## Optimization [18 marks]

Haruka has an eco-friendly bag in the shape of a cuboid with width 12 cm , length 36 cm and height of 9 cm . The bag is made from five rectangular pieces of cloth and is open at the top.


1a. Calculate the area of cloth, in $\mathrm{cm}^{2}$, needed to make Haruka's bag.
[2 marks]

1b. Calculate the volume, in $\mathrm{cm}^{3}$, of the bag.

Nanako decides to make her own eco-friendly bag in the shape of a cuboid such that the surface area is minimized.

The width of Nanako's bag is $x \mathrm{~cm}$, its length is three times its width and its height is $y \mathrm{~cm}$.
diagram not to scale


The volume of Nanako's bag is $3888 \mathrm{~cm}^{3}$.

1c. Use this value to write down, and simplify, the equation in $x$ and $y$ for the[2 marks] volume of Nanako's bag.

1d. Write down and simplify an expression in $x$ and $y$ for the area of cloth, $A,[2$ marks] used to make Nanako's bag.

1e. Use your answers to parts (c) and (d) to show that $A=3 x^{2}+\frac{10368}{x}$.

1f. Find $\frac{\mathrm{d} A}{\mathrm{~d} x}$.

1g. Use your answer to part (f) to show that the width of Nanako's bag is 12 [3 marks] cm.

1h. The cloth used to make Nanako's bag costs 4 Japanese Yen (JPY) per $\mathrm{cm}^{2}$.

Find the cost of the cloth used to make Nanako's bag.

