# Trapezoids [65 marks]

**1a.** *[2 marks]*

The cross-sectional view of a tunnel is shown on the axes below. The line represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by , relative to an origin .

![](data:image/png;base64;base64,)

Point has coordinates , point has coordinates , and point has coordinates .

When the height of the tunnel is and when the height of the tunnel is . These points are shown as and on the diagram, respectively.

Write down the integral which can be used to find the cross-sectional area of the tunnel.

**1b.** *[2 marks]*

Hence find the cross-sectional area of the tunnel.

**2a.** *[3 marks]*

A function is given by .

Write down the derivative of .

**2b.** *[3 marks]*

Find the point on the graph of at which the gradient of the tangent is equal to 6.

**3a.** *[2 marks]*

The following diagram shows part of the graph of , . The shaded region *R* is bounded by the -axis, -axis and the graph of .

![](data:image/png;base64;base64,)

Write down an integral for the area of region *R*.

**3b.** *[1 mark]*

Find the area of region *R*.

**3c.** *[2 marks]*

The three points A(0, 0) , B(3, 10) and C(, 0) define the vertices of a triangle.

![](data:image/png;base64;base64,)

Find the value of , the -coordinate of C, such that the area of the triangle is equal to the area of region *R*.

**4a.** *[1 mark]*

Consider the curve .

Find an expression for .

**4b.** *[6 marks]*

Show that the normal to the curve at the point where  is .

**5a.** *[1 mark]*

The diagram shows the curve .

![](data:image/png;base64;base64,)

The equation of the vertical asymptote of the curve is .

Write down the value of .

**5b.** *[3 marks]*

Find .

**5c.** *[2 marks]*

At the point where , the gradient of the tangent to the curve is .

Find the value of .

**6a.** *[5 marks]*

The function  has a local maximum and a local minimum. The local maximum is at .

Show that .

**6b.** *[2 marks]*

Find the coordinates of the local **minimum**.

**6c.** *[2 marks]*

Write down the interval where the gradient of the graph of  is negative.

**6d.** *[5 marks]*

Determine the equation of the normal at in the form .

**7a.** *[2 marks]*

Consider the curve *y* = 5*x*3 − 3*x*.

Find .

**7b.** *[2 marks]*

The curve has a tangent at the point P(−1, −2).

Find the gradient of this tangent at point P.

**7c.** *[2 marks]*

Find the equation of this tangent. Give your answer in the form *y* = *mx* + *c*.

**8a.** *[2 marks]*

Let . The graph of  is shown in the following diagram.

![](data:image/png;base64;base64,)

Find .

**8b.** *[4 marks]*

Find the area of the region enclosed by the graph of , the *x*-axis and the lines *x* = 1 and *x* = 2 .

**9a.** *[4 marks]*

Consider the curve *y* = 2*x*3 − 9*x*2 + 12*x* + 2, for −1 < *x* < 3

Sketch the curve for −1 < *x* < 3 and −2 < *y* < 12.

**9b.** *[1 mark]*

A teacher asks her students to make some observations about the curve.

Three students responded.  
**Nadia** said *“The x-intercept of the curve is between −1 and zero”.*  
**Rick** said *“The curve is decreasing when x < 1 ”.*  
**Paula** said *“The gradient of the curve is less than zero between x = 1 and x = 2 ”.*

State the name of the student who made an **incorrect** observation.

**9c.** *[3 marks]*

Find .

**9d.** *[3 marks]*

Given that *y* = 2*x*3 − 9*x*2 + 12*x* + 2 = *k* has **three** solutions, find the possible values of *k*.

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