# Trapezoids [65 marks]

**1a.** *[2 marks]*

The cross-sectional view of a tunnel is shown on the axes below. The line $\left[AB\right]$ represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y=−0.1x^{3}+ 0.8x^{2}, 2\leq x\leq 8$, relative to an origin $O$.



Point $A$ has coordinates $\left(2, 0\right)$, point $B$ has coordinates $\left(2, 2.4\right)$, and point $C$ has coordinates $\left(8, 0\right)$.

When $x=4$ the height of the tunnel is $6.4 m$ and when $x=6$ the height of the tunnel is $7.2 m$. These points are shown as $D$ and $E$ on the diagram, respectively.

Write down the integral which can be used to find the cross-sectional area of the tunnel.

**1b.** *[2 marks]*

Hence find the cross-sectional area of the tunnel.

**2a.** *[3 marks]*

A function $f$ is given by $f(x)=4x^{3}+\frac{3}{x^{2}}−3, x\ne 0$.

Write down the derivative of $f$.

**2b.** *[3 marks]*

Find the point on the graph of $f$ at which the gradient of the tangent is equal to 6.

**3a.** *[2 marks]*

The following diagram shows part of the graph of $f(x)=\left(6−3x\right)\left(4+x\right)$, $x\in R$. The shaded region *R* is bounded by the $x$-axis, $y$-axis and the graph of $f$.



Write down an integral for the area of region *R*.

**3b.** *[1 mark]*

Find the area of region *R*.

**3c.** *[2 marks]*

The three points A(0, 0) , B(3, 10) and C($a$, 0) define the vertices of a triangle.



Find the value of $a$, the $x$-coordinate of C, such that the area of the triangle is equal to the area of region *R*.

**4a.** *[1 mark]*

Consider the curve $y=x^{2}−4x+2$.

Find an expression for $\frac{dy}{dx}$.

**4b.** *[6 marks]*

Show that the normal to the curve at the point where $x=1$ is $2y−x+3=0$.

**5a.** *[1 mark]*

The diagram shows the curve $y=\frac{x^{2}}{2}+\frac{2a}{x}, x\ne 0$.



The equation of the vertical asymptote of the curve is $x=k$.

Write down the value of $k$.

**5b.** *[3 marks]*

Find $\frac{dy}{dx}$.

**5c.** *[2 marks]*

At the point where $x=2$, the gradient of the tangent to the curve is $0.5$.

Find the value of $a$.

**6a.** *[5 marks]*

The function $f\left(x\right)=\frac{1}{3}x^{3}+\frac{1}{2}x^{2}+kx+5$ has a local maximum and a local minimum. The local maximum is at $x=−3$.

Show that $k=−6$.

**6b.** *[2 marks]*

Find the coordinates of the local **minimum**.

**6c.** *[2 marks]*

Write down the interval where the gradient of the graph of $f\left(x\right)$ is negative.

**6d.** *[5 marks]*

Determine the equation of the normal at $x=−2$ in the form $y=mx+c$.

**7a.** *[2 marks]*

Consider the curve *y* = 5*x*3 − 3*x*.

Find $\frac{dy}{dx}$.

**7b.** *[2 marks]*

The curve has a tangent at the point P(−1, −2).

Find the gradient of this tangent at point P.

**7c.** *[2 marks]*

Find the equation of this tangent. Give your answer in the form *y* = *mx* + *c*.

**8a.** *[2 marks]*

Let $f\left(x\right)=6x^{2}−3x$. The graph of $f$ is shown in the following diagram.



Find $∫\left(6x^{2}−3x\right)dx$.

**8b.** *[4 marks]*

Find the area of the region enclosed by the graph of $f$, the *x*-axis and the lines *x* = 1 and *x* = 2 .

**9a.** *[4 marks]*

Consider the curve *y* = 2*x*3 − 9*x*2 + 12*x* + 2, for −1 < *x* < 3

Sketch the curve for −1 < *x* < 3 and −2 < *y* < 12.

**9b.** *[1 mark]*

A teacher asks her students to make some observations about the curve.

Three students responded.
**Nadia** said *“The x-intercept of the curve is between −1 and zero”.*
**Rick** said *“The curve is decreasing when x < 1 ”.*
**Paula** said *“The gradient of the curve is less than zero between x = 1 and x = 2 ”.*

State the name of the student who made an **incorrect** observation.

**9c.** *[3 marks]*

Find $\frac{dy}{dx}$.

**9d.** *[3 marks]*

Given that *y* = 2*x*3 − 9*x*2 + 12*x* + 2 = *k* has **three** solutions, find the possible values of *k*.

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