Paper 2

Time allowed: 1 hour 30 minutes

Maximum number of marks: 80 marks

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You need a graphical display calculator for this paper.

1 [Maximum mark: 12]

In a flat country there are two ancient, straight, Roman roads. Relative to a co-ordinate system, the two roads have equations $y = 2x + 3$ and $7y + 4x = 75$. Units are in kilometres.

a The roads cross at a village at point A. Find the coordinates of point A. [2]

b Determine, with a reason, whether or not these two roads are perpendicular. [3]

c Point B lies on $y = 2x + 3$ and has $x = -2$. Point C lies on $7y + 4x = 75$ and has $y = 5$. Find the co-ordinates of (i) point B, (ii) point C. [2]

d Point M is the mid-point of the line section BC. Find the coordinates of point M. [2]

e A drone is to be flown from point B to point C at ground level. Calculate the distance it will travel. [3]

2 [Maximum mark: 16]

Two hundred married couples, of different age groups, were asked their preferences about a celebratory meal. Their responses are given in the table below.

Meal\Age	Young	Middle-aged	Elderly
Take-away	28	20	5
Eat out	20	35	22
Cook in	16	26	28

One of the couples is selected at random.

a Find the probability that they

i are elderly **ii** prefer take-away **iii** are middle-aged and prefer to eat out

iv are elderly, given that they prefer take-away

v prefer take-away, given that they are elderly. [5]

It is thought that meal preference and age group are dependent on one another.

b Devise and carry out a test, to test this hypothesis at the 5% level.

You should:

- state the name of the test being used
- state the hypotheses
- under the null hypothesis, give a table of expected frequencies in a similar format to that above and comment on these values with regard to the validity of the test
- state the number of degrees of freedom
- state the p-value.

With a reason, state the conclusion of the test. Give your answer in the context of the question. [11]

3 [Maximum mark: 11]

The leader of country A claims that its citizens are more intelligent than those of neighbouring country B. To test this claim, a random sample of 12 citizens from country A and 10 citizens from country B was taken, and their Intelligence Quotients (IQs) were measured. The results are as follows:

A: 98, 101, 90, 115, 87, 95, 102, 110, 96, 108, 100, 103

B: 103, 102, 95, 100, 98, 97, 105, 94, 104, 101

It can be assumed that the IQs of citizens from both countries are normally distributed with a common variance.

a Find the mean of the sample from

i country A

ii country B, giving your answers correct to 1 decimal place. [3]

b Devise and carry out a test to test the leader of country A's claim at the 10% level.

State the name of the test being used and the number of tails. State the hypotheses. State the p-value. With a reason, state the conclusion of the test, giving your answer in the context of the question. [8]

4 [Maximum mark: 12]

Paired bivariate data (x, y) is collected from 11 students, where x is their time to swim 100 m (measured in seconds) and y is their time to run 200 m (also measured in seconds). The data is given in the following table:

x	100	81	120	104	180	200	152	102	94	131	142
y	40	35	44	39	51	60	48	40	37	43	47

a Calculate the Pearson product moment correlation coefficient (r) for this data, and state what this value of r implies about the relationship between the swimming and running times. [5]

b **i** Calculate the equation of the linear regression line of y on x.

ii Write down the mean point $(\bar{x}, \bar{y})$ that the linear regression line of y on x must pass through.

iii A twelfth student had a swimming time of 110 seconds. Estimate their running time. [5]

c State two reasons why the equation found in part **b i** should not be used to estimate the swimming time of a student with a running time of 23 seconds. [2]

5 [Maximum mark: 15]

The masses of a species of large tortoises are normally distributed with a mean of 10 kg and a standard deviation of 2 kg.

a Find the probability that a tortoise, chosen at random, has a mass between 9 kg and 12 kg. [2]

b There is a famous tortoise called Michelle. It is known that 70% of the tortoises have a mass more than Michelle's. Find Michelle's mass. [3]

c Find the probability that a tortoise has a mass of more than 12 kg, given that it has a mass of more than 11 kg. [4]

It is known that that 6.68% of these tortoises have a mass greater than 13 kg.

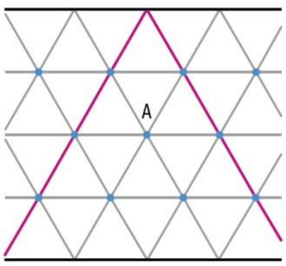
There are a very large number of these tortoises on an island. A sample of 100 of them is captured at random. Since they do not run very fast, it can be assumed that their masses are independent of one another.

d **i** Find the probability that exactly 6 of the sample have a mass greater than 13 kg.

ii Find the probability that at least 3 of the sample have a mass greater than 13 kg. [6]

6 [Maximum mark: 14]

In a very large country, mobile phone transmitter masts are placed at the vertices of an equilateral triangle grid, as shown in the diagram:



The sides of the equilateral triangles are each 20 km long.

a **i** Copy the diagram and sketch in the lines to show the Voronoi diagram cell about the mast labelled *A*.

ii Describe the shape of this cell. [5]

b Find the area of this cell, which will be the area of the ground that is controlled by mast *A*. [4]

c Find the furthest distance that a person could be from a mast. [3]

d Find the **exact** value of the ratio

area of equilateral triangle formed by three adjacent masts : area of a cell found in part **b**.
[2]

Markscheme

1 a Solving $y = 2x + 3$ and $7y + 4x = 75$ gives $A = (3, 9)$ — M1 A1

[2 marks]

b $y = 2x + 3$ has gradient of 2. $7y + 4x = 75$ has gradient of $-\frac{4}{7}$ — A1 A1

$-\frac{4}{7} \neq -\frac{1}{2}$ so not perpendicular. — R1

[3 marks]

c i $B = (-2, -1)$ **ii** $C = (10, 5)$ — A1 A1

[2 marks]

d $M = \left(\frac{-2+10}{2}, \frac{-1+5}{2}\right) = (4, 2)$ — M1 A1

[2 marks]

e $\sqrt{(10 - -2)^2 + (5 - -1)^2} = \sqrt{12^2 + 6^2} = 13.4\text{km (3 s.f.)}$ — M1 A2

[3 marks]

[Total: 12 marks]

2 a i $\frac{55}{200} = \frac{11}{40}$ **ii** $\frac{53}{200}$ **iii** $\frac{35}{200} = \frac{7}{40}$ — A1 A1 A1

iv $\frac{5}{53}$ **v** $\frac{5}{55} = \frac{1}{11}$ — A1 A1

[5 marks]

b ψ^2 test for independence — A1

H_0 : meal preference and age group are independent — A1

H_1 : meal preference and age group are dependent — A1

Meal\Age	Young	Middle aged	Elderly
Take-away	16.96	21.465	14.575
Eat out	24.64	31.185	21.175
Cook in	22.40	28.35	19.25

A2

All these values are >5, making the test valid. — R1

4 degrees of freedom — A1

p-value$= 3.24 \times 10^{-4} < 0.05$ — A2

So we reject H_0 and conclude that meal preference and age group are dependent.

R1 A1

[11 marks]

[Total: 16 marks]

3 a i $\bar{x}_A = 100.4$ **ii** $\bar{x}_B = 99.9$ — (M1) A1 A1

[3 marks]

b 2 sample t-test; 1 tailed — A1 A1

$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A > \mu_B$ — A1 A1

$p = 0.427\text{(3 s.f.)}$ — A2

$0.427 > 0.10$ so we accept $H_0 : \mu_A = \mu_B$ — R1

We do not believe the leader's claim that his citizens are more intelligent. — A1

[8 marks]

[Total: 11 marks]

4 a $r = 0.979$ (3 s.f.) — A2

Strong, positive, linear correlation — A1 A1 A1

[5 marks]

b **i** $y = 0.187x + 20.1$ A1 A1

ii $(128, 44)$ A1

iii $y = 0.187(110) + 20.1 = 40.7$ (3 s.f.) M1 A1

[5 marks]

c The line of x on y should be used instead when estimating a swimming time from a running time. R1

Using the line to estimate a swimming time when the running time is 23 seconds would be extrapolation a long way away from the given data. R1

[5 marks]

[Total: 12 marks]

5 **a** $X \sim N(10, 2^2)$, $P(9 < X < 12) = 0.533$ (3 s.f.) M1 A1

[2 marks]

b Let Michelle's mass be M.

$P(X > M) = 0.7 \Rightarrow P(X \le M) = 0.3$ M1 A1

$\Rightarrow M = 8.95...$ Michelle's weight is 8.95 kg (3 s.f.) A1

[3 marks]

c $P(X > 12 | X > 11) = \dfrac{P(X > 12)}{P(X > 11)} = \dfrac{1 - P(X \le 12)}{1 - P(X \le 11)}$ M1 A1

$= \dfrac{0.15865...}{0.30853...} = 0.514$ (3 s.f.) A2

[4 marks]

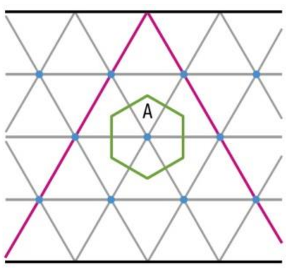
d **i** $Y \sim B(100, 0.0668)$, $P(Y = 6) = 0.159$ (3 s.f.) M1 A2

ii $P(Y \ge 3) = 1 - P(Y \le 2) = 0.967$ (3 s.f.) M1 A2

[6 marks]

[Total: 15 marks]

6 **a** **i**



A3

ii A regular hexagon A1 A1

[5 marks]

b Cell consists of 6 smaller equilateral triangles each of height 10 km R1

For each small triangle, $\frac{1}{2}$ base length $= 10\tan 30 = \dfrac{10}{\sqrt{3}}$ A1

Total area of regular hexagon $= 6 \times 10 \times \dfrac{10}{\sqrt{3}} = 200\sqrt{3} = 346\,\text{km}^2$ (3 s.f.) M1 A1

[4 marks]

c Furthest distance will be length of hypotenuse of right-angled triangle (half of one smaller equilateral triangle) R1

$= \dfrac{10}{\cos 30} = \dfrac{20}{\sqrt{3}} = 11.5\,\text{km}$ (3 s.f.) M1 A1

[3 marks]

d Original equilateral triangle has area $= \frac{1}{2} \times 20 \times 20 \sin 60 = 100\sqrt{3}$ M1

Area of cell found in (b) is $200\sqrt{3}$, so ratio is $100\sqrt{3} : 200\sqrt{3}$, which is 1:2 A1

[2 marks]

[Total: 14 marks]