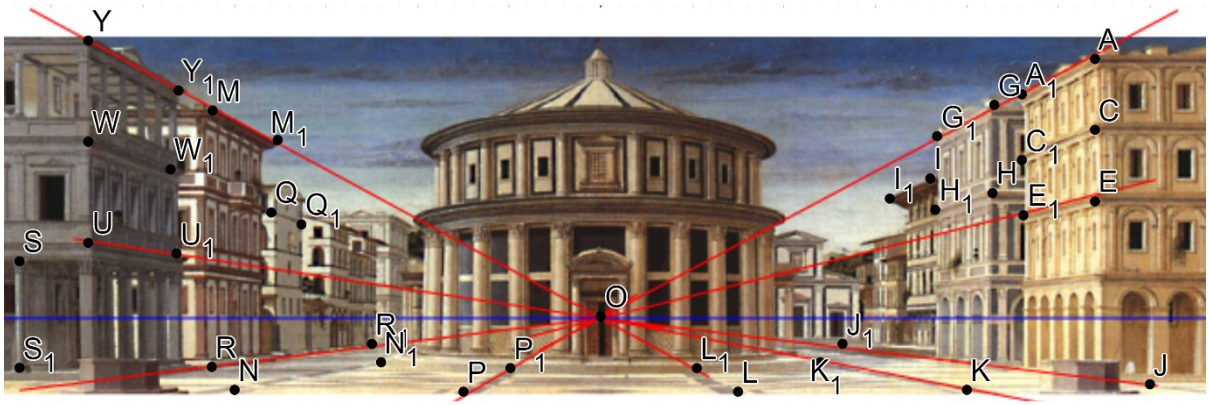
4.2 Gradient of lines and its applications

Convergence lines are used in perspective to help the artist introduce the notion of depth. To investigate the gradient of those lines a researcher has labelled and mapped the positions of



Here are the coordinate of each point:

A(5.4, 3.8)	A_1(4.6, 3.385)	B(7.2, 3.8)	C(5.4, 3.03)
C_1(4.6, 2.73)	D(7.2, 3.03)	E(5.4, 2.25)	E_1(4.6, *e*)
F(7.2, 2.25)	G(4.3, 3.323)	G_1(3.6, 2.87)	H(4.28, *h*)
H_1(3.6, 2.13)	I(3.6, 2.5)	I_1(3.1, *i*)	J(6, 0.26)
J_1(2.64, j)	K(4, 0.2)	K_1(2.4, 0.52)	L(1.5, 0.19)
L_1(1.05, 0.43)	M(−4.24, 3.27)	M_1(−3.5, 2.875)	N(−4, 0.2)
N_1(*n* , 0.52)	O(0, 1)	P(−1.5, 0.19)	P_1(−1.05, 0.433)
Q(−3.6, 2.13)	Q_1(−3.27, 2.03)	R(−4.25, *r*)	R_1(−2.5, 0.69)
S(−6.35, 1.6)	S_1(−6.35, 0.44)	T(−7.1, 1.6)	T_1(−7.1, 0.44)
U(−5.6, 1.8)	U_1(*u* , 1.66)	V(−7.7, 1.8)	W(−5.6, 2.9)
W_1(w, 2.56)	X(−7.7, 2.9)	Y(−5.6, 4)	Y_1(−4.6, 3.46)
Z(−7.7, 4)			

1 **a** Calculate the gradients of the line in the foreground:

i	AB	**ii**	CD	**iii**	EF
iv	ZY	**v**	XW	**vi**	VU
vii	AE	**viii**	BF	**ix**	YU

b Hence state the pattern for the gradient of

i horizontal lines **ii** vertical lines

2 **a** Calculate the gradient of lines building lines showing the depth:

i AA_1, **ii** AO **iii** A_1O **iv** CC_1

v CO **vi** C_1O **vii** KK_1 **viii** KO

ix K_1O **x** PP_1 **xi** PO **xii** P_1O

xiii YY_1 **xiv** YO **xv** Y_1O **xvi** MM_1

xvii MO **xviii** M_1O

b Hence

i state the pattern for the gradients of the segments of same line

ii discuss whether any 2 different perspective lines could have the same gradient.

c Explain whether

i Y,Y_1, M and M_1 are collinear (on the same line)

ii W, Q, Q_1 and O are collinear

iii A,A_1,G,$G_1$and PP_1 are collinear

d as all perspective line meet in O calculate

i *e* **ii** *i* **iii** *u* **iv** *j*

e Show that LL_1 is the reflection of the line PP_1 in the y axis.

f State the transformation between JO and R_1O hence find r

g Hence state the pattern for the gradient of line reflected in the y axis.

h As K is the reflection N in the y axis state the gradient of NN_1 and find *n*

i As Q_1 is the reflection H in the y axis state the gradient of II_1 and find *h*

3 Pipes must slope slightly downhill to drain properly. The standard slope is anywhere from ¼ inch to 3 inches per foot.

According to the International Plumbing Code, drainage pipes should be run with a uniform slope at the following minimum pitches:

	PIPE DIAMETER	**MINIMUM SLOPE**
i	2 1/2" or smaller	1/4" per foot
ii	3" to 6"	1/9" per foot
iii	8" or larger	1/17" per foot

a If you calculate the gradient of each slope, state whether your gradient is going to be positive or negative

b Calculate the gradient for each type of pipe using the fact that 1 foot = 12 inches

c Determine the percentage that would be written if these pipes were running as gutter on a road

d If a pipe has been installed with a gradient of

i −0.019, explain whether it should be reinstalled

ii −0.003, find the pipe diameter

Answers

1 a i 0 **ii** 0 **iii** 0

iv 0 **v** 0 **vi** 0

vii undefined **viii** undefined **ix** undefined

b i the gradient is always 0 **ii** the gradient is always undefined.

2 a i Gradient of $AA_1 = 0.519$ **ii** Gradient of $AO = 0.519$

iii Gradient of $OA_1 = 0.519$ **iv** Gradient of $CC_1 = 0.376$

v Gradient of $CO = 0.376$ **vi** Gradient of $OC_1 = 0.376$

vii Gradient of $KK_1 = -0.2$ **viii** Gradient of $KO = -0.2$

ix Gradient of $OK_1 = -0.2$ **xi** Gradient of $PP_1 = 0.54$

xii Gradient of $PO = 0.54$ **xiii** Gradient of $OP_1 = 0.54$

xiv Gradient of $YY_1 = -0.536$ **xv** Gradient of $YO = -0.536$

xvi Gradient of $OY_1 = -0.536$ **xvi** Gradient of $MM_1 = -0.536$

xvii Gradient of $MO = -0.536$ **xviii** Gradient of $OM_1 = -0.536$

b i they have the same gradient

ii no, if they have the same gradient they would be parallel and thus could not all intersect at O

c i they are collinear as Gradient of $YY_1 = -0.536$ =Gradient of MM_1

ii they are not collinear as Gradient of $WO = -0.34 \neq$ Gradient of $QQ_1 = -0.314$

iii AA_1 G G_1 are collinear as Gradient of $AA_1 = 0.519$ = Gradient of GG_1 but are not with PP_1 as Gradient of $PP_1 = 0.54$

d i As Gradient of EE_1 =Gradient of EO $EE_1 = 0.23$ $e = 206$,

ii Gradient of $II_1 = 0.417$,$i = 2.29$,

iii Gradient of $UU_1 = -0.143$,$u = -4.62$

iv Gradient of $JJ_1 = -0.123$ $j = 0.674$

v $w = 4.6$

e Gradient of $PP_1 = 0.54$, Gradient of $LL_1 = -0.54$

f reflection in the y axis so $r = 0.48$

g the gradient changes its sign

h as K = N gradient of $NN_1 = 0.2$ and $n = -2.4$

i as Gradient of $QQ_1 = -0.314$ Gradient of $HH_1 = 0.314$ and $h = 2.34$

3 a negative as your slope goes down

b i −0.021 **ii** −0.009 **iii** −0.005

c i 2.08% **ii** 0.92% **iii** 0.49%

d i yes as the minimum slope is 1/4" per foot which means that the actual gradient ≤ -0.021

ii 3" to 6" as $-0.009 < -0.003 < -0.005$