

# Mathematics assessed student work



International Baccalaureate<sup>®</sup> Baccalauréat International Bachillerato Internacional



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#### Diploma Programme Mathematics assessed student work

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#### **IB** mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.



RISK

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

#### As IB learners we strive to be:

INKER

#### **INQUIRERS**

OWI FDG

ATORS

We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

#### **KNOWLEDGEABLE**

We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

#### **THINKERS**

We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

#### COMMUNICATORS

We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

#### PRINCIPLED

We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

#### **OPEN-MINDED**

We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

#### CARING

We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

#### **RISK-TAKERS**

We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

#### BALANCED

We understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

#### REFLECTIVE

We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.



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#### Overview

The intention of this section is to demonstrate the overall marking standards that are required, as well as to illustrate how the achievement levels for each criterion should be awarded. The assessment was undertaken by experienced moderators.

The comments and marks provided for criteria A–E are those of these moderators. All the explorations have been marked using both the standard level (SL) and higher level (HL) criteria. The requirements for criteria A–D are identical for SL and HL; it is only criterion E (use of mathematics) that is different. It is essential that teachers of both subjects and levels standardize their marking of the exploration before submitting their marks.

Some of the explorations have separate comments for mathematics: analysis and approaches and for mathematics: applications and interpretation. This is to illustrate how the marking may be affected based on whether a student has met the mathematics used during their course or has chosen to use mathematics not from their course. Generally, this may affect the mark given in criterion C (personal engagement).

Teachers may simply wish to see how an exploration was marked. Using the links, teachers can navigate to the comments and to the annotated student work. The annotated work shows, for each criterion, where in the exploration evidence can be seen for reaching a decision on awarding the mark for that criterion.

Alternatively, teachers may wish to mark the student work themselves. Using the links, the student work can also be viewed in its original format. Teachers can then compare their own marking to that of the moderator and go on to look at the annotated student work.

The comments should be read in conjunction with the explorations and the annotations on the explorations.

The table below categorizes the explorations by the "best fit" for the topic of the exploration and shows the marks given for SL (first number) and HL (second number). When a cell is split in two, the top part shows the marks for mathematics: analysis and approaches (analysis) and the bottom part shows the marks for mathematics: applications and interpretation (applications).

Samples 30 to 49 were written by students following one of the DP mathematics courses first taught in 2019. The samples are grouped as follows:

30–34	DP mathematics: applications and interpretation SL
35–39	DP mathematics: analysis and approaches SL
40–44	DP mathematics: applications and interpretation HL
45–49	DP mathematics: analysis and approaches HL

Example number	Title	Number and algebra	Functions (and modelling)	Geometry and trigonometry	Probability and statistics	Calculus	Other
1	Horse's jump		SL 11 marks				
			HL 9 marks				
2	Infant mortality				SL 16 marks		
					HL 14 marks		
3	Packaging and			SL 14 marks			
	geometrical shapes			HL12 marks			

Example number	Title	Number and algebra	Functions (and modelling)	Geometry and trigonometry	Probability and statistics	Calculus	Othe
4	Rowing speeds				SL 16 marks HL 15 marks		
5	Social media				SL 9 marks HL 7 marks		
6	Path of quickest descent		SL 9 marks HL 8 marks				
7	Bayes' Theorem and baseball				SL 10 marks HL 9 marks		
8	Modelling stools					SL 13 marks HL 11 marks	
9	The Cantor set						SL 12 marks HL 11 marks
10	Human population growth		Analysis SL16 marks Analysis HL 15 marks Applications SL16 marks				
			Applications HL 15 marks				
11	Pursuit curves					SL 15 marks HL 14 marks	
12	Microwave popcorn		SL 20 marks HL 20 marks				
13	Ice cream					SL 15 marks HL 13 marks	
14	Zeno's arrow paradox					SL 18 marks HL 17 marks	
15	Chinese remainder theorem	SL 12 marks HL 11 marks					
16	Solar panels			SL 20 marks HL 20 marks			
17	Hyperboloids			SL 19 marks HL 19 marks			
18	Prime number theories	SL 10 marks HL 8 marks					

Example number	Title	Number and algebra	Functions (and modelling)	Geometry and trigonometry	Probability and statistics	Calculus	Other
19	Optimisation at the cottage	aigebra	inoucling)	ligonometry	Statistics	SL 11 marks HL 9 marks	
20	Encrypting messages	Analysis SL17 marks Analysis HL					
		15 marks Application s SL16 marks Application s HL 14 marks					
21	Gabriel Dawe's rainbow coloured curves		SL 19 marks HL 18 marks				
22	Titus tunnel bridge		SL 19 marks HL 18 marks				
23	Transport fare charges in Madrid		SL 15 marks HL 14 marks				
24	Golf and kinematics		SL 7 marks HL 7 marks				
25	Mathematics in barcodes	SL 13 marks HL 12 marks					
26	Probability in the courtroom				SL 19 marks HL 18 marks		
27	Converting piano music to guitar music		SL 11 marks HL 10 marks				
28	The Chinese postman problem: analysis	Analysis SL 14 marks HL 14 marks					
29	Real-world applications of the prisoner's dilemma				SL 10 marks HL 9 marks		
30	Airline overbooking				SL 19 marks HL 17 marks		
31	Volleyball spike			SL 16 marks HL 14 marks			
32	Optimum age for 100m sprint				SL 15 marks HL 14 marks		

Example	Title	Number	Functions	Geometry	Probability	Calculus	Othe
number		and algebra	(and modelling)	and trigonometry	and statistics		
33	Volume of a chess					SL 17 marks	
	pawn					HL 15 marks	
34	Sentence length				SL 14 marks		
	and genre of novel				HL 13 marks		
35	Paracetamol					SL 20 marks	
	tablet optimization					HL 18 marks	
36	Internal		SL 18 marks				
	temperature of cookies		HL 16 marks				
37	Towers of Hanoi	SL 16 marks					
	extension	HL 14 marks					
38	Morphine in					SL 15 marks	
	pharmacokinetics					HL 13 marks	
39	Voters of				SL 17 marks		
	tomorrow				HL 15 marks		
40	Radioactive					SL 19 marks	
	modelling					HL 18 marks	
41	Travelling			SL 15 marks			
	salesman and a trip around India			HL 13 marks			
42	Terrarium design		SL 15 marks				
	for garden spiders		HL 13 marks				
43	Perishable and				SL 17 marks		
	non-perishable goods				HL 16 marks		
44	Ethnicity spread				SL 17 marks		
	and crime rate				HL 16 marks		
45	Radii of curvature					SL 18 marks	
						HL 17 marks	
46	Robotic arm		SL 18 marks				
			HL 17 marks				
47	Efficiency of sun			SL 14 marks			
	visors			HL 13 marks			
48	Path of a violin			SL 18 marks			
	bow frog			HL 17 marks			
49	How can we make					SL 16 marks	
	it better?					HL 15 marks	

### Example 1: Horse's jump

# Example 2: Infant mortality

## Example 3: Packaging and geometrical shapes

### Example 4: Rowing speeds

### Example 5: Social media

### Example 6: Path of quickest descent

## Example 7: Bayes' Theorem and baseball

### Example 8: Modelling stools

### Example 9: The Cantor set

## Example 10: Human population growth

### Example 11: Pursuit curves

### Example 12: Microwave popcorn

### Example 13: Ice cream

### Example 14: Zeno's arrow paradox

### Example 15: Chinese remainder theorem

## Example 16: Solar panels

### Example 17: Hyperboloids

### Example 18: Prime number theories

### Example 19: Optimisation at the cottage

### Example 20: Encrypting messages

#### Example 21: Gabriel Dawe's rainbow coloured curves

### Example 22: Titus tunnel bridge

### Example 23: Transport fare charges in Madrid

### Example 24: Golf and kinematics

### Example 25: Mathematics in barcodes

### Example 26: Probability in the courtroom

### Example 27: Converting piano music to guitar music

# Example 28: The Chinese postman problem: analysis

Subject: Mathematics: analysis and approaches Paper component: Internal assessment, standard level (SL) and higher level (HL) Student work (PDF) Annotated student work (PDF) Examiner's comments coversheet (PDF)

# Example 29: Real-world applications of the prisoner's dilemma

# Example 30: Airline overbooking

#### Example 31: Volleyball spike

#### Example 32: Optimum age for 100m sprint

# Example 33: Volume of a chess pawn

#### Example 34: Sentence length and genre of novel

#### Example 35: Paracetamol tablet optimization

# Example 36: Internal temperature of cookies

#### Example 37: Towers of Hanoi extension

#### Example 38: Morphine in pharmacokinetics

# Example 39: Voters of tomorrow

#### Example 40: Radioactive modelling

# Example 41: Travelling salesman and a trip around India

#### Example 42: Terrarium design for garden spiders

# Example 43: Perishable and non-perishable goods

#### Example 44: Ethnicity spread and crime rate

#### Example 45: Radii of curvature

#### Example 46: Robotic arm

#### Example 47: Efficiency of sun visors

#### Example 48: Path of a violin bow frog

#### Example 49: How can we make it better?