NAME ______________________ DATE ______________

4–6 Factoring Quadratic Polynomials

Factoring I

Objective: To factor quadratic polynomials.

Vocabulary

Quadratic polynomial (or second-degree polynomial) A polynomial of the form $ax^2 + bx + c$ $(a \neq 0)$. The term ax^2 is the *quadratic term*, bx is the *linear term*, and c is the *constant term*. Examples: $x^2 + 6$ $\quad 2x^2 + 3x$ $\quad x^2 - 4x + 3$

Quadratic trinomial A quadratic polynomial for which a, b, and c are all nonzero integers. Examples: $x^2 + 5x + 6$ $\quad 2x^2 - x + 3$

Irreducible polynomial A polynomial that has more than one term and cannot be expressed as a product of polynomials of lower degree taken from a given factor set. Examples: $3x^2 + 3x - 9$ $\quad x^2 + 2x + 4$

Prime polynomial An irreducible polynomial with integral coefficients is prime if the GCF of its coefficients is 1. Examples: $x^2 + 2x + 4$ $\quad 2x^2 - 3x + 5$

Factor completely a polynomial Express the polynomial as a product of factors where each factor is either a monomial, a prime polynomial, or a power of a prime polynomial.

Example: $3x^3 + 12x^2 + 12x = 3x(x^2 + 4x + 4)$ ← not factored completely

$3x^3 + 12x^2 + 12x = 3x(x + 2)^2$ ← factored completely

Example 1 Factor $x^2 + 5x - 14$.

Solution Recall that multiplying two binomials often gives a trinomial. The FOIL method can be used to multiply two binomials:

$$(2x + 3)(x - 5) = \overset{F}{(2x)(x)} + \overset{O \qquad I}{(-10x + 3x)} + \overset{L}{(3)(-5)}$$

$$= 2x^2 - 7x - 15$$

So one way to factor a trinomial is to try using the FOIL method in reverse.

1. $\overset{F}{x^2} + 5x - 14 = (x \quad)(x \quad)$ — $x^2 = (x)(x)$. There is only one possible product that equals the quadratic term.

2. $x^2 + 5x - \overset{L}{14} = (\ \ ?)(\ \ ?)$ — $(-1)(14)$, $(1)(-14)$, $(-2)(7)$, $(2)(-7)$. Four possible products equal the constant term.

The possible factorizations are:

$(x - 1)(x + 14)$	$(x + 1)(x - 14)$	$(x - 2)(x + 7)$	$(x + 2)(x - 7)$
$O + I = 13x$	$O + I = -13x$	$O + I = 5x$	$O + I = -5x$

3. The linear term of the given trinomial is $5x$. Of the four possible factorizations above, only $(x - 2)(x + 7)$ gives the required linear term.

$$\therefore x^2 + 5x - 14 = (x - 2)(x + 7)$$

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4-6 Factoring Quadratic Polynomials *(continued)*

Example 2 Factor $9x^2 - 15x + 4$.

Solution

1. $9x^2 - 15x + 4 = (x \quad)(9x \quad)$ or $(3x \quad)(3x \quad)$

$(x)(9x)$ $(3x)(3x)$ ← Two possible products equal the quadratic term.

2. $9x^2 - 15x + 4 = (\quad ?)(\quad ?)$

$(1)(4)$ $(-1)(-4)$
$(2)(2)$ $(-2)(-2)$

Only two of these products are possible since the coefficient of the linear term is negative.

The possible factorizations are:

$(x - 1)(9x - 4)$, $O + I = -13x$
$(x - 4)(9x - 1)$, $O + I = -37x$
$(x - 2)(9x - 2)$, $O + I = -20x$
$(3x - 1)(3x - 4)$, $O + I = -15x$
$(3x - 2)(3x - 2)$, $O + I = -12x$

3. The linear term of the given trinomial is $-15x$. Of the possible factorizations above, only $(3x - 1)(3x - 4)$ gives the required linear term.

$\therefore 9x^2 - 15x + 4 = (3x - 1)(3x - 4)$

Example 3 Factor completely: **a.** $x^2 + 2x + 3$ **b.** $4x^4 + 10x^3 - 6x^2$

Solution

a. $x^2 + 2x + 3 = (x \quad ?)(x \quad ?)$

$(1)(3)$ $(-1)(-3)$

Only one of these products is possible since the coefficient of the linear term is positive.

The only possible factorization $(x + 1)(x + 3)$ does not check since $O + I = 3x + x = 4x$ and the required linear term is $2x$.

$\therefore x^2 + 2x + 3$ is *prime*.

b. $4x^4 + 10x^3 - 6x^2 = 2x^2(2x^2 + 5x - 3)$ Factor out $2x^2$.
$= 2x^2(2x - 1)(x + 3)$ Reverse the FOIL method.

Factor completely. If the polynomial is prime, say so.

1. $x^2 - 7x + 6$
2. $k^2 + 7k + 12$
3. $y^2 - 11y + 24$
4. $p^2 + 2p - 3$
5. $z^2 + 5z + 6$
6. $a^2 - 5a - 6$
7. $t^2 + 9t - 22$
8. $v^2 - 11v - 60$
9. $x^2 + 4x + 5$
10. $s^2 + 5s - 36$
11. $h^2 - 15h + 36$
12. $x^2 + 3x - 8$
13. $8u^2 + 6u + 1$
14. $12y^2 - y - 1$
15. $2p^2 + p - 3$
16. $3r^2 + r - 10$
17. $6n^2 + n - 2$
18. $4w^2 - 11w + 6$
19. $2t^2 - 4t - 70$
20. $3x^2 + 9x - 30$
21. $2k^3 - 2k^2 - 112k$
22. $5z^3 - 25z^2 + 30z$
23. $27x^4 + 48x^3 - 12x^2$
24. $36x^4 + 44x^3 + 24x^2$

Factoring 2

Written Exercises

Factor completely. If the polynomial is prime, say so.

A

1. $x^2 - 9x + 8$

2. $t^2 + 9t + 14$

3. $z^2 - 11z + 18$

4. $u^2 - 10u + 9$

5. $r^2 + 12r + 20$

6. $y^2 - 5y + 6$

7. $p^2 - 8p + 9$

8. $h^2 - 10h + 24$

9. $s^2 - 20s + 36$

10. $z^2 - 9z + 12$

11. $x^2 + x - 12$

12. $t^2 + 2t - 15$

13. $t^2 - 2t - 35$

14. $s^2 - 6s - 27$

15. $3z^2 + 4z + 1$

16. $5v^2 + 4v - 1$

17. $8 + 2s - s^2$

18. $21 - 4x - x^2$

19. $x^2 - xy - 30y^2$

20. $p^2 + 2pq - 24q^2$

21. $u^2 - 8uv - 12v^2$

22. $h^2 - 8hk - 15k^2$

23. $2t^2 + 5t - 3$

24. $3x^2 - 8x + 5$

25. $3p^2 - 7p - 6$

26. $4r^2 + 8r + 3$

27. $6x^2 - 7xy - 3y^2$

28. $6s^2 + st - 5t^2$

29. $2h^2 + 7hk - 15k^2$

30. $2u^2 + uv - 21v^2$

B

31. $6x^2 + 7x - 10$

32. $4y^2 - 17y + 15$

33. $4t^2 - 9t + 6$

34. $25u^2 - 20u + 4$

35. $12p^2 - 32pq - 5q^2$

36. $4r^2 + 16rs - 10s^2$

37. $4x^3 + 8x^2y - 5xy^2$

38. $4x^2 + 3xy - 15y^2$

39. $4pq^4 - 32pq$

40. $81uv^3 + 3u^4$

41. $r^4 - 16s^4$

42. $x^6 - 64y^6$

43. $x^4 - 3x^2 - 4$

44. $z^4 - 10z^2 + 9$