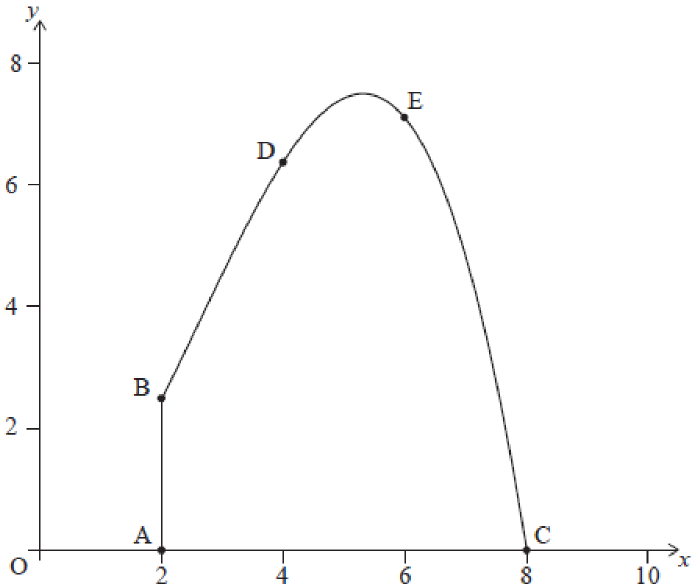
Trapezoids *[65 marks]*

The cross-sectional view of a tunnel is shown on the axes below. The line $[AB]$ represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y = -0.1x^3 + 0.8x^2, 2 \leq x \leq 8$, relative to an origin O.



Point A has coordinates $(2, 0)$, point B has coordinates $(2, 2.4)$, and point C has coordinates $(8, 0)$.

When $x = 4$ the height of the tunnel is $6.4\ m$ and when $x = 6$ the height of the tunnel is $7.2\ m$. These points are shown as D and E on the diagram, respectively.

1a. Write down the integral which can be used to find the cross-sectional area of the tunnel. *[2 marks]*

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1b. Hence find the cross-sectional area of the tunnel. *[2 marks]*

A function f is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3, x \neq 0$.

2a. Write down the derivative of f. *[3 marks]*

2b. Find the point on the graph of f at which the gradient of the tangent is equal to 6. *[3 marks]*

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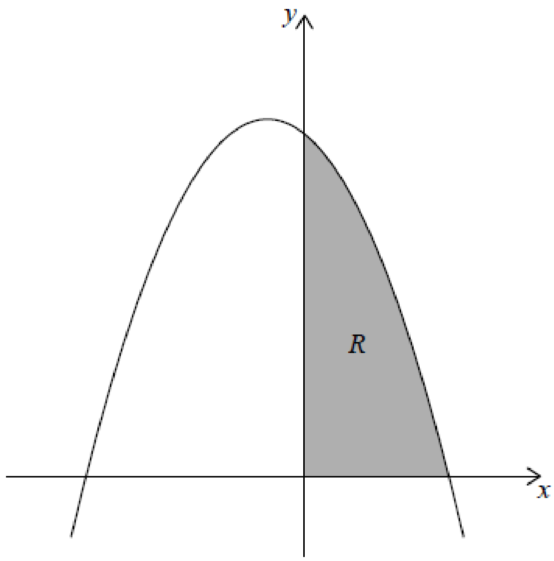
The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$.
The shaded region *R* is bounded by the x-axis, y-axis and the graph of f.



3a. Write down an integral for the area of region *R*. *[2 marks]*

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3b. Find the area of region *R*. *[1 mark]*

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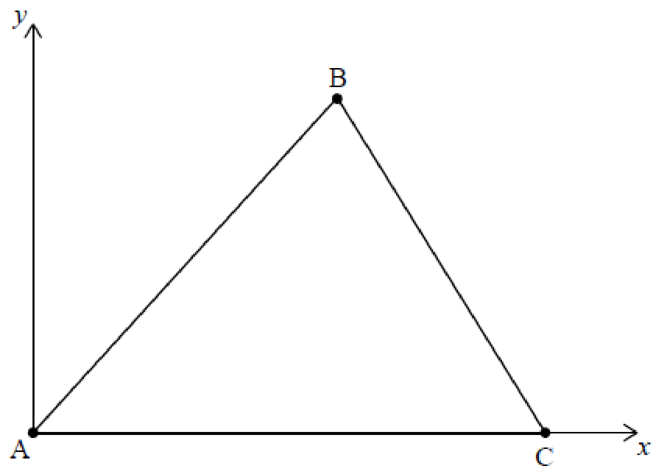
3c. The three points A(0, 0) , B(3, 10) and C(a, 0) define the vertices of a triangle. *[2 marks]*



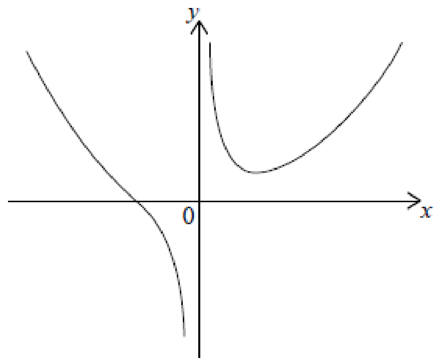
Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region *R*.

Consider the curve $y = x^2 - 4x + 2$.

4a. Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[1 mark]*

4b. Show that the normal to the curve at the point where $x = 1$ is $2y - x + 3 = 0$. *[6 marks]*

The diagram shows the curve $y = \frac{x^2}{2} + \frac{2a}{x}, x \neq 0$.



The equation of the vertical asymptote of the curve is $x = k$.

5a. Write down the value of k. *[1 mark]*

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5b. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[3 marks]*

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5c. At the point where $x = 2$, the gradient of the tangent to the curve is 0.5. *[2 marks]*

Find the value of a.

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The function $f\left(x\right)=\frac{1}{3}x^{3}+\frac{1}{2}x^{2}+kx+5$ has a local maximum and a local minimum. The local maximum is at $x=-3$.

6a. Show that $k=-6$. *[5 marks]*

6b. Find the coordinates of the local **minimum**. *[2 marks]*

6c. Write down the interval where the gradient of the graph of $f\left(x\right)$ is negative. *[2 marks]*

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6d. Determine the equation of the normal at $x = -2$ in the form $y = mx + c$. *[5 marks]*

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Consider the curve $y = 5x^3 - 3x$.

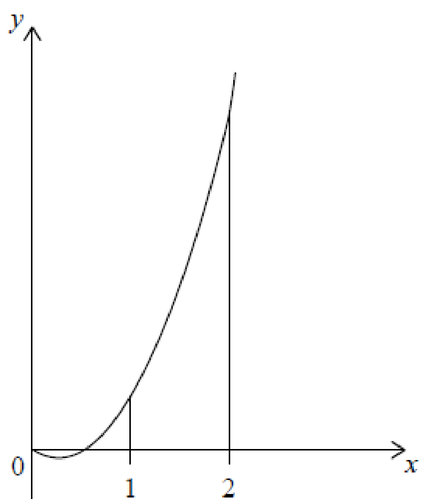
7a. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[2 marks]*

The curve has a tangent at the point P$(-1, -2)$.

7b. Find the gradient of this tangent at point P. *[2 marks]*

7c. Find the equation of this tangent. Give your answer in the form $y = mx + c$. *[2 marks]*

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



8a. Find $\int \left(6x^2 - 3x\right) \mathrm{d}x$. *[2 marks]*

8b. Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. *[4 marks]*

Consider the curve $y = 2x^3 - 9x^2 + 12x + 2$, for $-1 < x < 3$

9a. Sketch the curve for $-1 < x < 3$ and $-2 < y < 12$. *[4 marks]*

9b. A teacher asks her students to make some observations about the curve. *[1 mark]*

Three students responded.
Nadia said *"The x-intercept of the curve is between −1 and zero".*
Rick said *"The curve is decreasing when x < 1 ".*
Paula said *"The gradient of the curve is less than zero between x = 1 and x = 2 ".*

State the name of the student who made an **incorrect** observation.

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9c. Find $\frac{dy}{dx}$. *[3 marks]*

9d. Given that $y = 2x^3 - 9x^2 + 12x + 2 = k$ has **three** solutions, find the possible values of k. *[3 marks]*