## Mathematics assessed student work

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## Diploma Programme

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## IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.


## Overview

The intention of this section is to demonstrate the overall marking standards that are required, as well as to illustrate how the achievement levels for each criterion should be awarded. The assessment was undertaken by experienced moderators.

The comments and marks provided for criteria A-E are those of these moderators. All the explorations have been marked using both the standard level $(\mathrm{SL})$ and higher level $(\mathrm{HL})$ criteria. The requirements for criteria A-D are identical for SL and HL; it is only criterion E (use of mathematics) that is different. It is essential that teachers of both subjects and levels standardize their marking of the exploration before submitting their marks.
Some of the explorations have separate comments for mathematics: analysis and approaches and for mathematics: applications and interpretation. This is to illustrate how the marking may be affected based on whether a student has met the mathematics used during their course or has chosen to use mathematics not from their course. Generally, this may affect the mark given in criterion $C$ (personal engagement).

Teachers may simply wish to see how an exploration was marked. Using the links, teachers can navigate to the comments and to the annotated student work. The annotated work shows, for each criterion, where in the exploration evidence can be seen for reaching a decision on awarding the mark for that criterion.

Alternatively, teachers may wish to mark the student work themselves. Using the links, the student work can also be viewed in its original format. Teachers can then compare their own marking to that of the moderator and go on to look at the annotated student work.
The comments should be read in conjunction with the explorations and the annotations on the explorations.

The table below categorizes the explorations by the "best fit" for the topic of the exploration and shows the marks given for SL (first number) and HL (second number). When a cell is split in two, the top part shows the marks for mathematics: analysis and approaches (analysis) and the bottom part shows the marks for mathematics: applications and interpretation (applications).
Samples 30 to 49 were written by students following one of the DP mathematics courses first taught in 2019. The samples are grouped as follows:

| $30-34$ | DP mathematics: applications and interpretation SL |
| :--- | :--- |
| $35-39$ | DP mathematics: analysis and approaches SL |
| $40-44$ | DP mathematics: applications and interpretation HL |
| $45-49$ | DP mathematics: analysis and approaches HL |


| Example <br> number | Title | Number <br> and <br> algebra | Functions <br> (and <br> modelling) | Geometry <br> and <br> trigonometry | Probability <br> and <br> statistics | Calculus | Other |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Horse's jump |  | SL 11 marks <br> HL 9 marks |  |  |  |  |
| 2 | Infant mortality |  |  | SL 16 marks <br> HL 14 marks |  |  |  |
| 3 | Packaging and <br> geometrical <br> shapes |  |  | SL 14 marks |  |  |  |
| HL12 marks |  |  |  |  |  |  |  |


| Example number | Title | Number and algebra | Functions (and modelling) | Geometry and trigonometry | Probability and statistics | Calculus | Other |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | Rowing speeds |  |  |  | SL 16 marks HL 15 marks |  |  |
| 5 | Social media |  |  |  | SL 9 marks HL 7 marks |  |  |
| 6 | Path of quickest descent |  | SL 9 marks HL 8 marks |  |  |  |  |
| 7 | Bayes' Theorem and baseball |  |  |  | SL 10 marks HL 9 marks |  |  |
| 8 | Modelling stools |  |  |  |  | SL 13 marks HL 11 marks |  |
| 9 | The Cantor set |  |  |  |  |  | SL 12 <br> marks <br> HL 11 <br> marks |
| 10 | Human population growth |  | Analysis SL16 marks Analysis HL 15 marks <br> Applications SL16 marks Applications HL 15 marks |  |  |  |  |
| 11 | Pursuit curves |  |  |  |  | SL 15 marks HL 14 marks |  |
| 12 | Microwave popcorn |  | SL 20 marks <br> HL 20 marks |  |  |  |  |
| 13 | Ice cream |  |  |  |  | SL 15 marks HL 13 marks |  |
| 14 | Zeno's arrow paradox |  |  |  |  | SL 18 marks HL 17 marks |  |
| 15 | Chinese remainder theorem | SL 12 marks HL 11 marks |  |  |  |  |  |
| 16 | Solar panels |  |  | SL 20 marks HL 20 marks |  |  |  |
| 17 | Hyperboloids |  |  | SL 19 marks HL 19 marks |  |  |  |
| 18 | Prime number theories | SL 10 marks HL 8 marks |  |  |  |  |  |


| Example number | Title | Number and algebra | Functions (and modelling) | Geometry and trigonometry | Probability and statistics | Calculus | Other |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | Optimisation at the cottage |  |  |  |  | SL 11 marks HL 9 marks |  |
| 20 | Encrypting messages | Analysis SL17 marks <br> Analysis HL 15 marks <br> Application s SL16 marks <br> Application s HL 14 marks |  |  |  |  |  |
| 21 | Gabriel Dawe's rainbow coloured curves |  | SL 19 marks HL 18 marks |  |  |  |  |
| 22 | Titus tunnel bridge |  | SL 19 marks HL 18 marks |  |  |  |  |
| 23 | Transport fare charges in Madrid |  | SL 15 marks HL 14 marks |  |  |  |  |
| 24 | Golf and kinematics |  | SL 7 marks HL 7 marks |  |  |  |  |
| 25 | Mathematics in barcodes | SL 13 marks HL 12 marks |  |  |  |  |  |
| 26 | Probability in the courtroom |  |  |  | SL 19 marks HL 18 marks |  |  |
| 27 | Converting piano music to guitar music |  | SL 11 marks HL 10 marks |  |  |  |  |
| 28 | The Chinese postman problem: analysis | Analysis SL 14 marks HL 14 marks |  |  |  |  |  |
| 29 | Real-world applications of the prisoner's dilemma |  |  |  | SL 10 marks HL 9 marks |  |  |
| 30 | Airline overbooking |  |  |  | SL 19 marks HL 17 marks |  |  |
| 31 | Volleyball spike |  |  | SL 16 marks HL 14 marks |  |  |  |
| 32 | Optimum age for 100m sprint |  |  |  | SL 15 marks HL 14 marks |  |  |

$\left.\begin{array}{|c|l|l|l|l|l|l|l|}\hline \begin{array}{l}\text { Example } \\ \text { number }\end{array} & \begin{array}{l}\text { Title } \\ \text { Number } \\ \text { and } \\ \text { algebra }\end{array} & \begin{array}{l}\text { Functions } \\ \text { (and } \\ \text { modelling) }\end{array} & \begin{array}{l}\text { Geometry } \\ \text { and } \\ \text { trigonometry }\end{array} & \begin{array}{l}\text { Probability } \\ \text { and } \\ \text { statistics }\end{array} & \text { Calculus } & \text { Other } \\ \hline 33 & \begin{array}{l}\text { Volume of a chess } \\ \text { pawn }\end{array} & & & \text { SL 17 marks }\end{array}\right\}$

## Example 1: Horse's jump

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 2: Infant mortality

Subject: Mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 3: Packaging and geometrical shapes

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 4: Rowing speeds

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level and (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 5: Social media

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
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## Example 6: Path of quickest descent

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 7: Bayes' Theorem and baseball

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 8: Modelling stools

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
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Examiner's comments coversheet (PDF)

## Example 9: The Cantor set

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 10: Human population growth

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 11: Pursuit curves

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
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## Example 12: Microwave popcorn

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 13: Ice cream

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
Annotated student work (PDF)
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## Example 14: Zeno's arrow paradox

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 15: Chinese remainder theorem

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and (HL)
Student work (PDF)
Annotated student work (PDF)
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## Example 16: Solar panels

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 17: Hyperboloids

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 18: Prime number theories

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 19: Optimisation at the cottage

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment (IA), standard level (SL) and higher level (HL)
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## Example 20: Encrypting messages

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 21: Gabriel Dawe's rainbow coloured curves

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and (HL)
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## Example 22: Titus tunnel bridge

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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Examiner's comments coversheet (PDF)

## Example 23: Transport fare charges in Madrid

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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Examiner's comments coversheet (PDF)

## Example 24: Golf and kinematics

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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Annotated student work (PDF)
Examiner's comments coversheet (PDF)

## Example 25: Mathematics in barcodes

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 26: Probability in the courtroom

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 27: Converting piano music to guitar music

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 28: The Chinese postman problem: analysis

Subject: Mathematics: analysis and approaches
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 29: Real-world applications of the prisoner's dilemma

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
Student work (PDF)
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## Example 30: Airline overbooking

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 31: Volleyball spike

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 32: Optimum age for 100 m sprint

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 33: Volume of a chess pawn

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 34: Sentence length and genre of novel

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 35: Paracetamol tablet optimization

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 36: Internal temperature of cookies

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 37: Towers of Hanoi extension

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 38: Morphine in pharmacokinetics

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 39: Voters of tomorrow

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 40: Radioactive modelling

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## Example 41: Travelling salesman and a trip around India

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## Example 42: Terrarium design for garden spiders

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## Example 43: Perishable and non-perishable goods

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
Paper component: Internal assessment, standard level (SL) and higher level (HL)
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## Example 44: Ethnicity spread and crime rate

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 45: Radii of curvature

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 46: Robotic arm

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 47: Efficiency of sun visors

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 48: Path of a violin bow frog

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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## Example 49: How can we make it better?

Subject: Mathematics: analysis and approaches and mathematics: applications and interpretation
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