FUNDAMENTAL THEOREM OF CALCULUS

Given $\frac{dy}{dx}=3x^2+4x-5$ with the initial condition $y(2)=-1$. Find $y(3)$.

Method 1: Integrate $y=\int(3x^2+4x-5)dx$, and use the initial condition to find C. Then write the particular solution, and use your particular solution to find $y(3)$.

Method 2: Use the Fundamental Theorem of Calculus: $\int_a^b f'(x)dx=f(b)-f(a)$

Sometimes there is no antiderivative so we have to use Method 2 and our graphing calculator.

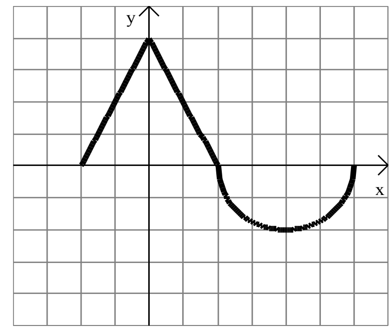
Ex. $f'(x)=\sin(x^2)$ and $f(2)=-5$. Find $f(1)$.

Ex. The graph of f' consists of two line segments and a semicircle as shown on the right. Given that $f(-2)=5$, find:

(a) $f(0)$

(b) $f(2)$

(c) $f(6)$



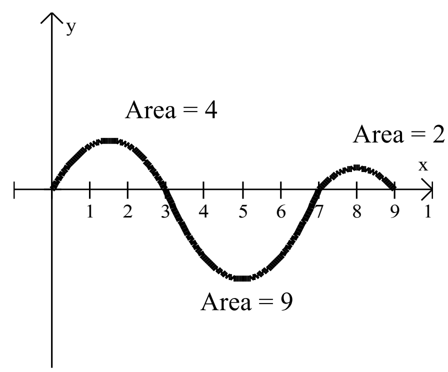
Graph of f'

Ex. The graph of f' is shown. Use the figure and the fact that $f(3)=5$ to find:

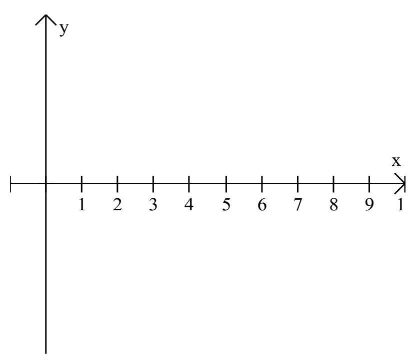
(a) $f(0)$

(b) $f(7)$

(c) $f(9)$



Then sketch the graph of *f*.



Ex. A pizza with a temperature of 95°C is put into a 25°C room when $t=0$. The pizza's temperature is decreasing at a rate of $r(t)=6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t=5$ minutes.

CALCULUS
WORKSHEET ON FUNDAMENTAL THEOREM OF CALCULUS

Work the following on **notebook paper**.
Work problems 1 - 3 by both methods.

1. $y' = 2 + \frac{1}{x^2}$ and $y(1) = 6$. Find $y(3)$.

2. $f'(x) = \cos(2x)$ and $f(0) = 3$. Find $f\left(\frac{\pi}{4}\right)$.

3. Water flows into a tank at a rate of $\frac{dW}{dt} = \frac{1}{75}\left(600 + 20t - t^2\right)$, where $\frac{dW}{dt}$ is measured in gallons per hour and t is measured in hours. If there are 150 gallons of water in the tank at time $t = 0$, how many gallons of water are in the tank when $t = 24$?

Work problems 4 – 8 using the Fundamental Theorem of Calculus and your calculator.

4. $f'(x) = \cos\left(x^3\right)$ and $f(0) = 2$. Find $f(1)$.

5. $f'(x) = e^{-x^2}$ and $f(5) = 1$. Find $f(2)$.

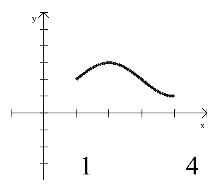
6. A particle moving along the x-axis has position $x(t)$ at time t with the velocity of the particle $v(t) = 5\sin\left(t^2\right)$. At time $t = 6$, the particle's position is (4, 0). Find the position of the particle when $t = 7$.

7. Let $F(t)$ represent a bacteria population which is 4 million at time $t = 0$. After t hours, the population is growing at an instantaneous rate of 2^t million bacteria per hour. Find the total increase in the bacteria population during the first three hours, and find the population at $t = 3$ hours.

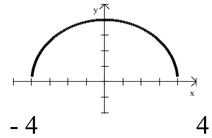
8. A particle moves along a line so that at any time $t \geq 0$ its velocity is given by $v(t) = \frac{t}{1+t^2}$. At time $t = 0$, the position of the particle is $s(0) = 5$. Determine the position of the particle at $t = 3$.

Use the Fundamental Theorem of Calculus and the given graph.

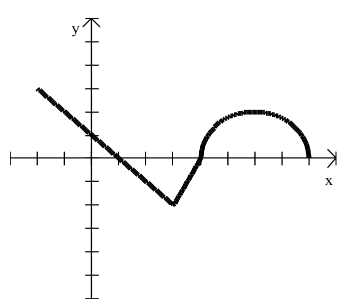
9. The graph of f' is shown on the right.
$\int_1^4 f'(x)\,dx = 6.2$ and $f(1) = 3$. Find $f(4)$.



10. The graph of f' is the semicircle shown on the right.
Find $f(-4)$ given that $f(4) = 7$.



11. The graph of f', consisting of two line segments and a semicircle, is shown on the right. Given that $f(-2) = 5$, find:
(a) $f(1)$ (b) $f(4)$ (c) $f(8)$



12. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x)=\frac{1+e^x}{x^2}$ Find $f(3.1)$

13. (Multiple Choice) If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1)=5$, then $f(4)=$

(A) 4.988 (B) 5 (C) 5.016 (D) 5.376 (E) 5.629

<u>Answers to Worksheet on the First Fundamental Theorem of Calculus</u>

1. $\frac{32}{3}$
2. $\frac{7}{2}$
3. 357.36 gallons
4. 2.932
5. 0.996
6. 3.837
7. 10.099 million, 14.099 million
8. 6.151
9. 9.2
10. $7-8\pi$
11. (a) 9.5 (b) 6.5 (c) $6.5+2\pi$
12. 6.238
13. D