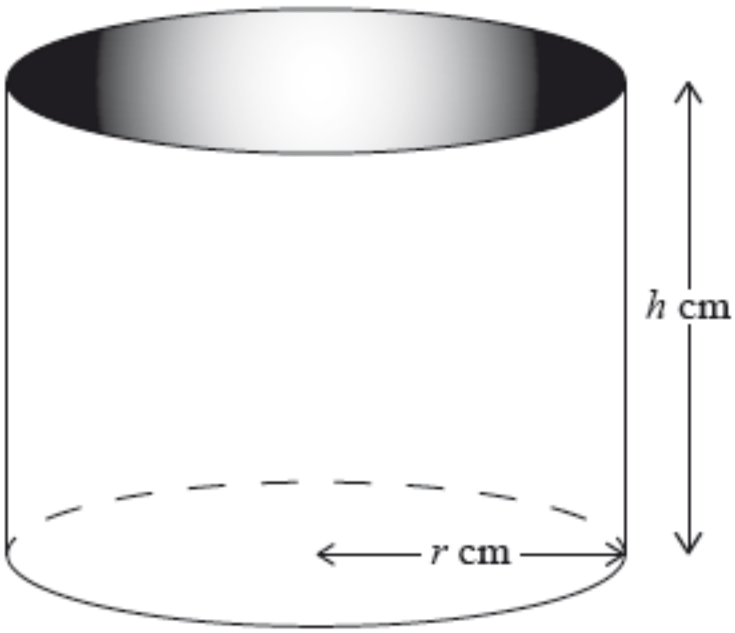
OP2 [54 marks]

1a. *[2 marks]*

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

Write down a formula for A, the surface area to be coated.

1b. *[1 mark]*

The volume of the water container is 0.5 m^3.

Express this volume in cm^3.

1c. *[1 mark]*

Write down, in terms of r and h, an equation for the volume of this water container.

1d. *[2 marks]*

Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$.

1e. *[3 marks]*

The water container is designed so that the area to be coated is minimized.

Find $\frac{dA}{dr}$.

1f. *[3 marks]*

Using your answer to part (e), find the value of r which minimizes A.

1g. *[2 marks]*

Find the value of this minimum area.

1h. *[3 marks]*

One can of water-resistant material coats a surface area of 2000 cm^2.

Find the least number of cans of water-resistant material that will coat the area in part (g).

2a. *[3 marks]*

A company sells fruit juices in cylindrical cans, each of which has a volume of 340 cm^3. The surface area of a can is A cm^2 and is given by the formula

$A = 2\pi r^2 + \frac{680}{r}$,

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

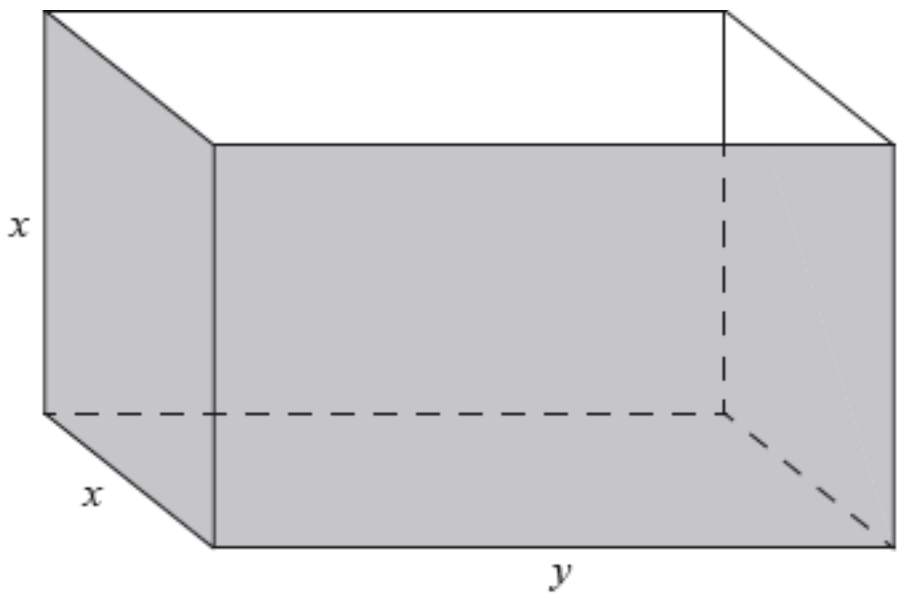
Find $\frac{dA}{dr}$

2b. *[3 marks]*

Calculate the value of r that minimizes the surface area of a can.

3a. *[4 marks]*

Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is 36 m^3.

Let $A(x)$ be the outside surface area of the container.

Show that $A(x) = \frac{108}{x} + 2x^2$.

3b. *[2 marks]*

Find $A'(x)$.

3c. *[5 marks]*

Given that the outside surface area is a minimum, find the height of the container.

3d. *[5 marks]*

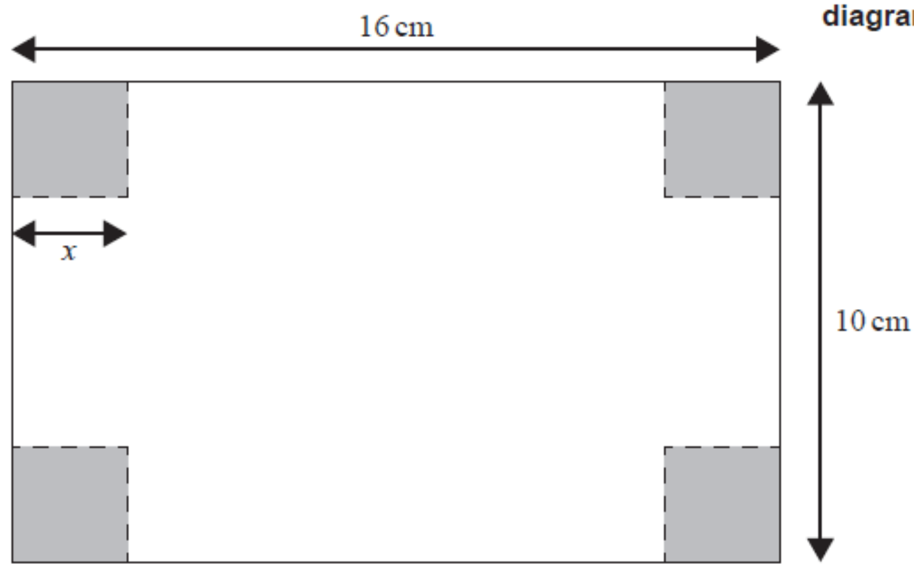
Fred paints the outside of the container. A tin of paint covers a surface area of 10 m^2 and costs $20. Find the total cost of the tins needed to paint the container.

4a. *[2 marks]*

Hugo is given a rectangular piece of thin cardboard, 16 cm by 10 cm. He decides to design a tray with it.

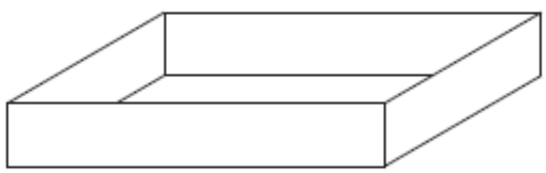
He removes from each corner the shaded squares of side x cm, as shown in the following diagram.

diagram not to scale



The remainder of the cardboard is folded up to form the tray as shown in the following diagram.

diagram not to scale



Write down, **in terms of** x , the length and the width of the tray.

4b. *[4 marks]*

(i) State whether x can have a value of 5. Give a reason for your answer.

(ii) Write down the interval for the possible values of x .

4c. *[2 marks]*

Show that the volume, V cm^3, of this tray is given by

$$V = 4x^3 - 52x^2 + 160x.$$

4d. *[3 marks]*

Find $\frac{dV}{dx}$.

4e. *[4 marks]*

Sketch the graph of $V = 4x^3 - 51x^2 + 160x$, for the possible values of x found in part (b)(ii), and $0 \leq V \leq 200$. Clearly label the maximum point.