

**4-5 Factoring Polynomials**

## Factoring 3

**Objective:** To factor polynomials by using the GCF, by recognizing special products, and by grouping terms.

**Vocabulary**

**Factor a polynomial** To express a polynomial as a product of other polynomials.

**Greatest monomial factor** The GCF of the terms of a polynomial.

**Special factoring patterns**

Perfect square trinomials	$a^2 + 2ab + b^2 = (a + b)^2$	$a^2 - 2ab + b^2 = (a - b)^2$
Difference of squares	$a^2 - b^2 = (a + b)(a - b)$	
Sum of cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	
Difference of cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	

**Example 1** Factor: a.  $3x^4 - 6x^3 + 12x^2$       b.  $8a^3b - 12a^2b^2$

**Solution** a.  $3x^4 - 6x^3 + 12x^2 = 3x^2(x^2 - 2x + 4)$  ← The GCF of the terms is  $3x^2$ .

b.  $8a^3b - 12a^2b^2 = 4a^2b(2a - 3b)$  ← The GCF of the terms is  $4a^2b$ .

**Factor each polynomial.**

1.  $y^2 + y$

2.  $4x - 28$

3.  $8a^4 - 14a^2b$

4.  $6x^2 - 8x^3 - 10x^4$

5.  $10x^8 + 15x^7 - 35x^5$

6.  $11a^3b - 22a^2b^2 + 55ab^3$

**Example 2** Factor: a.  $z^2 + 8z + 16$       b.  $9x^2 - 6xy + y^2$       c.  $36m^2 - 49n^2$

**Solution** a.  $z^2 + 8z + 16 = z^2 + 2(z)(4) + (4)^2$  ← perfect square trinomial  
 $= (z + 4)^2$

b.  $9x^2 - 6xy + y^2 = (3x)^2 - 2(3x)(y) + y^2$  ← perfect square trinomial  
 $= (3x - y)^2$

c.  $36m^2 - 49n^2 = (6m)^2 - (7n)^2$  ← difference of squares  
 $= (6m + 7n)(6m - 7n)$

**Example 3** Factor  $2a^5 - 162a$ .

**Solution** Always begin by looking for the GCF of the terms. If the GCF is not 1, then factor the GCF out. The GCF of  $2a^5$  and  $-162a$  is  $2a$ .

$$\begin{aligned}
 2a^5 - 162a &= 2a(a^4 - 81) \\
 &= 2a[(a^2)^2 - (9)^2] \leftarrow \text{difference of squares} \\
 &= 2a(a^2 + 9)(a^2 - 9) \\
 &= 2a(a^2 + 9)(a^2 - 3^2) \leftarrow \text{difference of squares} \\
 &= 2a(a^2 + 9)(a + 3)(a - 3)
 \end{aligned}$$



**4–5 Factoring Polynomials (continued)**

Factor each polynomial.

- |                       |                       |                              |
|-----------------------|-----------------------|------------------------------|
| 7. $x^2 + 10x + 25$   | 8. $a^2 - 16a + 64$   | 9. $4y^2 - 12y + 9$          |
| 10. $4b^2 + 28b + 49$ | 11. $x^2 - 16$        | 12. $y^2 - 100$              |
| 13. $4k^2 - 25$       | 14. $9m^2 - 64$       | 15. $25x^2 + 20xy + 4y^2$    |
| 16. $81p^2 - 49q^2$   | 17. $3x^2 + 12x + 12$ | 18. $5c^3 + 30c^2d + 45cd^2$ |
| 19. $rt^2 - r$        | 20. $4x^2y - 36y$     | 21. $16n^4 - 1$              |

**Example 4** Factor: a.  $a^3 - 8$                       b.  $27x^3 + 1$ **Solution** a.  $a^3 - 8 = a^3 - 2^3 = (a - 2)(a^2 + 2a + 4)$  ← difference of cubesb.  $27x^3 + 1 = (3x)^3 + 1^3 = (3x + 1)(9x^2 - 3x + 1)$  ← sum of cubes

Factor each polynomial.

- |               |                |                 |                    |
|---------------|----------------|-----------------|--------------------|
| 22. $x^3 + 1$ | 23. $64 - a^3$ | 24. $t^3 + 125$ | 25. $1000c^3 - 27$ |
|---------------|----------------|-----------------|--------------------|

**Example 5** Factor: a.  $2a^3 - 3a^2 - 4a + 6$                       b.  $12x^3 + 4x^2y - 3x - y$ **Solution** a. The first and second terms have a common factor of  $a^2$ , and the third and fourth terms have a common factor of  $-2$ . Factor by grouping terms.

$$\begin{aligned} 2a^3 - 3a^2 - 4a + 6 &= (2a^3 - 3a^2) + (-4a + 6) \\ &= a^2(2a - 3) - 2(2a - 3) \quad \text{Common factor is } 2a - 3. \\ &= (a^2 - 2)(2a - 3) \quad \text{Factor out } (2a - 3). \end{aligned}$$

b. The first and third terms have a common factor of  $3x$ , and the second and fourth terms have a common factor of  $y$ . Factor by grouping terms.

$$\begin{aligned} 12x^3 + 4x^2y - 3x - y &= (12x^3 - 3x) + (4x^2y - y) \\ &= 3x(4x^2 - 1) + y(4x^2 - 1) \\ &= (3x + y)(4x^2 - 1) \quad \text{← difference of squares} \\ &= (3x + y)(2x + 1)(2x - 1) \end{aligned}$$

Factor each polynomial.

- |                              |                             |                             |
|------------------------------|-----------------------------|-----------------------------|
| 26. $a(b + 2) - 3(b + 2)$    | 27. $m(n - 2) - (2 - n)$    | 28. $20a^3 - 5a^2 + 8a - 2$ |
| 29. $10y^3 + 10y^2 + 3y + 3$ | 30. $9a^2b - 8a^2 - 9b + 8$ | 31. $5x^2y - 7x^2 - 7 + 5y$ |

**Mixed Review Exercises**

Write as a simplified polynomial.

- |                         |                       |                      |
|-------------------------|-----------------------|----------------------|
| 1. $(a + 2)^2$          | 2. $(3a - 2)(4a + 3)$ | 3. $r^2s^2(4r - 5s)$ |
| 4. $(5a - 2) - (3 - a)$ | 5. $(x^2 - 2)(x + 5)$ | 6. $(-a)^2(2a^2)^3$  |



# Factoring 4

Factor each polynomial.

5.  $16k^2 - 1$
7.  $4y^2 + 20y + 25$
9.  $16x^2 - 25$
11.  $121s^2 - 66st + 9t^2$
13.  $36p^2 - 49q^2$
15.  $st^2 - s$
17.  $t^3 - 27$
19.  $16r^4s + 2rs^4$
21.  $x(y - 3) + 2(y - 3)$
23.  $x(y - 3) + 2(3 - y)$
25.  $pq - 2q + 2p - 4$
27.  $ab - 2 - 2b + a$
29.  $x^2 - 6x + 9 - 4y^2$
31.  $u^2 - v^2 + 2v - 1$
33.  $x^4 - 2x^2y + y^2$
35.  $a^6 + b^3$
37.  $16s^4 - 81$
39.  $x^6 - y^6$
41.  $u^2 - v^2 - 2u - 2v$
43.  $(p + q)^3 - (p - q)^3$
45.  $s^3 + t^3 + s^2t + st^2$
47.  $(a + b)^6 - (a - b)^6$
6.  $121x^2 - 1$
8.  $9s^2 - 24s + 16$
10.  $4h^2 - 81$
12.  $16x^2 + 40xy + 25y^2$
14.  $9x^4 - 16z^2$
16.  $p^3q - pq$
18.  $8p^3 + 1$
20.  $3x^2y^4 - 81x^2y$
22.  $u(v - 1) - 2(v - 1)$
24.  $u(v - 1) - 2(1 - v)$
26.  $xy - 2y - x + 2$
28.  $4ab + 1 - 2a - 2b$
30.  $z^2 + 2z + 1 - w^2$
32.  $x^2 - y^2 - 4y - 4$
34.  $4u^4v^2 + 4u^2v + 1$
36.  $250x^2 - 2x^5$
38.  $p^4 - q^4$
40.  $64 - z^6$
42.  $a^2 - b^2 + a - b$
44.  $(x + y)^3 + (x - y)^3$
46.  $u^3 - v^3 - u^2v + uv^2$
48.  $(a + b)^4 - (a - b)^4$



# Factoring 5

For use after Section 4-6 of text ALGEBRA AND TRIGONOMETRY, Structure and Method, Book 2

NAME \_\_\_\_\_ DATE \_\_\_\_\_ SCORE \_\_\_\_\_

## Factoring Quadratic Polynomials

Factor completely. If the polynomial is prime, say so.

1.  $b^2 + 8b + 15$  \_\_\_\_\_

2.  $x^2 - 5x + 4$  \_\_\_\_\_

3.  $y^2 + 4y - 5$  \_\_\_\_\_

4.  $a^2 - a - 6$  \_\_\_\_\_

5.  $3 - 4c + c^2$  \_\_\_\_\_

6.  $z^2 + 4z - 12$  \_\_\_\_\_

7.  $m^2 + 3m - 2$  \_\_\_\_\_

8.  $6 - 5e - e^2$  \_\_\_\_\_

9.  $10 - 3d - d^2$  \_\_\_\_\_

10.  $n^2 - 3n + 4$  \_\_\_\_\_

11.  $2g^2 - 8g - 24$  \_\_\_\_\_

12.  $60 - 5h - 5h^2$  \_\_\_\_\_

13.  $45 + 24n + 3n^2$  \_\_\_\_\_

14.  $2k^2 + 2k - 60$  \_\_\_\_\_

15.  $p^3 - 4p^2 - 5p$  \_\_\_\_\_

16.  $20r + 6r^2 - 2r^3$  \_\_\_\_\_

17.  $2a^2 + 5a - 3$  \_\_\_\_\_

18.  $6b^2 + b - 2$  \_\_\_\_\_

19.  $9s^2 + 6s - 8$  \_\_\_\_\_

20.  $10t^2 - 19t + 6$  \_\_\_\_\_

21.  $30 - 13c - 10c^2$  \_\_\_\_\_

22.  $6d^2 + 35d - 6$  \_\_\_\_\_

23.  $u^2 - uv - 12v^2$  \_\_\_\_\_

24.  $6x^2 - 5xy + y^2$  \_\_\_\_\_

25.  $12x^2 + 12x - 45$  \_\_\_\_\_

26.  $18y^2 + 54y + 28$  \_\_\_\_\_

27.  $6m^3 - 19m^2 + 15m$  \_\_\_\_\_

28.  $20n^3 - 32n^2 + 12n$  \_\_\_\_\_

29.  $20e^4 - 23e^3 + 6e^2$  \_\_\_\_\_

30.  $12f^3 - 14f^2 - 40f$  \_\_\_\_\_

