IB Optimization 1

15. A company sells fruit juices in cylindrical cans, each of which has a volume of 340 cm^3. The surface area of a can is $A\text{ cm}^2$ and is given by the formula

$$A = 2\pi r^2 + \frac{680}{r},$$

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

(a) Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

(b) Calculate the value of r that minimizes the surface area of a can. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

IB Optimization 2

15. A company sells fruit juices in cylindrical cans, each of which has a volume of 340 cm^3. The surface area of a can is $A\text{ cm}^2$ and is given by the formula

$$A = 2\pi r^2 + \frac{680}{r},$$

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

(a) Find $\frac{dA}{dr}$. [3]

(b) Calculate the value of r that minimizes the surface area of a can. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

IB Optimization 3

15. A cuboid has a rectangular base of width x cm and length $2x$ cm. The height of the cuboid is h cm. The total length of the edges of the cuboid is 72 cm.

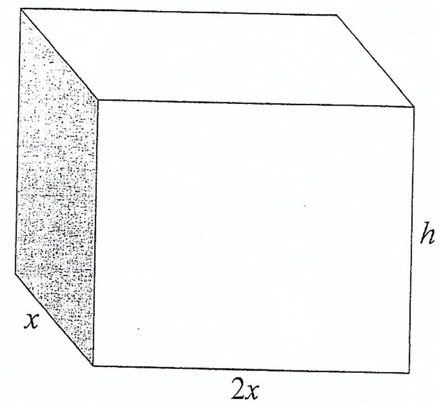


diagram not to scale

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

(a) Find the value of a. [3]

(b) Find the value of x that makes the volume a maximum. [3]

Working:

Answers:

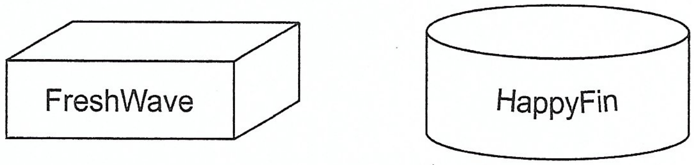
(a) . . . .

(b) . . . .

IB Optimization 4

4. FreshWave brand tuna is sold in cans that are in the shape of a cuboid with length 8 cm, width 5 cm and height 3.5 cm. HappyFin brand tuna is sold in cans that are cylindrical with diameter 7 cm and height 4 cm.

diagram not to scale



(a) Find the volume, in cm^3, of a can of

(i) FreshWave tuna;

(ii) HappyFin tuna. [4]

The price of tuna per cm^3 is the same for each brand. A can of FreshWave tuna costs 90 cents.

(b) Calculate the price, in cents, of a can of HappyFin tuna. [2]

Working:

Answers:

(a) (i) ....

(ii) ....

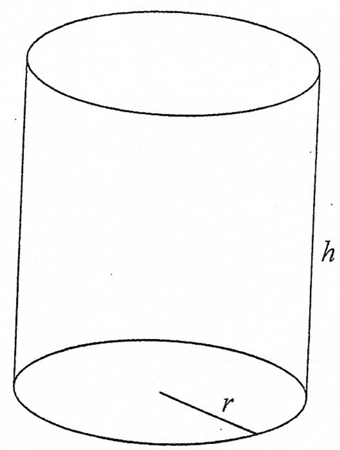
(b) ....

IB Optimization 5

6. *[Maximum mark: 22]*

Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm³.

diagram not to scale



Nadia decides to make the radius, r, of the bin 5 cm.

(a) Calculate

(i) the area of the base of the wastepaper bin;

(ii) the height, h, of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin. *[7 marks]*

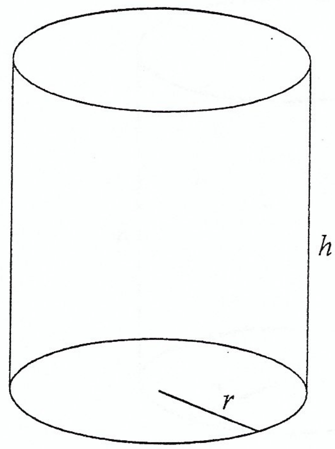
(b) State whether Nadia's design is practical. Give a reason. *[2 marks]*

(This question continues on the following page)

(Question 6 continued)

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

diagram not to scale



Let the radius of the base of Merryn's wastepaper bin be r, and let its height be h.

(c) Write down an equation in h and r, using the given volume of the bin. *[1 mark]*

(d) Show that the total external surface area, A, of the bin is $A = \eth r^2 + \frac{16000}{r}$. *[2 marks]*

(e) Write down $\frac{dA}{dr}$. $A = \pi r^2 + \frac{16000}{r}$ *[3 marks]*

(f) (i) Find the value of r which minimizes the total external surface area of the wastepaper bin.

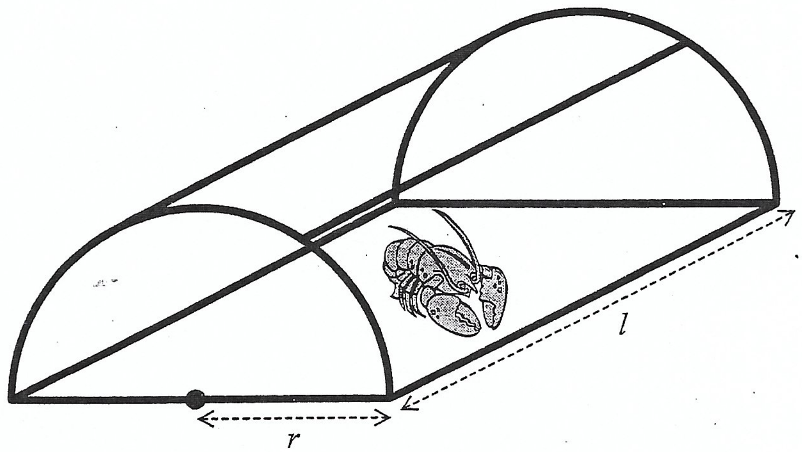
(ii) Calculate the value of h corresponding to this value of r. *[5 marks]*

(g) Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. *[2 marks]*

6. *[Maximum mark: 17]*

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.

diagram not to scale



The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

(a) Write down an expression for T in terms of r, l and π. *[3]*

The volume of the lobster trap is $0.75\,\text{m}^3$.

(b) Write down an equation for the volume of the lobster trap in terms of r, l and π. *[3]*

(c) Show that $T = (2\pi + 4)\,r + \dfrac{6}{\pi r^2}$. *[2]*

(d) Find $\dfrac{dT}{dr}$. *[3]*

The lobster trap is designed so that the length of steel used in its frame is a minimum.

(e) Show that the value of r for which T is a minimum is $0.719\,\text{m}$, correct to three significant figures. *[2]*

(f) Calculate the value of l for which T is a minimum. *[2]*

(g) Calculate the minimum value of T. *[2]*