Name____

Section 1.9 Inverse Functions

Objective: In this lesson you learned how to find inverse functions graphically and algebraically.

Important Vocabulary Define each term or concept.

Inverse function Let f and g be two functions. If $f(g(x)) = x$ for every x in the domain of g and $g(f(x)) = x$ for every x in the domain of f, then g is the inverse function of the function f. The function g is denoted by f^{-1}.

Horizontal Line Test A function f has an inverse if and only if no horizontal line intersects the graph of f at more that one point.

I. Inverse Functions (Pages 93–94)

What you should learn
How to find inverse functions informally and verify that two functions are inverse functions of each other

For a function f that is defined by a set of ordered pairs, to form the inverse function of f, . . . interchange the first and second coordinates of each of these ordered pairs.

For a function f and its inverse f^{-1}, the domain of f is equal to ______the range of f^{-1}______, and the range of f is equal to ______the domain of f^{-1}______.

To verify that two functions, f and g, are inverse functions of each other, . . . find $f(g(x))$ and $g(f(x))$. If both of these compositions are equal to the identity function x for every x in the domain of the inner function, then the functions are inverses of each other.

Example 1: Verify that the functions $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$ are inverse functions of each other.

II. The Graph of an Inverse Function (Page 95)

What you should learn
How to use graphs of functions to determine whether functions have inverse functions

If the point (a, b) lies on the graph of f, then the point (__b__, __a__) must lie on the graph of f^{-1} and vice versa. The graph of f^{-1} is a *reflection* of the graph of f in the line ______$y = x$______.

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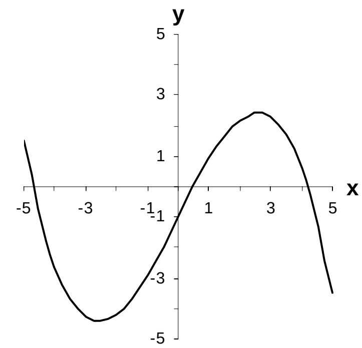
III. One-to-One Functions (Page 96)

To tell whether a function has an inverse function from its graph, . . . simply use the Horizontal Line Test, that is, check to see that no horizontal line intersects the graph of the function at more than one point.

> ***What you should learn***
> How to use the Horizontal Line Test to determine if functions are one-to-one

A function f is **one-to-one** if . . . each value of the dependent variable corresponds to exactly one value of the independent variable.

A function f has an inverse function if and only if f is ____one-to-one____.



Example 2: Does the graph of the function at the right have an inverse function? Explain.
No, it doesn't pass the Horizontal Line Test.

IV. Finding Inverse Functions Algebraically (Pages 97–98)

> ***What you should learn***
> How to find inverse functions algebraically

To find the inverse of a function f algebraically, . . .

1) Use the Horizontal Line Test to decide whether f has an inverse function.
2) In the equation for $f(x)$, replace $f(x)$ by y.
3) Interchange the roles of x and y, and solve for y.
4) Replace y by $f^{-1}(x)$ in the new equation.
5) Verify that f and f^{-1} are inverse functions of each other by showing that the domain of f is equal to the range of f^{-1}, the range of f is equal to the domain of f^{-1}, and $f(f^{-1}(x)) = x = f^{-1}(f(x))$.

Example 3: Find the inverse (if it exists) of $f(x) = 4x - 5$.
$f^{-1}(x) = 0.25x + 1.25$

> **Homework Assignment**
>
> Page(s)
>
> Exercises