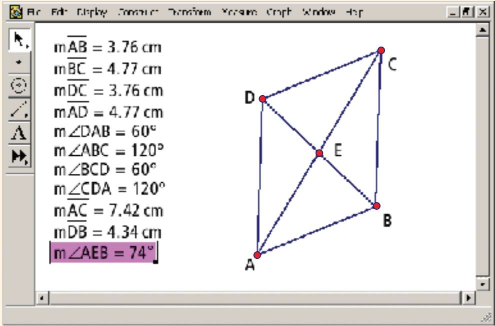
## Activity 2

1. Use the parallelogram you constructed in Activity 1. Move $A$ so that $AB = BC$. What type of special parallelogram results?



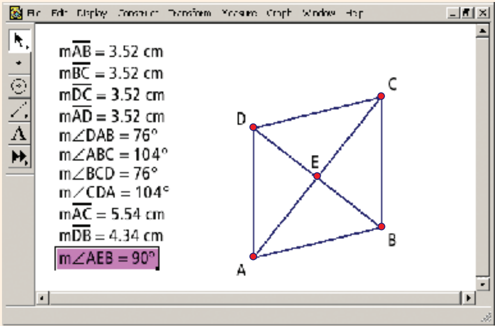

2. Move $A$ so that $AB \neq BC$.

3. Label the intersection of the diagonals as $E$. Measure $\angle AEB$.

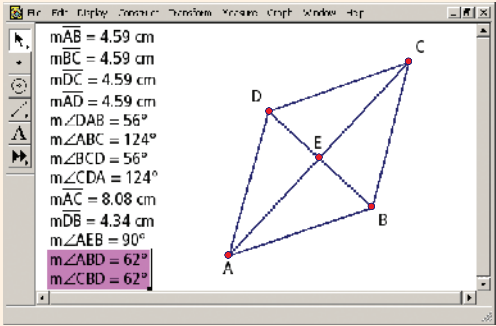



4. Move $A$ so that $m\angle AEB = 90°$. What type of special parallelogram results?

5. Move $A$ so that $m\angle AEB \neq 90°$.




6. Measure $\angle ABD$ and $\angle CBD$. Move $A$ so that $m\angle ABD = m\angle CBD$. What type of special parallelogram results?




## Try This

3. **Make a Conjecture** What are three conditions for a rhombus? Write your conjectures as conditional statements.

4. **Make a Conjecture** A square is both a rectangle and a rhombus. What conditions do you think must hold for a parallelogram to be a square?

# 25-4 Conditions for Special Parallelograms

***Essential Question:*** What information about a parallelogram allows you to conclude it is a rectangle, rhombus, or square?

***Objective***
Prove that a given quadrilateral is a rectangle, rhombus, or square.

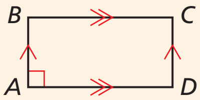
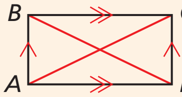
**Who uses this?**
Building contractors and carpenters can use the conditions for rectangles to make sure the frame for a house has the correct shape.

**Animated Math**

When you are given a parallelogram with certain properties, you can use the theorems below to determine whether the parallelogram is a rectangle.

**Know it! Note**

**Theorems** | **Conditions for Rectangles**

| | THEOREM | EXAMPLE |
|---|---|---|
| **25-4-1** | If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle. (▱ with one rt. ∠ → rect.) |  |
| **25-4-2** | If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (▱ with diags. ≅ → rect.) |  $\overline{AC} \cong \overline{BD}$ |

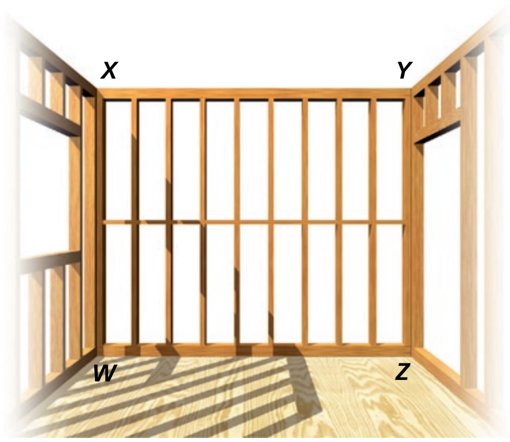
*You will prove Theorems 25-4-1 and 25-4-2 in Exercises 31 and 28.*

COMMON CORE GPS **EXAMPLE 1** MCC9-12.G.MG.1

my.hrw.com
Online Video Tutor

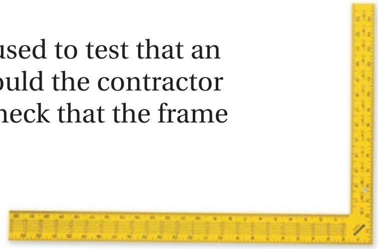
## *Carpentry Application*

**A contractor built a wood frame for the side of a house so that $\overline{XY} \cong \overline{WZ}$ and $\overline{XW} \cong \overline{YZ}$. Using a tape measure, the contractor found that $XZ = WY$. Why must the frame be a rectangle?**




Both pairs of opposite sides of $WXYZ$ are congruent, so $WXYZ$ is a parallelogram. Since $XZ = WY$, the diagonals of ▱$WXYZ$ are congruent. Therefore the frame is a rectangle by Theorem 25-4-2.

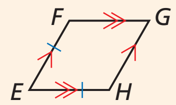
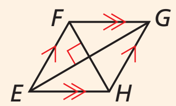
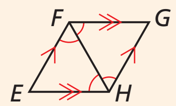
**CHECK IT OUT!** **1.** A carpenter's square can be used to test that an angle is a right angle. How could the contractor use a carpenter's square to check that the frame is a rectangle?



Below are some conditions you can use to determine whether a parallelogram is a rhombus.

**Know it! Note**

## Theorems: Conditions for Rhombuses

| THEOREM | EXAMPLE |
|---|---|
| **25-4-3** If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. (▱ with one pair cons. sides ≅ → rhombus) |  |
| **25-4-4** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (▱ with diags. ⊥ → rhombus) |  |
| **25-4-5** If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (▱ with diag. bisecting opp. ∠s → rhombus) |  |

*You will prove Theorems 25-4-3 and 25-4-4 in Exercises 32 and 30.*

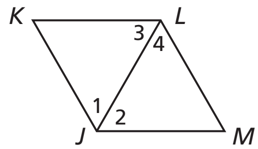
**Caution!**

In order to apply Theorems 25-5-1 through 25-5-5, the quadrilateral must be a parallelogram.

**PROOF**

### Theorem 25-4-5

**Given:** *JKLM* is a parallelogram. $\overline{JL}$ bisects ∠*KJM* and ∠*KLM*.
**Prove:** *JKLM* is a rhombus.




**Proof:**

| Statements | Reasons |
|---|---|
| **1.** *JKLM* is a parallelogram. $\overline{JL}$ bisects ∠*KJM* and ∠*KLM*. | **1.** Given |
| **2.** ∠1 ≅ ∠2, ∠3 ≅ ∠4 | **2.** Def. of ∠ bisector |
| **3.** $\overline{JL} \cong \overline{JL}$ | **3.** Reflex. Prop. of ≅ |
| **4.** △*JKL* ≅ △*JML* | **4.** ASA *Steps 2, 3* |
| **5.** $\overline{JK} \cong \overline{JM}$ | **5.** CPCTC |
| **6.** *JKLM* is a rhombus. | **6.** ▱ with one pair cons. sides ≅ → rhombus |

To prove that a given quadrilateral is a square, it is sufficient to show that the figure is both a rectangle and a rhombus. You will explain why this is true in Exercise 43.

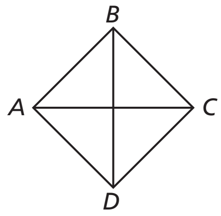
COMMON CORE GPS **EXAMPLE 2** MCC9-12.G.CO.11

my.hrw.com

Online Video Tutor

## Applying Conditions for Special Parallelograms

**Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.**




**A** **Given:** $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{AD}, \overline{AD} \perp \overline{DC}, \overline{AC} \perp \overline{BD}$

**Conclusion:** *ABCD* **is a square.**

**Step 1** Determine if *ABCD* is a parallelogram.

$\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{AD}$ — *Given*

*ABCD* is a parallelogram. — *Quad. with opp. sides ≅ → ▱*

**Step 2** Determine if *ABCD* is a rectangle.

$\overline{AD} \perp \overline{DC}$, so ∠*ADC* is a right angle. — *Def. of ⊥*

*ABCD* is a rectangle. — *▱ with one rt. ∠ → rect.*

**Step 3** Determine if *ABCD* is a rhombus.

$\overline{AC} \perp \overline{BD}$ — *Given*

*ABCD* is a rhombus. — *▱ with diags. ⊥ → rhombus*

**Step 4** Determine if *ABCD* is a square.

Since *ABCD* is a rectangle and a rhombus, it has four right angles and four congruent sides. So *ABCD* is a square by definition. The conclusion is valid.

**B** **Given:** $\overline{AB} \cong \overline{BC}$

**Conclusion:** *ABCD* **is a rhombus.**

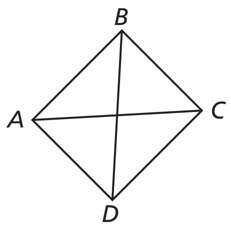
The conclusion is not valid. By Theorem 25-4-3, if one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. To apply this theorem, you must first know that *ABCD* is a parallelogram.

**Remember!**

You can also prove that a given quadrilateral is a rectangle, rhombus, or square by using the definitions of the special quadrilaterals.

**CHECK IT OUT!**

2. Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.
Given: ∠*ABC* is a right angle.
Conclusion: *ABCD* is a rectangle.




COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.GPE.5

my.hrw.com

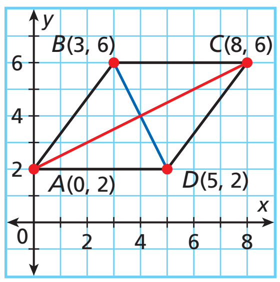
Online Video Tutor

## Identifying Special Parallelograms in the Coordinate Plane

**Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.**

**A** $A(0, 2), B(3, 6), C(8, 6), D(5, 2)$

**Step 1** Graph ▱*ABCD*.




**Step 2** Determine if *ABCD* is a rectangle.

$$AC = \sqrt{(8-0)^2 + (6-2)^2} = \sqrt{80} = 4\sqrt{5}$$

$$BD = \sqrt{(5-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

Since $4\sqrt{5} \neq 2\sqrt{5}$, *ABCD* is not a rectangle. Thus *ABCD* is not a square.

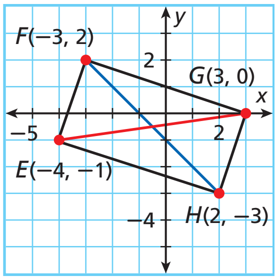
**Step 3** Determine if *ABCD* is a rhombus.

slope of $\overline{AC} = \frac{6-2}{8-0} = \frac{1}{2}$ slope of $\overline{BD} = \frac{2-6}{5-3} = -2$

Since $\left(\frac{1}{2}\right)(-2) = -1$, $\overline{AC} \perp \overline{BD}$. *ABCD* is a rhombus.

**B** $E(-4, -1), F(-3, 2), G(3, 0), H(2, -3)$

**Step 1** Graph ▱*EFGH*.




**Step 2** Determine if *EFGH* is a rectangle.

$EG = \sqrt{[3-(-4)]^2 + [0-(-1)]^2}$

$= \sqrt{50} = 5\sqrt{2}$

$FH = \sqrt{[2-(-3)]^2 + (-3-2)^2}$

$= \sqrt{50} = 5\sqrt{2}$

Since $5\sqrt{2} = 5\sqrt{2}$, the diagonals are congruent.
*EFGH* is a rectangle.

**Step 3** Determine if *EFGH* is a rhombus.

slope of $\overline{EG} = \frac{0-(-1)}{3-(-4)} = \frac{1}{7}$

slope of $\overline{FH} = \frac{-3-2}{2-(-3)} = \frac{-5}{5} = -1$

Since $\left(\frac{1}{7}\right)(-1) \neq -1$, $\overline{EG} \not\perp \overline{FH}$.

So *EFGH* is a not a rhombus and cannot be a square.

**CHECK IT OUT!** **Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.**

**3a.** $K(-5, -1), L(-2, 4), M(3, 1), N(0, -4)$

**3b.** $P(-4, 6), Q(2, 5), R(3, -1), S(-3, 0)$

MCC.MP.6 MATHEMATICAL PRACTICES

## THINK AND DISCUSS

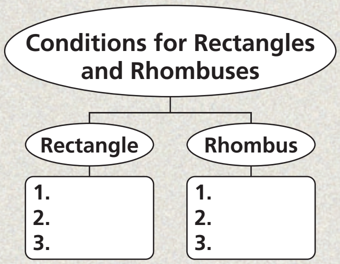
**1.** What special parallelogram is formed when the diagonals of a parallelogram are congruent? when the diagonals are perpendicular? when the diagonals are both congruent and perpendicular?

**2.** Draw a figure that shows why this statement is not necessarily true: If one angle of a quadrilateral is a right angle, then the quadrilateral is a rectangle.

**3.** A rectangle can also be defined as a parallelogram with a right angle. Explain why this definition is accurate.

Know it! Note

**4.** **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write at least three conditions for the given parallelogram.



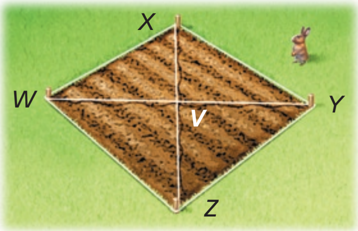
# 25-4 Exercises

my.hrw.com
Homework Help

## GUIDED PRACTICE

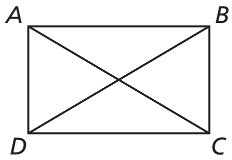
SEE EXAMPLE 1

1. **Gardening** A city garden club is planting a square garden. They drive pegs into the ground at each corner and tie strings between each pair. The pegs are spaced so that $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. How can the garden club use the diagonal strings to verify that the garden is a square?




SEE EXAMPLE 2

**Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.**




2. Given: $\overline{AC} \cong \overline{BD}$
   Conclusion: $ABCD$ is a rectangle.
3. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \perp \overline{BC}$
   Conclusion: $ABCD$ is a rectangle.

SEE EXAMPLE 3

**Multi-Step** **Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.**

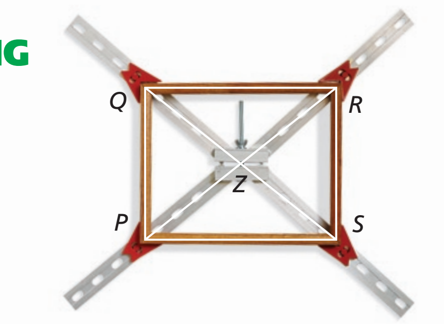
4. $P(-5, 2)$, $Q(4, 5)$, $R(6, -1)$, $S(-3, -4)$
5. $W(-6, 0)$, $X(1, 4)$, $Y(2, -4)$, $Z(-5, -8)$

## PRACTICE AND PROBLEM SOLVING

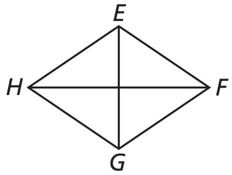
| Independent Practice | |
|---|---|
| For Exercises | See Example |
| 6 | 1 |
| 7–8 | 2 |
| 9–10 | 3 |

my.hrw.com
Online Extra Practice

6. **Crafts** A framer uses a clamp to hold together the pieces of a picture frame. The pieces are cut so that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$. The clamp is adjusted so that $PZ$, $QZ$, $RZ$, and $SZ$ are all equal. Why must the frame be a rectangle?




**Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.**

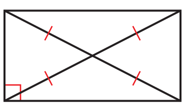


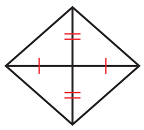

7. Given: $\overline{EG}$ and $\overline{FH}$ bisect each other. $\overline{EG} \perp \overline{FH}$
   Conclusion: $EFGH$ is a rhombus.
8. Given: $\overline{FH}$ bisects $\angle EFG$ and $\angle EHG$.
   Conclusion: $EFGH$ is a rhombus.

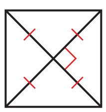
**Multi-Step** **Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.**

9. $A(-10, 4)$, $B(-2, 10)$, $C(4, 2)$, $D(-4, -4)$
10. $J(-9, -7)$, $K(-4, -2)$, $L(3, -3)$, $M(-2, -8)$

**Tell whether each quadrilateral is a parallelogram, rectangle, rhombus, or square. Give all the names that apply.**

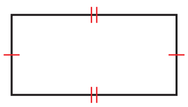
11. 

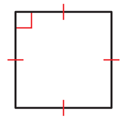
12. 

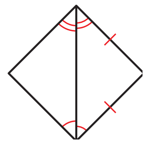
13. 

Peter Van Steen/HMH Photo

**Tell whether each quadrilateral is a parallelogram, rectangle, rhombus, or square. Give all the names that apply.**

**14.** 

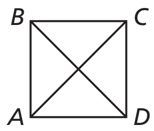
**15.** 

**16.** 

**17.** **ERROR ANALYSIS** In ▱$ABCD$, $\overline{AC} \cong \overline{BD}$. Which conclusion is incorrect? Explain the error.

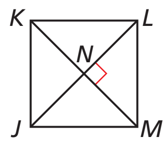
**A** *ABCD* is a rectangle.

**B** *ABCD* is a square.

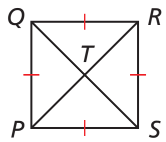



**H.O.T.** **Give one characteristic of the diagonals of each figure that would make the conclusion valid.**

**18.** Conclusion: *JKLM* is a rhombus.




**19.** Conclusion: *PQRS* is a square.




**The coordinates of three vertices of ▱*ABCD* are given. Find the coordinates of *D* so that the given type of figure is formed.**

**20.** $A(4, -2)$, $B(-5, -2)$, $C(4, 4)$; rectangle

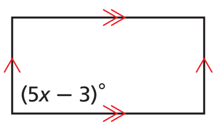
**21.** $A(-5, 5)$, $B(0, 0)$, $C(7, 1)$; rhombus

**22.** $A(0, 2)$, $B(4, -2)$, $C(0, -6)$; square

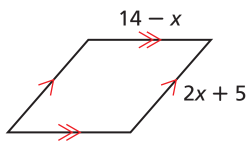
**23.** $A(2, 1)$, $B(-1, 5)$, $C(-5, 2)$; square

**Find the value of *x* that makes each parallelogram the given type.**

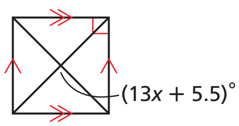
**24.** rectangle




**25.** rhombus



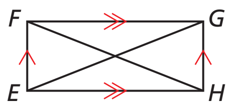

**26.** square




**27.** **Critical Thinking** The diagonals of a quadrilateral are perpendicular bisectors of each other. What is the best name for this quadrilateral? Explain your answer.

**28.** Complete the two-column proof of Theorem 25-4-2 by filling in the blanks.

**Given:** *EFGH* is a parallelogram.
$\overline{EG} \cong \overline{HF}$
**Prove:** *EFGH* is a rectangle.




**Proof:**

| Statements | Reasons |
|---|---|
| 1. *EFGH* is a parallelogram. $\overline{EG} \cong \overline{HF}$ | 1. Given |
| 2. $\overline{EF} \cong \overline{HG}$ | 2. **a.** ___?___ |
| 3. **b.** ___?___ | 3. Reflex. Prop. of ≅ |
| 4. $\triangle EFH \cong \triangle HGE$ | 4. **c.** ___?___ |
| 5. $\angle FEH \cong$ **d.** ___?___ | 5. **e.** ___?___ |
| 6. $\angle FEH$ and $\angle GHE$ are supplementary. | 6. **f.** ___?___ |
| 7. **g.** ___?___ | 7. ≅ ∠s supp. → rt. ∠s |
| 8. *EFGH* is a rectangle. | 8. **h.** ___?___ |

**Real-World Connections**

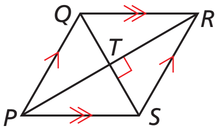
**29.** A state fair takes place on a plot of land given by the coordinates $A(-2, 3)$, $B(1, 2)$, $C(2, -1)$, and $D(-1, 0)$.

**a.** Show that the opposite sides of quadrilateral *ABCD* are parallel.

**b.** A straight path connects *A* and *C*, and another path connects *B* and *D*. Use slopes to prove that these two paths are perpendicular.

**c.** What can you conclude about *ABCD*? Explain your answer.

**30.** Complete the paragraph proof of Theorem 25-4-4 by filling in the blanks.



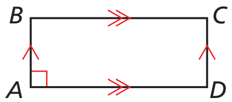

**Given:** *PQRS* is a parallelogram. $\overline{PR} \perp \overline{QS}$

**Prove:** *PQRS* is a rhombus.

**Proof:**

It is given that *PQRS* is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{PT} \cong$ **a.** ___?___ . By the Reflexive Property of Congruence, $\overline{QT} \cong$ **b.** ___?___ . It is given that $\overline{PR} \perp \overline{QS}$, so $\angle QTP$ and $\angle QTR$ are right angles by the definition of **c.** ___?___ . Then $\angle QTP \cong \angle QTR$ by the **d.** ___?___ . So $\triangle QTP \cong \triangle QTR$ by **e.** ___?___ , and $\overline{QP} \cong$ **f.** ___?___ , by CPCTC. By Theorem 25-4-3, if one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a **g.** ___?___ . Therefore *PQRS* is rhombus.

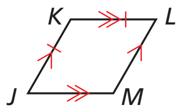
H.O.T. **31.** Write a two-column proof of Theorem 25-4-1.




**Given:** *ABCD* is a parallelogram. $\angle A$ is a right angle.

**Prove:** *ABCD* is a rectangle.

H.O.T. **32.** Write a paragraph proof of Theorem 25-4-3.




**Given:** *JKLM* is a parallelogram. $\overline{JK} \cong \overline{KL}$

**Prove:** *JKLM* is a rhombus.

H.O.T. **33.** **Algebra** Four lines are represented by the equations below.

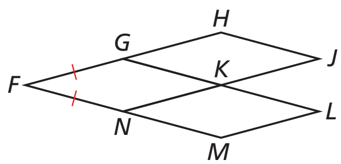
$\ell$: $y = -x + 1$ $\quad$ $m$: $y = -x + 7$ $\quad$ $n$: $y = 2x + 1$ $\quad$ $p$: $y = 2x + 7$

**a.** Graph the four lines in the coordinate plane.

**b.** Classify the quadrilateral formed by the lines.

**c.** **What if...?** Suppose the slopes of lines $n$ and $p$ change to 1. Reclassify the quadrilateral.

H.O.T. **34.** Write a two-column proof.




**Given:** *FHJN* and *GLMF* are parallelograms.
$\overline{FG} \cong \overline{FN}$

**Prove:** *FGKN* is a rhombus.

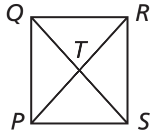
**35.** **Write About It** Write a biconditional statement based on the theorems about the diagonals of rectangles. Write a biconditional statement based on the theorems about the diagonals of rhombuses. Can you write a biconditional statement based on the theorems about opposite angles in parallelograms? Explain your answer.

**Construction** Use the diagonals to construct each figure. Then use the theorems from this lesson to explain why your method works.

**36.** rectangle $\qquad$ **37.** rhombus $\qquad$ **38.** square

## TEST PREP

**39.** In ▱$PQRS$, $\overline{PR}$ and $\overline{QS}$ intersect at $T$. What additional information is needed to conclude that $PQRS$ is a rectangle?



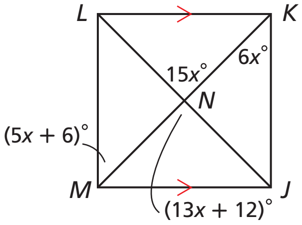

Ⓐ $\overline{PT} \cong \overline{QT}$ Ⓒ $\overline{PT} \perp \overline{QT}$

Ⓑ $\overline{PT} \cong \overline{RT}$ Ⓓ $\overline{PT}$ bisects $\angle QPS$.

**40.** Which of the following is the best name for figure $WXYZ$ with vertices $W(-3, 1)$, $X(1, 5)$, $Y(8, -2)$, and $Z(4, -6)$?

Ⓕ Parallelogram Ⓖ Rectangle Ⓗ Rhombus Ⓙ Square

**41.** **Extended Response**




**a.** Write and solve an equation to find the value of $x$.

**b.** Is $JKLM$ a parallelogram? Explain.

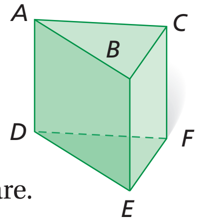
**c.** Is $JKLM$ a rectangle? Explain.

**d.** Is $JKLM$ a rhombus? Explain.

## CHALLENGE AND EXTEND

**42.** **Given:** $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$

**Prove:** $EBCF$ is a rectangle.




**43.** **Critical Thinking** Consider the following statement: If a quadrilateral is a rectangle and a rhombus, then it is a square.

**a.** Explain why the statement is true.

**b.** If a quadrilateral is a rectangle, is it necessary to show that all four sides are congruent in order to conclude that it is a square? Explain.

**c.** If a quadrilateral is a rhombus, is it necessary to show that all four angles are right angles in order to conclude that it is a square? Explain.

**44.** **Cars** As you turn the crank of a car jack, the platform that supports the car rises. Use the diagonals of the parallelogram to explain whether the jack forms a rectangle, rhombus, or square.

## FOCUS ON MATHEMATICAL PRACTICES

MATHEMATICAL PRACTICES

H.O.T. **45.** **Properties** Give the most specific name for the parallelogram with the given properties.

**a.** diagonals are congruent and perpendicular

**b.** diagonals bisect each other and are congruent

**c.** diagonals are perpendicular

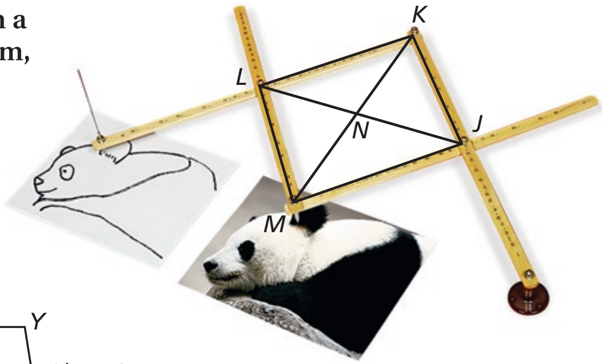
H.O.T. **46.** **Justify** Coco made a skating rink in her back yard. The rink is a quadrilateral $PQRS$ where $\overline{PQ}$ is parallel to $\overline{RS}$, $\overline{PQ}$ is congruent to $\overline{RS}$, and $\overline{PR}$ is congruent to $\overline{QS}$. What type of quadrilateral is her rink? Justify your answer.

# Ready to Go On?

**my.hrw.com**
Assessment and Intervention

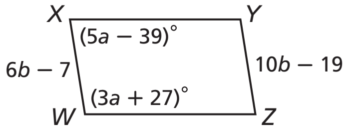
## 25-1 Properties of Parallelograms

**A pantograph is used to copy drawings. Its legs form a parallelogram. In ▱*JKLM*, *LM* = 17 cm, *KN* = 13.5 cm, and m∠*KJM* = 102°. Find each measure.**




1. *KM*
2. *KJ*
3. *MN*
4. m∠*JKL*
5. m∠*JML*
6. m∠*KLM*
7. Three vertices of ▱*ABCD* are *A*(−3, 1), *B*(5, 7), and *C*(6, 2). Find the coordinates of vertex *D*.

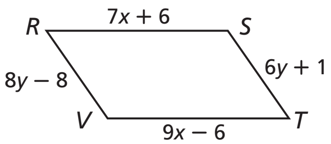
***WXYZ* is a parallelogram. Find each measure.**



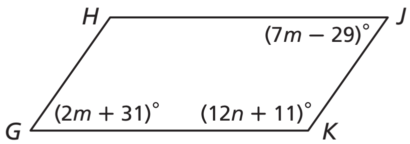

8. *WX*
9. *YZ*
10. m∠*X*
11. m∠*W*

## 25-2 Conditions for Parallelograms

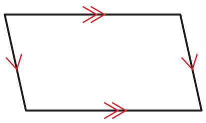
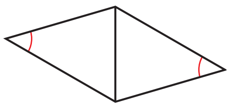
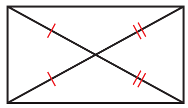
12. Show that *RSTV* is a parallelogram for $x = 6$ and $y = 4.5$.




13. Show that *GHJK* is a parallelogram for $m = 12$ and $n = 9.5$.

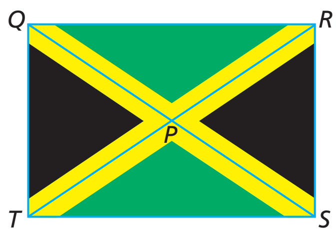



**Determine if each quadrilateral must be a parallelogram. Justify your answer.**

14. 
15. 
16. 
17. Show that a quadrilateral with vertices *C*(−9, 4), *D*(−4, 8), *E*(2, 6), and *F*(−3, 2) is a parallelogram.

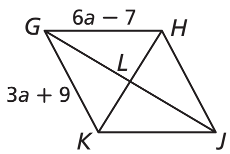
## 25-3 Properties of Special Parallelograms

**The flag of Jamaica is a rectangle with stripes along the diagonals. In rectangle *QRST*, *QS* = 80.5, and *RS* = 36. Find each length.**

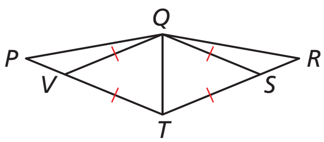



18. *SP*
19. *QT*
20. *TR*
21. *TP*

***GHJK* is a rhombus. Find each measure.**

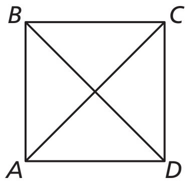



22. *HJ*
23. m∠*HJG* and m∠*GHJ* if m∠*JLH* = $(4b - 6)°$ and m∠*JKH* = $(2b + 11)°$
24. **Given:** *QSTV* is a rhombus. $\overline{PT} \cong \overline{RT}$
**Prove:** $\overline{PQ} \cong \overline{RQ}$



## 25-4 Conditions for Special Parallelograms

**Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.**




**25.** Given: $\overline{AC} \perp \overline{BD}$
Conclusion: *ABCD* is a rhombus.

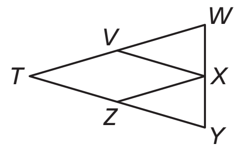
**26.** Given: $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$, $\overline{AB} \parallel \overline{CD}$
Conclusion: *ABCD* is a rectangle.

**Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.**

**27.** $W(-2, 2)$, $X(1, 5)$, $Y(7, -1)$, $Z(4, -4)$

**28.** $M(-4, 5)$, $N(1, 7)$, $P(3, 2)$, $Q(-2, 0)$

**29. Given:** $\overline{VX}$ and $\overline{ZX}$ are midsegments of $\triangle TWY$. $\overline{TW} \cong \overline{TY}$
**Prove:** *TVXZ* is a rhombus.

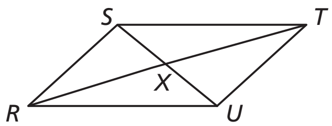



# PARCC Assessment Readiness

COMMON CORE GPS

## Selected Response

**1.** The diagram shows the parallelogram-shaped component that attaches a car's rearview mirror to the car. In parallelogram *RSTU*, $UR = 25$, $RX = 16$, and $m\angle STU = 42.4°$. Find *ST*, *XT*, and $m\angle RST$.




Ⓐ $ST = 16$, $XT = 25$, $m\angle RST = 42.4°$

Ⓑ $ST = 25$, $XT = 16$, $m\angle RST = 47.8°$

Ⓒ $ST = 25$, $XT = 16$, $m\angle RST = 137.6°$

Ⓓ $ST = 5$, $XT = 4$, $m\angle RST = 137.6°$

**2.** Use the diagonals to determine whether a parallelogram with vertices $A(-1, -2)$, $B(-2, 0)$, $C(0, 1)$, and $D(1, -1)$ is a rectangle, rhombus, or square. Give all the names that apply.

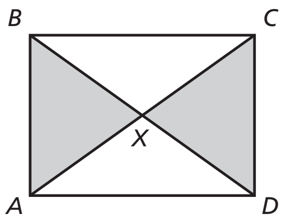
Ⓕ rectangle, rhombus, square

Ⓖ rectangle, rhombus

Ⓗ rectangle

Ⓙ square

**3.** An artist designs a rectangular quilt piece with different types of ribbon that go from the corner to the center of the quilt. The dimensions of the rectangle are $AB = 10$ inches and $AC = 14$ inches. Find *BX*.




Ⓐ $BX = 7$ inches

Ⓑ $BX = 10$ inches

Ⓒ $BX = 5$ inches

Ⓓ $BX = 14$ inches

## Mini-Task

**4.** Two vertices of a parallelogram are $A(2, 3)$ and $B(8, 11)$, and the intersection of the diagonals is $X(7, 6)$. Find the coordinates of the other two vertices.