Paper 1

Time allowed: 1 hour 30 minutes

Maximum number of marks: 80 marks

Answer all the questions.

All numerical answers must be given exactly or correct to three significant figures, unless otherwise stated in the question.

Answers should be supported by working and/or explanations. Where an answer is incorrect, some marks may be awarded for a correct method, provided this is shown clearly.

You need a graphical display calculator for this paper.

1 [Maximum mark: 6]

The dwarf planet Pluto can be considered as a sphere with radius of 1.1883×10^3 km.

a Find the volume of Pluto in km^3, expressing you answer in the form $a \times 10^k, 1 \leq a < 10, a \in \mathbb{Z}$ where

i a is given to 3 s.f.

ii a is given to the nearest integer. [4]

b Find the absolute percentage error in using part **a ii** rather than part **a i**. [2]

2 [Maximum mark: 5]

In this question give monetary answers to 2 decimal places.

Anna is going to invest money in a bank which pays 5% compound interest per year, compounded annually.

a Calculate how much Anna should deposit now, so that it will be worth £1000 in 10 years' time. [2]

Doris also invests money into an account paying 5% compound interest per year, compounded annually. She is going to deposit £X in the bank initially. In 5 years' time she will take out £1000, and leave the remainder in for another 6 years. At this time, she will then take out another £1000, which will leave her account empty.

b Calculate the value of X. [3]

3 [Maximum mark: 8]

Donald is making a decorative wall from bricks. Let u_n be the number of bricks in the nth row, starting from the top. The increasing sequence $\{u_n\}$ is an arithmetic progression and has a second term of 5 and an eleventh term of 41.

a Find the first term and the common difference. [3]

b **i** Find and simplify a formula for the nth term, u_n .

ii Hence find the value of n for which $u_n = 549$. [2]

c **i** Find and simplify a formula for the sum of the first n terms, S_n .

ii Hence, find the smallest value of n for which $S_n > 1000$. [3]

4 [Maximum mark: 5]

A surveyor is marking out an ornamental garden.

She starts at point A and walks in a straight line due North for 40 m to point B . She then turns clockwise and walks in a straight line for 50 m to point C. Here, she turns clockwise again and walks in a straight line for 60 m, which takes her back to point A exactly. Find the bearing of point C from point B . [5]

5 [Maximum mark: 8]

On a particular day, due to the tides, the depth of water, d metres, at the entrance to a harbour is modelled by

$$d = 9 + 6\cos(30t),$$

where t is time in hours after midnight.

A yacht with a keel can only enter or exit the harbour if the depth of the water is greater than 5 m.

a Find the time intervals when the yacht can enter or exit the harbour, for $0 \le t \le 24$. Give the answers in hours to 2 decimal places. [6]

b State the period of the above function. [2]

6 [Maximum mark: 6]

Marten has four cards with the numbers $1, 2, 3, 4$ on them. Kiki has four cards with the numbers $2, 3, 4, 5$ on them. They both select one of their cards at random.

a Find the probability that both their cards have the same number. [2]

b Find the probability that the sum of the two numbers is 5. [2]

c Find the probability that Kiki's number is greater than Marten's number. [2]

7 [Maximum mark: 5]

A small firm manufactures specialist, hand-built, sports cars. Let x be the number of cars that it produces in a month and let $£P$ be the profit that the firm makes that month. The variables are connected by the equation

$$P = -10x^4 + 500x^2 - 300 \text{ for } 0 \le x \le 7.$$

Note that x does **not** have to be an integer, as the firm can be part-way through completing a car at the end of a month.

a Find the profit made if 3 cars are produced. [2]

b Find the maximum profit that can be achieved in a month and the number of cars they should build in order to achieve this. [3]

8 [Maximum mark: 5]

A set of data, with values given in ascending order, is as follows:

$2, 3, 4, 6, 7, 9, 10, 11, 14, 17, x$.

a Given that x is an outlier, find an inequality that x must satisfy. [3]

b If this outlier is removed, state how the mean will change (without calculating it). [1]

c In general, if outliers are removed from a set of data, state what will happen to the standard deviation. [1]

9 [Maximum mark: 6]

The function $f(x)$ has the properties that $f(1) = 2$ and $\frac{\mathrm{d}f}{\mathrm{d}x} = 3x^2 + 8x + 7$.

Find the function $f(x)$. [6]

10 [Maximum mark: 8]

The area of a sector of a circle is $\frac{9\pi}{8}$ cm^2 and the arc length of this sector is $\frac{3\pi}{4}$ cm.

a Find

i the radius of the circle

ii the angle subtended at the centre of the circle by this sector. [6]

b Find the total distance around the perimeter of this sector, giving your answer to the nearest centimetre. [2]

11 [Maximum mark: 6]

The probability distribution given below is for a discrete random variable X that represents the gain of a player in a game.

X	-2	-1	0	1	2	a
$P(X = x)$	0.2	0.1		0.2	0.1	0.01

a Find $P(X=0)$. [2]

b Given that it is a fair game, find the value of the gain a. [4]

12 [Maximum mark: 6]

Consider the curve given by the equation $y(x)=x^3$. Find the coordinates of the point where the normal to the curve, at the point where $x=1$, intercepts the x-axis. [6]

13 [Maximum mark: 6]

Consider the graph $y(x)=x^2-1$.

a Find the value of $a>0$ such that $\int_0^a y(x)\,\mathrm{d}x=0$. [4]

b Use a sketch to give an explanation for this result. [2]

Markscheme

1 **a** $\frac{4\pi\left(1.1883\times10^{3}\right)^{3}}{3}=7.02856\ldots\times10^{9}$ M1 A1

i $7.03\times10^{9}\,\text{km}^{3}$ (3 s.f.) A1

ii $7\times10^{9}\,\text{km}^{3}$ A1

[4 marks]

b $\left|\frac{7-7.03}{7.03}\right|\times100\%=0.427\%$ M1 A1

[2 marks]
[Total: 6 marks]

2 **a** $Y(1.05)^{10}=1000\Rightarrow Y=£613.91$ (2 d.p.) M1 A1

[2 marks]

b Require $(1.05^{5}X-1000)1.05^{6}=1000$ M1

$\Rightarrow 1.05^{11}X=\left(1+1.05^{6}\right)1000\Rightarrow X=1368.21$ (2 d.p.) A1 A1

[3 marks]
[Total: 5 marks]

3 **a** $a+d=5,\quad a+10d=41$ M1

Solving gives $a=1,\ d=4$ A1 A1

[3 Marks]

b **i** $u_n=1+(n-1)4=4n-3$ A1

ii Solving $4n-3=549$ gives $n=138$ A1

[2 Marks]

c **i** $S_n=\frac{n}{2}(2+(n-1)4)=2n^2-n$ A1

ii Solving (e.g. with "table" on GDC) $2n^2-n>1000$ gives $n=23$ ($S_n=1035$) M1 A1

[3 marks]
[Total: 8 marks]

4 $60^2=40^2+50^2-2\times40\times50\cos B$ M1 A1

$\Rightarrow 5=40\cos B\Rightarrow B=82.819\ldots$ A1

Bearing is $180-82.819=97.2^{\circ}$ (3 s.f.) M1 A1

[Total: 5 marks]

5 **a** (From graph on calculator) $0\le t<4.39,\ 7.61<t<16.39,\ 19.61<t\le24$ A2 A2 A2

[6 marks]

b $\frac{360}{30}=12$ M1 A1

[2 marks]
[Total: 8 marks]

6 (By use of a lattice diagram or similar)

a $\frac{3}{16}$ A2

[2 marks]

b $\frac{3}{16}$ A2

[2 marks]

c	$\frac{10}{16}=\frac{5}{8}$	A2
		[2 marks]
		[Total: 6 marks]
7 a	Using a graph on the calculator, or substitution into the equation gives	(M1)
	£3390	A1
		[2 marks]
b	Identifying the maximum on the graph of P against x gives	(M1)
	£5950 by producing 5 cars	A1 A1
		[3 marks]
		[Total: 5 marks]
8 a	IQR = 14-4=10	A1
	so x>14+1.5×10⟹x>29	M1 A1
		[3 marks]
b	The mean will decrease.	R1
		[1 mark]
c	The standard deviation will decrease.	R1
		[1 mark]
		[Total: 5 marks]
9	$f(x)=\int 3x^2+8x+7\,dx = x^3+4x^2+7x+c$	M1 A1 A1
	Through $(1,2) \Rightarrow 1+4+7+c=2 \Rightarrow c=-10$	M1 A1
	$f(x)=x^3+4x^2+7x-10$	A1
		[6 marks]
		[Total: 6 marks]
10 a	Let α be angle subtended by the sector at the centre of the circle.	
	Then $\pi r^2\frac{\alpha}{360}=\frac{9\pi}{8}$ and $2\pi r\frac{\alpha}{360}=\frac{3\pi}{4}$	A1 A1
i	Dividing first expression by second gives $\frac{r}{2}=\frac{3}{2} \Rightarrow r=3\,\text{cm}$	M1 A1
ii	Substituting $r=3\,\text{cm}$ in first equation gives $\frac{\alpha}{360}=\frac{1}{8} \Rightarrow \alpha=45^{\circ}$	M1 A1
		[6 marks]
b	$\frac{3\pi}{4}+6=8.356\ldots=8$ cm (nearest cm)	M1 A1
		[2 marks]
		[Total: 8 marks]
11 a	As probabilities must add up to 1, $P(X=0)=0.39$	R1 A1
		[2 marks]
b	Fair game $\Rightarrow$ $E(X)=0$	R1
	$\Rightarrow -2\times0.2-1\times0.1+0\times0.39+1\times0.2+2\times0.1+0.01\times a=0$	M1 A1
	$\Rightarrow a=10$	A1
		[4 marks]
		[Total: 6 marks]
12	$\frac{dy}{dx}=3x^2$, $\left.\frac{dy}{dx}\right\|_{x=1}=3$	M1 A1

Gradient of normal is $-\frac{1}{3}$, so equation of normal is $y=-\frac{1}{3}x+c$ A1 M1

Normal passes through $(1,1)\Rightarrow 1=-\frac{1}{3}+c\Rightarrow c=\frac{4}{3}$ M1

Intercept is $\left(0,\frac{4}{3}\right)$ A1

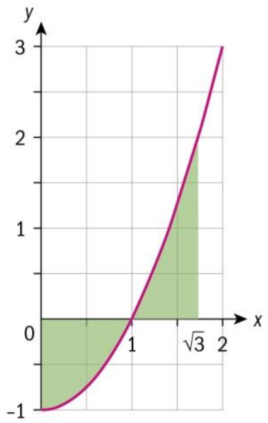
[6 marks]
[Total: 6 marks]

13 a $\int_0^a x^2-1\,\mathrm{d}x=\left[\frac{x^3}{3}-x\right]_0^a=\frac{a^3}{3}-a$ M1 A1

$a\left(\frac{a^2}{3}-1\right)=0\Rightarrow a^2=3\Rightarrow a=\sqrt{3}$ M1 A1

[4 marks]

b



A1

$y(x)$ is below the x-axis for $0<x<1$, so $\int_0^1 y(x)\,\mathrm{d}x$ is negative

$y(x)$ is above the x-axis for $1<x<\sqrt{3}$, so $\int_1^{\sqrt{3}} y(x)\,\mathrm{d}x$ is positive

These areas are equal in magnitude and opposite in sign, so they cancel out. R1

[2 marks]
[Total: 6 marks]