AP Calculus BC Name ________________________

6.5 – Logistic Growth

Answers to MC discussed in class…..ask if you have questions.

1. What are all the horizontal asymptotes of all the solutions of the logistic differential equation $\frac{dy}{dx} = y\left(8 - \frac{y}{1000}\right)$?

a. $y = 0$ b. $y = 8$ c. $y = 8{,}000$ d. $y = 0$ and $y = 8$ e. $y = 0$ and $y = 8{,}000$

2. Suppose $P(t)$ denotes the size of an animal population at time t and its growth is described by the differential equation $\frac{dP}{dt} = 0.002P(1000 - P)$. The population is growing fastest

a. initially b. when $P = 500$ c. when $P = 1000$ d. when $\frac{dP}{dt} = 0$ e. when $\frac{d^2P}{dt^2} > 0$

3. Which of the following statements characterize the logistic growth of a population whose limiting value is L?

I. The rate of growth increases initially.

II. The growth rate attains a maximum when the population equals L/2.

III. The growth rate approaches 0 as the population approaches L.

a. I b. II c. I and II d. II and III e. I, II, III (I may be true, but not necessarily)

4. (1998 BC 26) The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

A. 2,500 B. 3,000 C. 4,200 D. 5,000 E. 10,000

5. Find the carrying capacity for a population growth rate modeled by $\frac{dP}{dt} = 6P - 0.012P^2$.

a. 500 b. 50 c. 0.012 d. 0.002 e. None of these

1993 AB6 – No Calculator (not logistic, but similar)

Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to 800 – $P(t)$, where the constant of proportionality is k.

a. If $P(0) = 500$, find $P(t)$ in terms of t and k.

(a) $P'(t) = k(800 - P(t))$

$$\frac{dP}{800 - P} = k\,dt$$

$$-\ln|800 - P| = kt + C_0$$

$$|800 - P| = C_1 e^{-kt}$$

$$800 - 500 = C_1 e^0$$

$$C_1 = 300$$

Therefore $P(t) = 800 - 300e^{-kt}$

b. If $P(2) = 700$, find k.

(b) $P(2) = 700 = 800 - 300e^{-2k}$

$$k = \frac{\ln 3}{2} \approx 0.549$$

c. Find $\lim_{t\to\infty} P(t)$.

(c) $\lim_{t\to\infty}\left(800 - 300e^{-\frac{\ln 3}{2}t}\right) = 800$

1991 BC6 – No calculator

A certain rumor spreads through a community at the rate of $\frac{dy}{dt} = 2y(1-y)$, where y is the proportion of the population that has heard the rumor at time t.

a. What proportion of the community has heard the rumor when it is spreading the fastest?

(a) $2y(1-y) = 2y - 2y^2$ is largest when $2 - 4y = 0$

so proportion is $y = \frac{1}{2}$

b. If at time $t = 0$ ten percent of the people have heard the rumor, find y as a function of t.

(b) $\frac{1}{y(1-y)}dy = 2\,dt$

$$\int \frac{1}{y(1-y)}dy = \int 2\,dt$$

$$\int \frac{1}{y} + \frac{1}{1-y}dy = \int 2\,dt$$

$$\ln y - \ln(1-y) = 2t + C$$

$$\ln\frac{y}{1-y} = 2t + C$$

$$\frac{y}{1-y} = ke^{2t}$$

$$y(0) = 0.1 \Rightarrow k = \frac{1}{9}$$

$$y = \frac{e^{2t}}{9 + e^{2t}}$$

c. At what time t is the rumor spreading the fastest?

(c) $\frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1}{9}e^{2t}$

$$1 = \frac{1}{9}e^{2t}$$

$$t = \frac{1}{2}\ln 9 = \ln 3$$