AP Calculus BC

Logistics Functions

Name: _____   Date: _____

Show full solutions to each of the following. Do your work on separate paper. Attach it to this sheet. Turn it in by the end of the period.

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1. The growth rate of a population P of squirrels in a newly established wildlife preserve is modeled by the differential equation $\frac{dP}{dt} = 0.006P(240 - P)$, where t is measured in years.

    a. What is the carrying capacity for squirrels in this wildlife preserve?

    b. What is the squirrel population when the population is growing the fastest?

    c. What is the **rate of change** of the population when it is growing the fastest?

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2. The spread of a disease through a community can be modeled with the logistic equation $y = \frac{0.74}{1 + 34e^{-0.25t}}$ where y is the proportion of people who **are** infected after t days.

    a. What percentage of the people in the community will **not** become infected?

    b. On what day will the disease be spreading the fastest? What % of the community has the disease at this time?

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3. The spread of a disease through a community can be modeled with the logistic equation $y = \frac{5400}{3 + 99e^{-0.15t}}$ where y is the number of people infected after t days.

    a. Find C, k, and d. Be careful. Things aren't always what they seem.

Note: Assume the function is of the form $y = \frac{C}{1 + d \cdot e^{-kt}}$

    b. How many people are infected when the disease is spreading the fastest?

4. A population is modeled by a function P that satisfies the logistic differential equation
$$\frac{dP}{dt} = \frac{P}{8}\left(1 - \frac{P}{12}\right).$$

    a. Find the carrying capacity C.

    b. Find the constant k.

    c. What is the population at the time when the population is growing the fastest?

    d. Find the logistic model P(t) given an initial condition P(0) = 3.

5. A population of wild pigs just outside a small town is modeled by the function $P(t) = \dfrac{220}{1 + 72e^{-0.81t}}$

where t is measured in months.

    a. What is the carrying capacity?

    b. What is the initial population of pigs?

    c. What is the population after 2 months? After 4 months? Calculator ok. No work needed, just good notation. Round to the nearest pig.

    d. When does the population reach 60 pigs? You can use the calculator to verify your answer but show the equation work here. Round final answer to three decimal places.