Distributions practice *[136 marks]*

1a. *[2 marks]*

Markscheme

sketch of normal curve with shaded region to the right of the mean and correct values ***(M1)***

$0.0921 \quad (0.0920950\ldots)$ ***A1***

[2 marks]

1b. *[3 marks]*

Markscheme

EITHER

$(\mathrm{P}(x < 172))$

$0.906200\ldots$ ***(A1)***

$(0.906200\ldots - 0.68)$

$0.226200\ldots$ ***(A1)***

OR

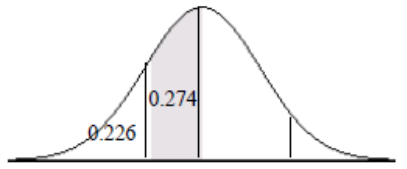
$(\mathrm{P}(163 < x < 172))$

$0.406200\ldots$ ***(A1)***

$0.5 - (0.68 - 0.406200\ldots)$ **OR** $0.5 + (0.68 - 0.406200\ldots)$

$0.226200\ldots$ **OR** $0.773799\ldots$ ***(A1)***

OR



(A1)(A1)

Note: Award ***A1*** for a normal distribution curve with a vertical line on each side of the mean and a correct probability of either 0.406 or 0.274 or 0.906 shown, ***A1*** for a probability of 0.226 seen.

THEN

$(k =)\ 158\ \mathrm{g}\ \ (157.867\ldots\ \mathrm{g})$ ***A1***

[3 marks]

2a. *[2 marks]*

Markscheme

$(\mathrm{E}(X)=)\ 10 \times 0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

2b. *[2 marks]*

Markscheme

recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

2c. *[3 marks]*

Markscheme

0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

3a. *[1 mark]*

Markscheme

$\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

3b. *[2 marks]*

Markscheme

$-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$= 1.83 \left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

3c. *[3 marks]*

Markscheme

$2 \times \frac{1}{18} \times \frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18} \times \frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$= \frac{1}{54} \left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

4a. *[2 marks]*

Markscheme

Let X be the random variable "distance from O".

$X \sim \mathrm{N}\left(10,\ 3^2\right)$

$\mathrm{P}(X < 13) = 0.841\ \ (0.841344\ldots)$ ***(M1)(A1)***

[2 marks]

4b. *[1 mark]*

Markscheme

$(\mathrm{P}(X > 15) =)\ \ 0.0478\ \ (0.0477903)$ ***A1***

[1 mark]

4c. *[2 marks]*

Markscheme

$\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228 \ \ (0.00228391\ldots)$ ***A1***

[2 marks]

4d. *[2 marks]*

Markscheme

$1 - (0.8143)^3$ ***(M1)***

$0.460 \ \ (0.460050\ldots)$ ***A1***

[2 marks]

4e. *[3 marks]*

Markscheme

METHOD 1

let Y be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10,\ 0.539949\ldots)$ ***(A1)***

$(\mathrm{P}(Y \geq 5) =)\ 0.717 \ \ (0.716650\ldots).$ ***A1***

METHOD 2

let Q be the random variable "number of times a point is not scored"

evidence of use of binomial distribution ***(M1)***

$Q \sim \mathrm{B}(10,\ 0.460050\ldots)$ ***(A1)***

$(\mathrm{P}(Q \leq 5) =)\ 0.717 \ \ (0.716650\ldots)$ ***A1***

[3 marks]

4f. *[2 marks]*

Markscheme

$\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$0.628 \ \ (0.627788\ldots)$ ***A1***

Note: Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

[2 marks]

4g. *[2 marks]*

Markscheme

$\frac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \frac{0.627788\ldots}{0.716650\ldots}\right)$ ***(M1)***

$0.876 \ \ (0.876003\ldots)$ ***A1***

[2 marks]

5a. *[2 marks]*

Markscheme

t	1	2	3	4	5	6
$\mathrm{P}(T = t)$	$\frac{1}{36}$ (0.027777…)	$\frac{3}{36}$ (0.083333…)	$\frac{5}{36}$ (0.138888…)	$\frac{7}{36}$ (0.194444…)	$\frac{9}{36}$ (0.25)	$\frac{11}{36}$ (0.305555…)

A2

Note: Award ***A1*** if three to five probabilities are correct.

[2 marks]

5b. *[1 mark]*

Markscheme

$\frac{32}{36}\left(\frac{8}{9}, 0.888888\ldots, 88.9\%\right)$ ***(A1)***

[1 mark]

5c. *[2 marks]*

Markscheme

use of conditional probability ***(M1)***

e.g. denominator of 32 **OR** denominator of $0.888888\ldots$, etc.

$\frac{11}{32}(0.34375, 34.4\%)$ ***A1***

[2 marks]

5d. *[2 marks]*

Markscheme

$\frac{1\times1+3\times2+5\times3+\ldots+11\times6}{36}$ ***(M1)***

$= \frac{161}{36}\left(4\frac{17}{36}, 4.47, 4.47222\ldots\right)$ ***A1***

[2 marks]

6a. *[2 marks]*

Markscheme

evidence of correct probability ***(M1)***

e.g sketch **OR** correct probability statement, $\mathrm{P}(X < 6.5)$

0.0151 ***A1***

[2 marks]

6b. *[1 mark]*

Markscheme

0.0228 ***A1***

Note: Answers should be given to 4 decimal place.

[1 mark]

6c. *[2 marks]*

Markscheme

multiplying **their** probability by 1000 ***(M1)***

451.7 ***A1***

[2 marks]

6d. *[1 mark]*

Markscheme

510.5 ***A1***

Note: Answers should be given to 4 sf.

[1 mark]

6e. *[2 marks]*

Markscheme

H_0 : stopping distances can be modelled by $\mathrm{N}\left(6.76, 0.12^2\right)$

H_1 : stopping distances cannot be modelled by $\mathrm{N}\left(6.76, 0.12^2\right)$ ***A1A1***

Note: Award ***A1*** for correct H_0, including reference to the mean and standard deviation. Award ***A1*** for the negation of their H_0.

[2 marks]

6f. *[3 marks]*

Markscheme

15.1 or 22.8 seen ***(M1)***

$0.0727 (0.0726542\ldots, 7.27\%)$ ***A2***

[3 marks]

6g. *[2 marks]*

Markscheme

$0.05 < 0.0727$ ***R1***

there is insufficient evidence to reject H_0 (or "accept H_0") ***A1***

Note: Do not award ***R0A1***.

[2 marks]

7a. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of summing probabilities to 1 ***(M1)***

eg $q + 4p^2 + p + 0.7 - 4p^2 = 1,\ \ 1 - 4p^2 - p - 0.7 + 4p^2$

$q = 0.3 - p$ ***A1 N2***

[2 marks]

7b. *[3 marks]*

Markscheme

correct substitution into $\mathrm{E}(X)$ formula ***(A1)***

eg $0 \times (0.3 - p) + 1 \times 4p^2 + 2 \times p + 3 \times (0.7 - 4p^2)$

valid approach to find when $\mathrm{E}(X)$ is a maximum ***(M1)***

eg max on sketch of $\mathrm{E}(X)$, $8p + 2 + 3 \times (-8p) = 0$, $\frac{-b}{2a} = \frac{-2}{2 \times (-8)}$

$p = \frac{1}{8}$ $(= 0.125)$ (exact) (accept $x = \frac{1}{8}$) ***A1 N3***

[3 marks]

7c. *[1 mark]*

Markscheme

2.225

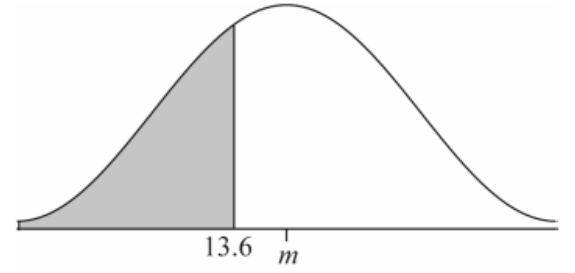
$\frac{89}{40}$ (exact), 2.23 ***A1 N1***

[1 mark]

8a. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1)

Note: Award ***(A1)*** for bell shaped curve with mean m **or** 13.6 indicated.
Award ***(A1)*** for approximately correct shaded region.

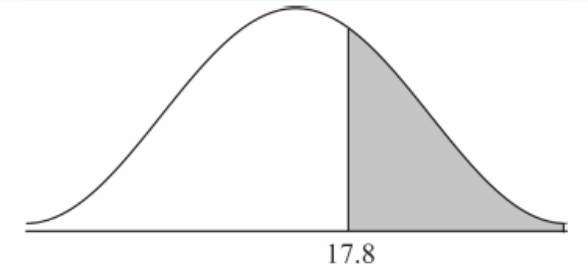
[2 marks]

8b. *[2 marks]*

Markscheme

$\mathrm{P}(T > 17.8) = 0.3$ ***(M1)***

OR



(M1)

Note: Award ***(M1)*** for correct probability equation using 0.3 **OR** correctly shaded diagram indicating 17.8. Strict or weak inequalities are accepted in parts (b), (c) and (d).

$\frac{13.6+17.8}{2}$ $\left(17.8 - \frac{17.8-13.6}{2}\right)$ **OR** $\left(13.6 + \frac{17.8-13.6}{2}\right)$ ***(M1)***

Note: Award ***(M0)(M1)*** for unsupported $\frac{13.6+17.8}{2}$ **OR** $\left(17.8 - \frac{17.8-13.6}{2}\right)$ **OR** $\left(13.6 + \frac{17.8-13.6}{2}\right)$ **OR** the midpoint of 13.6 and 17.8 is 15.7.
Award at most ***(M1)(M0)*** if the final answer is not seen. Award ***(M0)(M0)*** for using known values $m = 15.7$ and $\sigma = 4$ to validate $\mathrm{P}(T < 17.8) = 0.7$ or $\mathrm{P}(T < 13.6) = 0.3$.

15.7 ***(AG)***

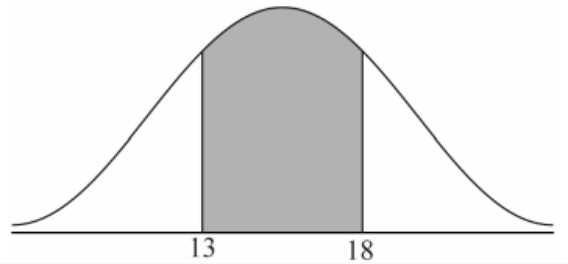
[2 marks]

8c. *[2 marks]*

Markscheme

$\mathrm{P}(13 \leq T \leq 18)$ ***(M1)***

OR



(M1)

Note: Award ***(M1)*** for correct probability equation **OR** correctly shaded diagram indicating 13 and 18.

$0.468 \ \ (46.8\%, \ 0.467516\ldots)$ ***(A1)(G2)***

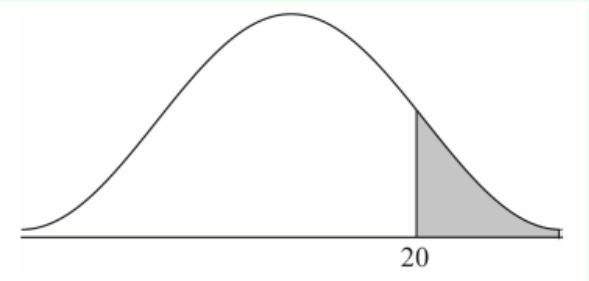
[2 marks]

8d. *[2 marks]*

Markscheme

$\mathrm{P}(T \geq 20)$ ***(M1)***

OR



(M1)

Note: Award ***(M1)*** for correct probability equation **OR** correctly shaded diagram indicating 20.

$0.141 \ \ (14.1\%, \ 0.141187\ldots)$ ***(A1)(G2)***

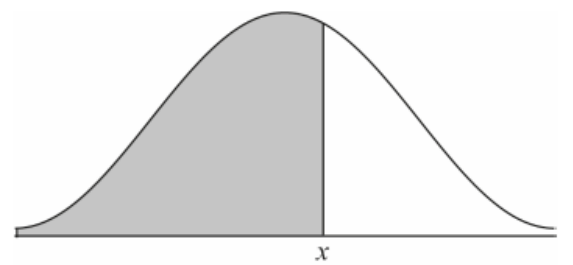
[2 marks]

8e. *[2 marks]*

Markscheme

$\mathrm{P}(T < t) = 0.6$ ***(M1)***

OR



(M1)

Note: Award ***(M1)*** for correct probability equation **OR** for a correctly shaded region with x indicated to the right-hand side of the mean.

$16.7 \ (16.7133\ldots)$ ***(A1)(G2)***

[2 marks]

8f. *[3 marks]*

Markscheme

$0.467516\ldots \times 0.141187\ldots \times 2$ ***(M1)(M1)***

OR

$(0.467516\ldots \times 0.141187\ldots) + (0.141187\ldots \times 0.467516\ldots)$ ***(M1)*** ***(M1)***

Note: Award ***(M1)*** for the multiplication of their parts (c)(i) and (c)(ii), ***(M1)*** for multiplying their product by 2 or for adding their products twice. Follow through from part (c).

$0.132 \ (13.2\%, \ 0.132014\ldots)$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(G0)*** for an unsupported final answer of $0.066007\ldots$

[3 marks]

8g. *[2 marks]*

Markscheme

$\frac{69}{102} \times 200$ ***(M1)***

Note: Award ***(M1)*** for correct probability multiplied by 200.

135 $(135.294\ldots)$ ***(A1)(G2)***

[2 marks]

8h. *[2 marks]*

Markscheme

$\left(\frac{67}{98} \times 200 =\right) 136.734\ldots$ ***(A1)***

Note: Award ***(M1)*** for 137 or $136.734\ldots$ seen.

Emlyn is incorrect, $135 < 137$ $(135.294\ldots < 136.734\ldots)$ ***(R1)***

Note: To award the final ***(R1)***, both the conclusion and the comparison must be seen. Award at most ***(A0)(R1)*****(ft)** for consistent incorrect methods in parts (f) and (g).

OR

$\left(\frac{67}{98} =\right) 0.684$ $(0.683673\ldots)$ $\left(\frac{69}{102} =\right) 0.676$ $(0.676470\ldots)$ ***(A1)***

Note: Award ***(A1)*** for both correct probabilities seen.

Emlyn is incorrect, $0.676 < 0.684$ ***(R1)***

Note: To award the final ***(R1)***, both the conclusion and the comparison must be seen. Award at most ***(A0)(R1)*****(ft)** for consistent incorrect methods in parts (f) and (g).

[2 marks]

9a. *[1 mark]*

Markscheme

$0.5\ \left(\frac{1}{2},\ 50\%\right)$ ***(A1) (C1)***

[1 mark]

9b. *[2 marks]*

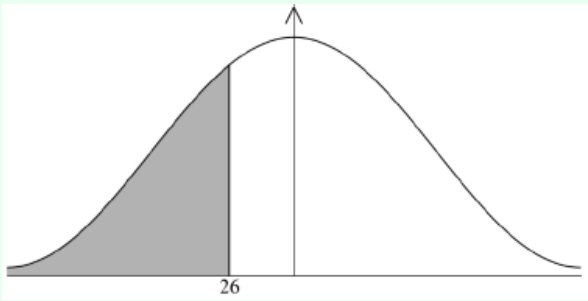
Markscheme

$\mathrm{P}(X \leq 26)$ ***(M1)***

Note: Award ***(M1)*** for a correct mathematical statement.

OR

Award ***(M1)*** for a diagram that shows the value 26 labelled to the left of the mean and the correct shaded region.



$3.45\ (0.344578\ldots,\ 34.5\%)$ ***(A1) (C2)***

[2 marks]

9c. *[3 marks]*

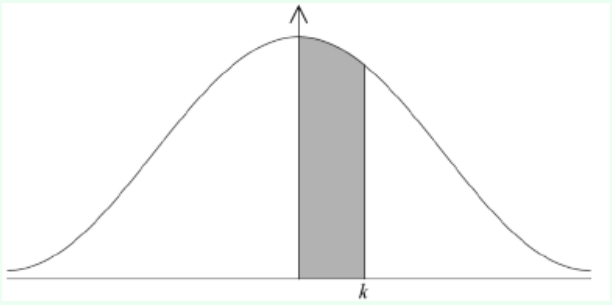
Markscheme

0.7 **OR** 0.3 (seen) ***(A1)***

Note: Award ***(A1)*** for 0.7 or 0.3 seen.

$\mathrm{P}(\text{time} < 7) = 0.7$ **OR** $\mathrm{P}(\text{time} > k) = 0.3$ ***(M1)***

Note: Award ***(M1)*** for a correct mathematical statement.
OR
Award ***(M1)*** for a diagram that shows k greater than the mean and shading in the region below k, above k, or between k and the mean.



$(k =)\ 30.6\ (30.6220\ldots)$ (minutes) ***(A1)*** ***(C3)***

Note: Accept "30 minutes and 37 seconds" or (from 3 sf k value) "30 minutes and 36 seconds".

[3 marks]

10a. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing area under curve = 1 ***(M1)***

eg $a + x + b = 1,\ 100 - a - b,\ 1 - a + b$

$\mathrm{P}(-1.6 < z < 2.4) = 1 - a - b\ \ (= 1 - (a + b))$ ***A1 N2***

[2 marks]

10b. *[4 marks]*

Markscheme

$\mathrm{P}(z > -1.6) = 1 - a$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|B),\ \mathrm{P}(B|A)$

correct working ***(A1)***

eg $\frac{\mathrm{P}(z<2.4 \cap z>-1.6)}{\mathrm{P}(z>-1.6)}$, $\frac{\mathrm{P}(-1.6<z<2.4)}{\mathrm{P}(z>-1.6)}$

$\mathrm{P}(z < 2.4 \,|\, z > -1.6) = \frac{1-a-b}{1-a}$ ***A1 N4***

Note: Do not award the final ***A1*** if correct answer is seen followed by incorrect simplification.

[4 marks]

10c. *[1 mark]*

Markscheme

$z = -1.6$ (may be seen in part (d)) ***A1 N1***

Note: Depending on the candidate's interpretation of the question, they may give $\frac{1-m}{s}$ as the answer to part (c). Such answers should be awarded the first ***(M1)*** in part (d), even when part (d) is left blank. If the candidate goes on to show $z = -1.6$ as part of their working in part (d), the ***A1*** in part (c) may be awarded.

[1 mark]

10d. *[6 marks]*

Markscheme

attempt to standardize x (do not accept $\frac{x-\mu}{\sigma}$) ***(M1)***

eg $\frac{1-m}{s}$ (may be seen in part (c)), $\frac{m-2}{s}$, $\frac{x-m}{\sigma}$

correct equation with each z-value ***(A1)(A1)***

eg $-1.6 = \frac{1-m}{s}$, $2.4 = \frac{2-m}{s}$, $m + 2.4s = 2$

valid approach (to set up equation in one variable) ***M1***

eg $2.4 = \frac{2-(1.6s+1)}{s}$, $\frac{1-m}{-1.6} = \frac{2-m}{2.4}$

correct working ***(A1)***

eg $1.6s + 1 = 2 - 2.4s$, $4s = 1$, $m = \frac{7}{5}$

$s = \frac{1}{4}$ ***A1 N2***

[6 marks]

11a. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Mean and standard deviation	Graph
Mean $= -2$; standard deviation $= 0.707$	C
Mean $= 0$; standard deviation $= 0.447$	D

(A1)(A1)

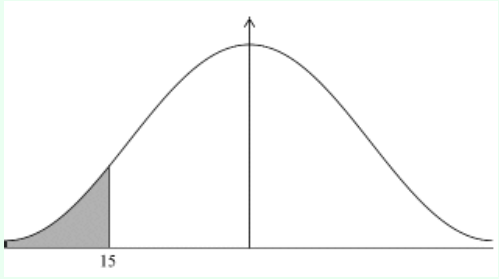
(C2)

Note: Award ***(A1)*** for each correct entry.

[2 marks]

11b. *[2 marks]*

Markscheme



(M1)

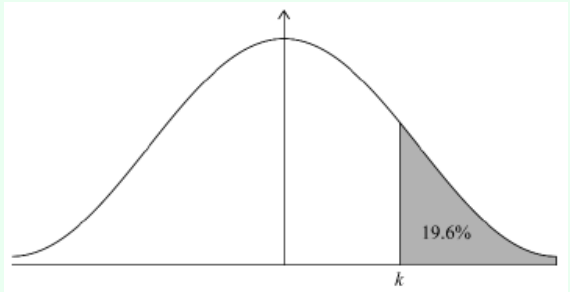
Note: Award ***(M1)*** for sketch with 15 labelled and left tail shaded **OR** for a correct probability statement, P($X < 15$).

0.0766 (0.0765637..., 7.66%) ***(A1)*** ***(C2)***

[2 marks]

11c. *[2 marks]*

Markscheme



(M1)

Note: Award ***(M1)*** for a sketch showing correctly shaded region to the right of the mean with 19.6% labelled (accept shading of the complement with 80.4% labelled) **OR** for a correct probability statement, P($X > k$) = 0.196 or P($X \leq k$) = 0.804.

23.0 (kg) (22.9959... (kg)) ***(A1)*** ***(C2)***

[2 marks]

12a. *[4 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg total probability = 1

correct equation ***(A1)***

eg $0.475 + 2k^2 + \frac{k}{10} + 6k^2 = 1,\ 8k^2 + 0.1k - 0.525 = 0$

$k = 0.25$ ***A2*** ***N3***

[4 marks]

12b. *[1 mark]*

Markscheme

$\mathrm{P}(X = 2) = 0.025$ ***A1*** ***N1***

[1 mark]

12c. *[3 marks]*

Markscheme

valid approach for finding $\mathrm{P}(X > 0)$ ***(M1)***

eg $1 - 0.475,\ 2(0.25^2) + 0.025 + 6(0.25^2),\ 1 - \mathrm{P}(X = 0),\ 2k^2 + \frac{k}{10} + 6k^2$

correct substitution into formula for conditional probability ***(A1)***

eg $\frac{0.025}{1-0.475}$, $\frac{0.025}{0.525}$

0.0476190

$\mathrm{P}(X = 2|X > 0) = \frac{1}{21}$ (exact), 0.0476 ***A1*** ***N2***

[3 marks]

13a. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of binomial distribution (may be seen in part (b)) ***(M1)***

$egnp$, 150×0.08

$k = 12$ ***A1*** ***N2***

[2 marks]

13b. *[2 marks]*

Markscheme

$\mathrm{P}(X = 12) = \binom{150}{12} (0.08)^{12} (0.92)^{138}$ ***(A1)***

0.119231

probability $= 0.119$ ***A1*** ***N2***

[2 marks]

13c. *[2 marks]*

Markscheme

recognition that $X \leqslant 11$ ***(M1)***

0.456800

$\mathrm{P}(X < 12) = 0.457$ ***A1*** ***N2***

[2 marks]

14a. *[1 mark]*

Markscheme

$\frac{6}{15}\left(0.4, \frac{2}{5}\right)$ ***A1***

[1 mark]

14b. *[2 marks]*

Markscheme

P($X = 8$) ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability. eg P($X = 8$), $X \sim \text{B}\left(20, \frac{6}{15}\right)$.

0.180 (0.179705...) ***A1***

[2 marks]

14c. *[3 marks]*

Markscheme

P(male) = $\frac{9}{15}(0.6)$ ***A1***

P($X \leq 9$) = 0.128 (0.127521...) ***(M1)A1***

Note: Award ***(M1)*** for evidence of correct approach *eg*, P($X \leq 9$).

[3 marks]

15a. *[2 marks]*

Markscheme

valid approach to find P(one red) (M1)

eg ${}_nC_a \times p^a \times q^{n-a}$, $\text{B}(n, p)$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\binom{3}{1}$

listing all possible cases for exactly one red (may be indicated on tree diagram)

P(1 red) = 0.444 $\left(= \frac{4}{9}\right)$ [0.444, 0.445] ***A1 N2***

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

15b. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $P(X = 2) + P(X = 3)$, $1 - P(X \leq 1)$, $\text{binomcdf}\left(3, \frac{1}{3}, 2, 3\right)$

correct working ***(A1)***

eg $\frac{2}{9} + \frac{1}{27}$, 0.222 + 0.037 , $1 - \left(\frac{2}{3}\right)^3 - \frac{4}{9}$

0.259259

P(at least two red) = $0.259\left(= \frac{7}{27}\right)$ ***A1 N3***

[3 marks] [5 maximum for parts (a.i) and (a.ii)]

15c. *[5 marks]*

Markscheme

recognition that winning \$10 means rolling exactly one green ***(M1)***

recognition that winning \$10 also means rolling at most 1 red ***(M1)***

eg "cannot have 2 or more reds"

correct approach ***A1***

eg $P(1G \cap 0R) + P(1G \cap 1R)$, $P(1G) - P(1G \cap 2R)$,

"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this ***A1*** for purely numerical expressions.

one correct probability for their approach ***(A1)***

eg $3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2$, $\frac{6}{27}$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\frac{1}{9}$, $\frac{2}{9}$

correct working leading to $\frac{1}{3}$ ***A1***

eg $\frac{3}{27} + \frac{6}{27}$, $\frac{12}{27} - \frac{3}{27}$, $\frac{1}{9} + \frac{2}{9}$

probability = $\frac{1}{3}$ ***AG N0***

[5 marks]

15d. *[1 mark]*

Markscheme

$x = \frac{7}{27}$, 0.259 (check ***FT*** from (a)(ii)) ***A1 N1***

[1 mark]

15e. *[2 marks]*

Markscheme

evidence of summing probabilities to 1 ***(M1)***

eg $\sum = 1,\ x + y + \frac{1}{3} + \frac{2}{9} + \frac{1}{27} = 1,\ 1 - \frac{7}{27} - \frac{9}{27} - \frac{6}{27} - \frac{1}{27}$

0.148147 (0.148407 if working with **their** x value to 3 sf)

$y = \frac{4}{27}$ (exact), 0.148 ***A1 N2***

[2 marks]

15f. *[3 marks]*

Markscheme

correct substitution into the formula for expected value ***(A1)***

eg $-w \cdot \frac{7}{27} + 10 \cdot \frac{9}{27} + 20 \cdot \frac{6}{27} + 30 \cdot \frac{1}{27}$

correct critical value (accept inequality) ***A1***

eg $w = 34.2857\ \left(= \frac{240}{7}\right),\ w > 34.2857$

$40 ***A1 N2***

[3 marks]