Section 3.2: Finding Limits Graphically and Numerically

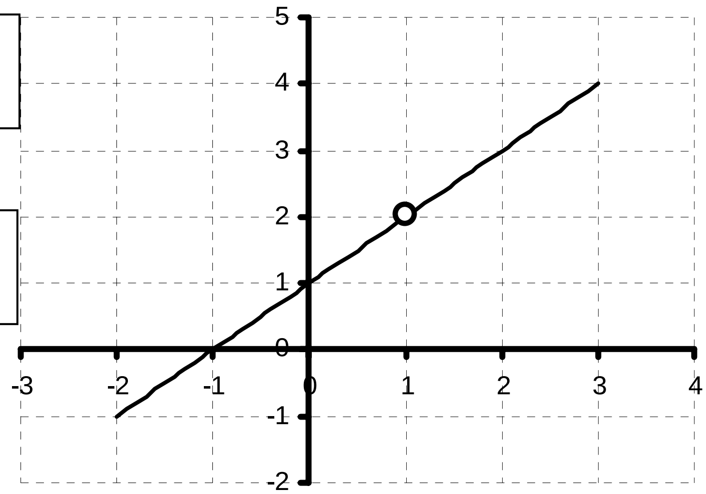
Consider the function $f(x) = \dfrac{x^2 - 1}{x - 1}$. What happens at $x = 1$?

We certainly can't find a function value there because $f(1)$ is undefined so the best we can do is to see what happens **near** the point $x = 1$. To do this we will graph the function. Since the function isn't easy to graph as it is written we will simply plot a few points:

x	$f(x)$
-1	0
0	1
.9	1.9
.99	1.99
1	undefined
1.01	2.01
1.1	2.1
2	3
3	4

Close to 2

Close to 2



There is a hole at $x = 1$ which is consistent with the fact that the function is undefined there. We can put in numbers into the function as **close** to 1 as we would like but we can't use 1 itself. Just saying that the function is undefined there doesn't tell us much about the function. What we would like to be able to answer is the question:

Q: What happens to $f(x)$ near $x = 1$?

A: As x gets **close** to 1 the value of $f(x)$ gets **close** to 2. We can see this from the graph. We read x across the horizontal axis the values for $f(x)$ are on the vertical axis. The graph is approaching the y value 2 as x gets **close** to 1.
We could also see this from the chart of values. You will notice that the $f(x)$ values are getting **close** to 2 as x is getting **closer** to 1.

We can write this mathematically the following way:

$\boxed{\lim_{x \to 1} f(x) = 2}$. We read this as "the limit as x approaches 1 of $f(x)$ is 2". There are the possible outcomes to a limit:

1. the limit doesn't exist
2. the limit exists and is a number
3. the limit exists and is $\pm\infty$

There are also different ways of finding a limit. We have seen two ways of finding the limit:

1. We tried numbers close to $x = 1$ and we checked what happened.
2. We looked at the graph and we saw what the function value was near $x = 1$.

Reading the limit off a graph is the **easiest** way to find the limit. Trying to create a table on numbers will work if the function behaves well. If it tends to change values very quickly this method may not be very accurate.

> **NOTE:** The most important thing to remember when solving for limits is that we only care about what is happening to the function **NEAR** the point and NOT what is happening at the point.

Here are some examples with tables:

Ex 1: $\lim_{x \to 2} \frac{x-2}{x^2-4}$. Once again this is asking the question, "What happens to the function as x gets close to 2". We need a table with values close to 2.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$\frac{x-2}{x^2-4}$	.25641	.250627	.25006	undefined	.249938	.249377	.243902

$\longrightarrow$ Close to 0.25 $\qquad$ $\longleftarrow$ Close to 0.25

You can tell that the values are getting close to $\boxed{0.25 \text{ or } 1/4}$.

Ex 2: $\lim_{x \to 4} \frac{\left(\frac{x}{x+1}\right) - \frac{4}{5}}{x-4}$

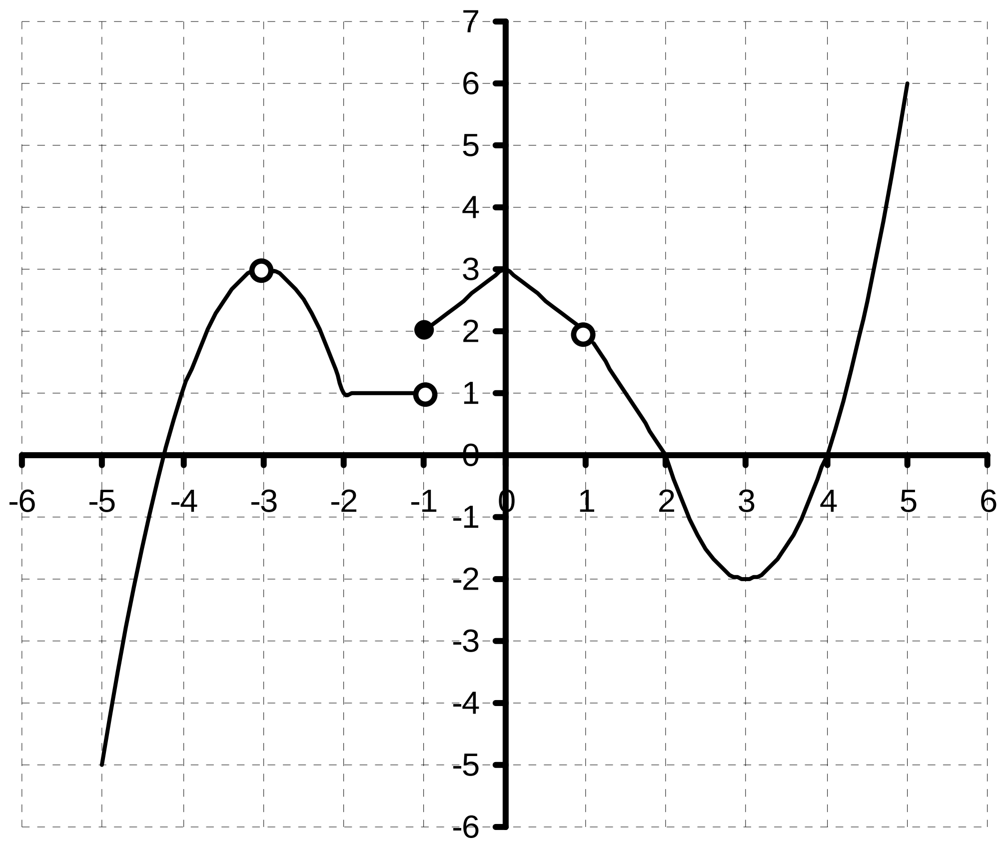
x	3.9	3.99	3.999	4	4.001	4.01	4.1
$\frac{\left(\frac{x}{x+1}\right) - \frac{4}{5}}{x-4}$	.040816	.04008	.04000	undefined	.03999	.03992	.03921

$\longrightarrow$ Close to 0.04 $\qquad$ $\longleftarrow$ Close to 0.04

Here the values are approaching $\boxed{0.04}$

Another way to find limits is graphically. We look at the graph and we know what happens to x so we read the limit from the values on the y axis.

Ex 3: Use the graph of $f(x)$ below to find the limits



a) $\lim\limits_{x \to -3} f(x) = \boxed{3}$

b) $\lim\limits_{x \to 0} f(x) = \boxed{3}$

c) $\lim\limits_{x \to -2} f(x) = \boxed{1}$

d) $\lim\limits_{x \to 1} f(x) = \boxed{2}$

e) $\lim\limits_{x \to -1} f(x) =$ Does Not Exist because the function does not come to one place at $x = 1$. From one side the function values are approaching 1 and from the other side the values are approaching 2.

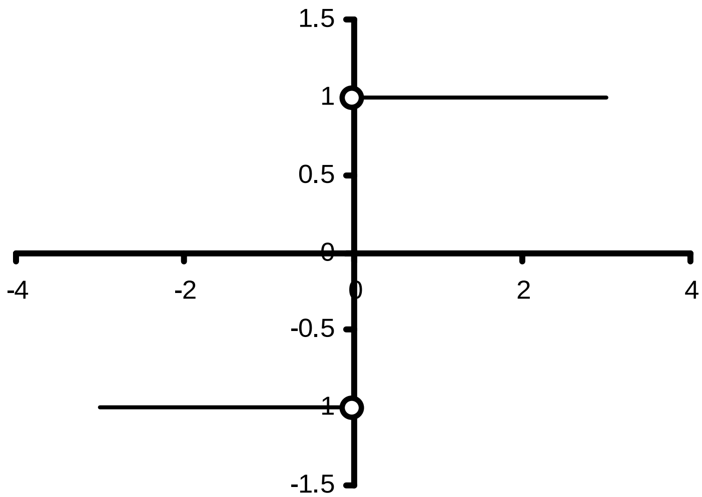
Ex 4: $\lim_{x \to 0} \frac{|x|}{x}$

We will do this problem 2 ways. By a chart and by a graph.

x	-3	-2	-1	0	1	2	3
$\frac{|x|}{x}$	-1	-1	-1	undefined	1	1	1

Close to -1

Close to 1



From either of these methods we can see that the function does not approach one unique value. From one side we are at -1 and from the other side we are at +1. Since these are not the same the limit does not exist.