IB Derivatives I

11. Consider the function $f(x) = ax^2 + c$.

(a) Find $f'(x)$. [1]

Point $\mathrm{A}(-2, 5)$ lies on the graph of $y = f(x)$. The gradient of the tangent to this graph at A is -6.

(b) Find the value of a. [3]

(c) Find the value of c. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

IB Derivatives 2

9. Consider the curve $y = x^2$.

(a) Write down $\frac{dy}{dx}$. *[1 mark]*

The point $P(3, 9)$ lies on the curve $y = x^2$.

(b) Find the gradient of the tangent to the curve at P. *[2 marks]*

(c) Find the equation of the normal to the curve at P. Give your answer in the form $y = mx + c$. *[3 marks]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

IB Derivatives 3

10. Let $f(x) = x^4$.

(a) Write down $f'(x)$. *[1]*

Point P $(2, 16)$ lies on the graph of f.

(b) Find the gradient of the tangent to the graph of $y = f(x)$ at P. *[2]*

(c) Find the equation of the normal to the graph at P. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. *[3]*

Working:

Answers:

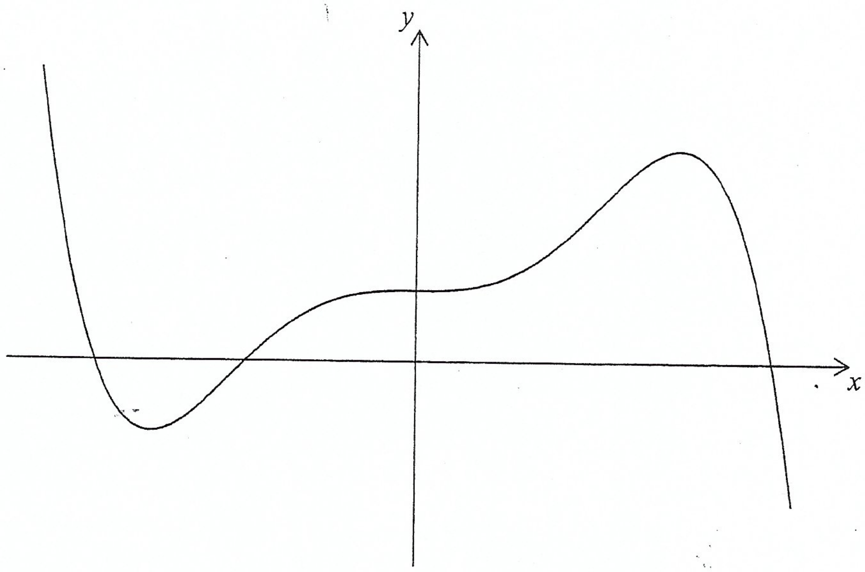
(a) . . . .

(b) . . . .

(c) . . . .

IB Derivatives 4

14. A sketch of the function $f(x) = 5x^3 - 3x^5 + 1$ is shown for $-1.5 \le x \le 1.5$ and $-6 \le y \le 6$.



(a) Write down $f'(x)$. *[2 marks]*

(b) Find the equation of the tangent to the graph of $y = f(x)$ at $(1, 3)$. *[2 marks]*

(c) Write down the coordinates of the second point where this tangent intersects the graph of $y = f(x)$. *[2 marks]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

IB Derivatives 5

15. Consider the curve $y = x^3 + kx$.

(a) Write down $\frac{dy}{dx}$. *[1]*

The curve has a local maximum at the point where $x = 2$.

(b) Find the value of k. *[3]*

(c) Find the value of y at this local maximum. *[2]*

Working:

Answers:

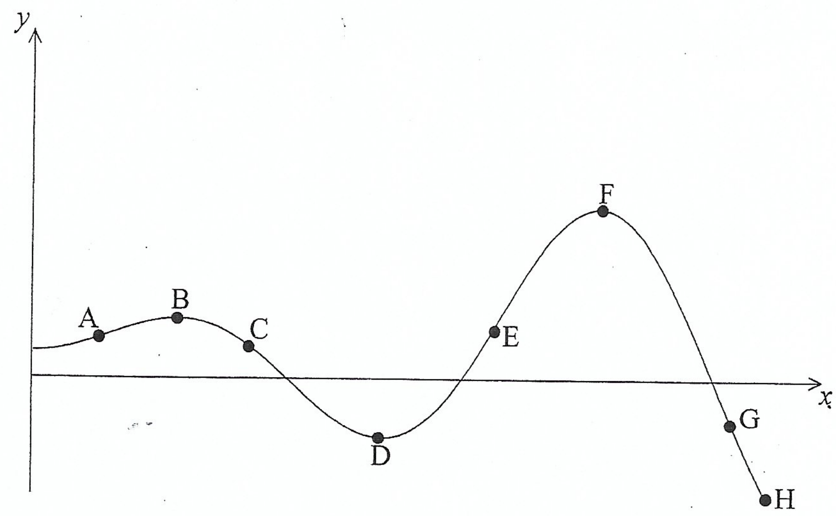
(a) ....

(b) ....

(c) ....

IB Derivatives 6

5. Consider the graph of the function $y = f(x)$ defined below.



Write down **all** the labelled points on the curve

(a) that are local maximum points; *[1 mark]*

(b) where the function attains its least value; *[1 mark]*

(c) where the function attains its greatest value; *[1 mark]*

(d) where the gradient of the tangent to the curve is positive; *[1 mark]*

(e) where $f(x) > 0$ and $f'(x) < 0$. *[2 marks]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

(d) ....

(e) ....

IB Derivatives 7

5. [Maximum mark: 20]

Consider the function $f(x)=\frac{96}{x^2}+kx$, where k is a constant and $x\neq 0$.

(a) Write down $f'(x)$. [3]

The graph of $y=f(x)$ has a local minimum point at $x=4$.

(b) Show that $k=3$. [2]

(c) Find $f(2)$. [2]

(d) Find $f'(2)$. [2]

(e) Find the equation of the normal to the graph of $y=f(x)$ at the point where $x=2$. Give your answer in the form $ax+by+d=0$ where $a, b, d\in\mathbb{Z}$. [3]

(f) Sketch the graph of $y=f(x)$, for $-5\leq x\leq 10$ and $-10\leq y\leq 100$. [4]

(g) Write down the coordinates of the point where the graph of $y=f(x)$ intersects the x-axis. [2]

(h) State the values of x for which $f(x)$ is decreasing. [2]

IB Derivatives 8

6. [Maximum mark: 16]

Consider the function $g(x) = x^3 + kx^2 - 15x + 5$.

(a) Find $g'(x)$. [3]

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

(b) (i) Show that $k = 6$.

(ii) Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form $y = mx + c$. [5]

(c) Use your answer to part (a) and the value of k, to find the x-coordinates of the stationary points of the graph of $y = g(x)$. [3]

(d) (i) Find $g'(-1)$.

(ii) Hence justify that g is decreasing at $x = -1$. [3]

(e) Find the y-coordinate of the local minimum. [2]

Calc

IB Derivatives 9

6. [Maximum mark: 12]

A function, f, is given by

$$f(x) = 4 \times 2^{-x} + 1.5x - 5.$$

(a) Calculate $f(0)$. [2]

(b) Use your graphic display calculator to solve $f(x) = 0$. [2]

(c) Sketch the graph of $y = f(x)$ for $-2 \le x \le 6$ and $-4 \le y \le 10$, showing the x and y intercepts. Use a scale of 2 cm to represent 2 units on both the horizontal axis, x, and the vertical axis, y. [4]

The function f is the derivative of a function g. It is known that $g(1) = 3$.

(d) (i) Calculate $g'(1)$.

(ii) Find the equation of the tangent to the graph of $y = g(x)$ at $x = 1$. Give your answer in the form $y = mx + c$. [4]

IB Derivatives 10

3. [Maximum mark: 13]

A distress flare is fired into the air from a ship at sea. The height, h, in metres, of the flare above sea level is modelled by the quadratic function

$$h(t) = -0.2t^2 + 16t + 12\,,\ t \geq 0\,,$$

where t is the time, in seconds, and $t = 0$ at the moment the flare was fired.

(a) Write down the height from which the flare was fired. [1]

(b) Find the height of the flare 15 seconds after it was fired. [2]

The flare fell into the sea k seconds after it was fired.

(c) Find the value of k. [2]

(d) Find $h'(t)$. [2]

(e) (i) Show that the flare reached its maximum height 40 seconds after being fired.

(ii) Calculate the maximum height reached by the flare. [3]

The nearest coastguard can see the flare when its height is more than 40 metres above sea level.

(f) Determine the total length of time the flare can be seen by the coastguard. [3]