

Mathematics: applications and interpretation Standard level Paper 1

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	Candidate session number								
1 hour 30 minutes									

Instructions to candidates

- · Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- · A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is [80 marks].





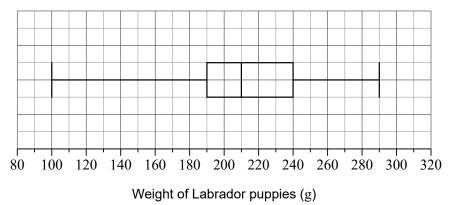
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

(a)

Palvinder breeds Labrador puppies at his farm. Over many years he recorded the weight (g) of the puppies.

The data is illustrated in the following box and whisker diagram.



Write down the median weight of the puppies.

[1]

(b) Write down the upper quartile.

[1]

(c) Find the interquartile range.

[2]

The weights of these Labrador puppies are normally distributed.

(d) Find the weight of the heaviest possible puppy that is not an outlier.

[2]

(This question continues on the following page)



(Question 1 continued)

i	
	1



The Osaka Tigers basketball team play in a multilevel stadium.



The most expensive tickets are in the first row. The ticket price, in Yen (X), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game				
1st row	6800 Yen			
2nd row	6550 Yen			
3rd row	6300 Yen			

(a)	Write down the value of the common difference, d	[1]
(b)	Calculate the price of a ticket in the 16th row.	[2]
(c)	Find the total cost of buying 2 tickets in each of the first 16 rows.	[3]



At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

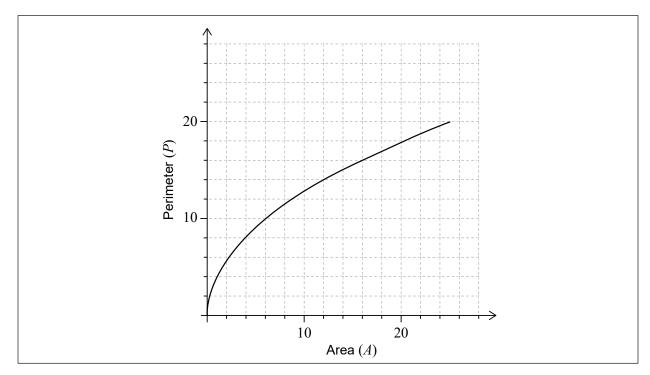
Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

The mean number of classes in which a student used the internet is 2.	
The mean number of diasses in which a student used the internet is 2.	
(b) Find the value of k .	[4]
It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every $10\mathrm{th}$ person on the list.	
(c) Identify the sampling technique used in the survey.	[1]



Turn over

The perimeter of a given square P can be represented by the function $P(A)=4\sqrt{A}$, $A\geq 0$, where A is the area of the square. The graph of the function P is shown for $0\leq A\leq 25$.



(a) Write down the value of P(25).

[1]

The range of P(A) is $0 \le P(A) \le n$.

(b) Hence write down the value of n.

[1]

(c) On the axes above, draw the graph of the inverse function, P^{-1} .

[3]

(d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$.

[1]

(This question continues on the following page)



(Question 4 continued)



5.	[Maximum]	mark:	61
IJ.	IIVIAXIIIIUIII	IIIain.	ΟI

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P=1350+400(1.25)^{-t}$, $t\geq 0$, where t is the number of days since the start of the migration season.

- (a) Find the population of the Bulbul birds,
 - (i) at the start of the migration season.
 - (ii) in the wetlands after 5 days.

[3]

(b) Calculate the time taken for the population to decrease below 1400.

[2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season.

[1]



As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week					
	0	1-2	3-4	>4		
Students	45	26	18	6		
Professors	15	8	5	12		
Staff and Administration	16	13	10	6		

Jing conducted a χ^2 test for independence at a 5 % level of significance.

(a)	State the null hypothesis.	[1]
(b)	Calculate the <i>p</i> -value for this test.	[2]
(c)	State, giving a reason, whether the null hypothesis should be accepted.	[2]



Turn over

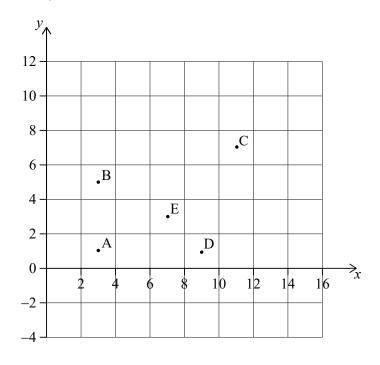
[2]

7. [Maximum mark: 6]

Points A(3,1), B(3,5), C(11,7), D(9,1) and E(7,3) represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



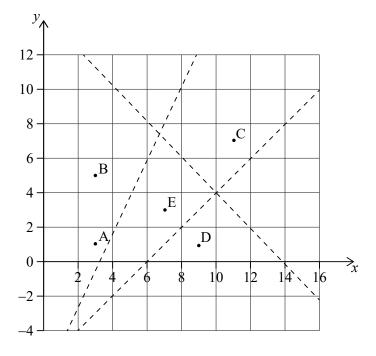
(a) Calculate the gradient of the line segment AE.

(This question continues on the following page)



(Question 7 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



- (b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form ax + by + d = 0 where $a, b, d \in \mathbb{Z}$. [3]
- (c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]



Turn over

[2]

8. [Maximum mark: 4]

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m $^{-2}$). The intensity level is given by the following formula.

$$L = 10 \log_{10}(S \times 10^{12}), S \ge 0$$

(a) An orchestra has a sound intensity of $6.4 \times 10^{-3} \, \mathrm{W \, m^{-2}}$. Calculate the intensity level, L of the orchestra.

(b) A rock concert has an intensity level of $112 \, \mathrm{dB}$. Find the sound intensity, S. [2]



Ms Calhoun measures the heights of students in her mathematics class. She is interested to see if the mean height of male students, $\mu_{\rm l}$, is the same as the mean height of female students, μ_2 . The information is recorded in the table.

Male height (cm)	150	148	143	152	151	149	147	
Female height (cm)	148	152	154	147	146	153	152	150

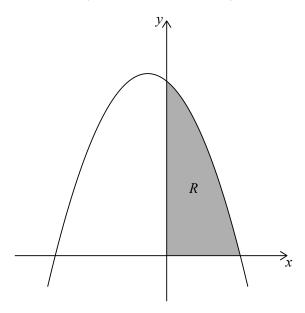
At the 10% level of significance, a *t*-test was used to compare the means of the two groups.

		data is wo gro	s assumed to be normally distributed and the standard deviations are equal between pups.	
	(a)	(i)	State the null hypothesis.	
		(ii)	State the alternative hypothesis.	[2]
	(b)	Calc	ulate the p -value for this test.	[2]
	(c)	State	e, giving a reason, whether Ms Calhoun should accept the null hypothesis.	[2]



Turn over

The following diagram shows part of the graph of f(x) = (6 - 3x)(4 + x), $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, y-axis and the graph of f.



(a) Write down an integral for the area of region R.

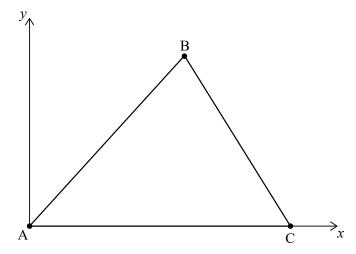
[2]

(b) Find the area of region R.

[1]

[2]

The three points A(0, 0), B(3, 10) and C(a, 0) define the vertices of a triangle.



(c) Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region R.

(This question continues on the following page)

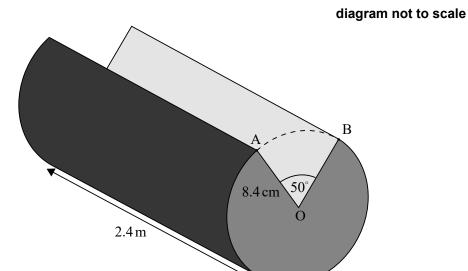


(Question 10 continued)



Turn over

Helen is building a cabin using cylindrical logs of length $2.4\,\mathrm{m}$ and radius $8.4\,\mathrm{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.



Find the volume of this log.



Jae Hee plays a game involving a biased six-sided die.

The faces of the die are labelled -3, -1, 0, 1, 2 and 5.

The score for the game, X, is the number which lands face up after the die is rolled.

The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
P(X=x)	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a)	Find the exact value of n	

[1]

Jae Hee plays the game once.

(b) Calculate the expected score.

[2]

Jae Hee plays the game twice and adds the two scores together.

	/ _ \	Find the			I	-4-1	- 4 2
1		i Find the	nronaniiity	i Jae Hee	nas a t	ntai score	$\Omega I - 1$
М	\cdot	1 1114 1110	probability	ouc i icc	nas a t	otal socie	O1 2.

[3]



Turn over

13.	[Maximum mark: 6]

Mr Burke teaches a mathematics class with $15~{\rm students}$. In this class there are $6~{\rm female}$ students and $9~{\rm male}$ students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question.

[1]

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times.

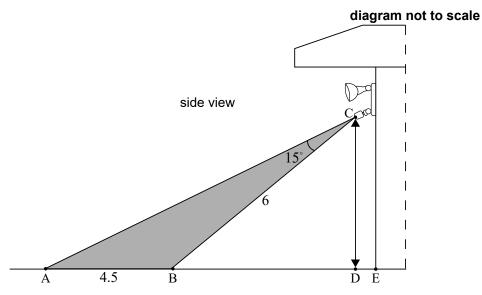
[2]

(c) Find the probability he will choose a male student at most $9 \ \mathrm{times}$.

[3]



Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is $A.5 \, m$ and the distance from B to C is $6 \, m$. Angle $A \, CB$ is $15 \, C$.



(a) Find CÂB. [3]

Point B on the ground is $5\,m$ from point E at the entrance to Ollie's house. He is $1.8\,m$ tall and is standing at point D, below the sensor. He walks towards point B.

(b)	Find the distance Ollie is from the entrance to his house when he first activates	
	the sensor.	[5]



Please **do not** write on this page.

Answers written on this page will not be marked.





Markscheme

Specimen paper

Mathematics: applications and interpretation

Standard level

Paper 1



Instructions to Examiners

Abbreviations

- **M** Marks awarded for attempting to use a correct **Method**.
- A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- **R** Marks awarded for clear **Reasoning**.
- **AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg M1, A2.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award MO followed by A1, as A mark(s) depend on the preceding M mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies *M2*, *A3*, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working.
 However, if further working indicates a lack of mathematical understanding do not award the final
 A1. An exception to this may be in numerical answers, where a correct exact value is followed by
 an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part,
 and correct FT working shown, award FT marks as appropriate but do not award the final A1 in
 that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 (incorrect decimal value)	Award the final A1 (ignore the further working)
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final A1
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final A1

3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks without brackets can only be awarded for work that is seen.

4 Follow through marks (only applied after an error is made)

Follow through (FT) marks are awarded where an incorrect answer from one part of a question is used correctly in subsequent part(s) or subpart(s). Usually, to award FT marks, there must be working present and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then FT marks should be awarded if appropriate.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer FT marks.
- If the error leads to an inappropriate value (e.g. probability greater than 1, use of r > 1 for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the MR leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- The *MR* penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by METHOD 1, METHOD 2, etc.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, accept equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- Rounding errors: only applies to final answers not to intermediate steps.
- Level of accuracy: when this is not specified in the question the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/CAS functionality are not allowed.

Calculator notation

The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) 210g

(b) 240g

A1

[1 mark]

[1 mark]

(c) 240-190 (M1) = 50 g A1 [2 marks]

2. (a) (d =) - 250

(b) $(u_{16} =)6800 + (16 - 1)(-250)$ M1 (¥)3050 A1

(c) $(S_{16} =) \left(\frac{16}{2}\right) (2 \times 6800 + (16 - 1)(-250)) \times 2$

Note: Award *M1* for correct substitution into arithmetic series formula. Award *M1* for multiplication by 2 seen.

OR

 $(S_{16} =) \left(\frac{16}{2}\right) (6800 + 3050) \times 2$ M1M1

Note: Award *M1* for correct substitution into arithmetic series formula. Award *M1* for multiplication by 2 seen.

(¥)158000 (157600) **A1**

[3 marks]

Total [6 marks]

Total [6 marks]

[1 mark]

[2 marks]

3. (a) discrete A1

[1 mark]

(b) $\frac{24+60+3k+40+15+6}{88+k} = 2$ **M1A1**

Note: Award *M1* for substitution into the formula for the mean, award *A1* for a correct equation.

attempt to solve their equation (M1)

k = 31 A1 [4 marks]

(c) systematic A1

Total [6 marks]

[1 mark]

4. (a) 20

A1 [1 mark]

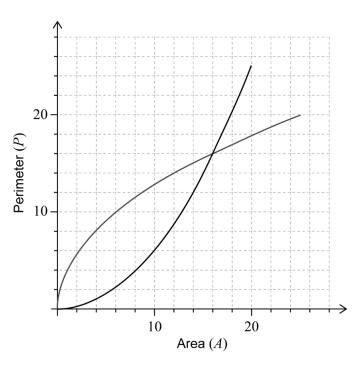
(b) n = 20

A1

Note: Follow through from part (a).

[1 mark]

(c)



(M1)A1A1

Note: Award *(M1)* for reflection in the line P = A, award *A1* for endpoint at (20, 25), award *A1* for passing through (16, 16).

[3 marks]

(d) when the perimeter is 8, the area is 4

-

A1

[1 mark]

Total [6 marks]

Total [5 marks]

5.	(a)	(i)	1750	A1	
		(ii)	$1350 + 400 (1.25)^{-5}$	(M1)	
			=1480	A1	
	Note	: Ac	cept 1481.		
					[3 marks]
	(b)	1400	$0 = 1350 + 400(1.25)^{-t}$	(M1)	
		9.32	(days (9.31885) (days))	A1	[O manulca]
					[2 marks]
	(c)	1350		A1	
	Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.				
					[1 mark]
				Total	[6 marks]
6.	(a)		ber of salad meals per week is independent of a person's position e university	A1	
	Note	: Ac	cept "not associated" instead of independent.		
			· · · · · · · · · · · · · · · · · · ·		[1 mark]
	(b)	0.02	01 (0.0201118)	A2	
					[2 marks]
	(c)	0.02	01 < 0.05	R1	
		the r	null hypothesis is rejected	A1	[2 marks]
					[Z IIIai KS]
	Note	aw	ard $(R1)$ for a correct comparison of their p -value to the test level, ard $(A1)$ for the correct interpretation from that comparison. not award $(R0)(A1)$.		

7. (a)
$$\frac{3-1}{7-3}$$
 (M1)
$$= 0.5$$
 A1 [2 marks]

(b) y-2=-2(x-5) (A1)(M1)

Note: Award *(A1)* for their -2 seen, award *(M1)* for the correct substitution of (5, 2) and their normal gradient in equation of a line.

$$2x + y - 12 = 0$$
 A1 [3 marks]

(c) every point in the cell is closer to E than any other snow shelter A1 [1 mark]

Total [6 marks]

8. (a)
$$10\log_{10}\left(6.4\times10^{-3}\times10^{12}\right)$$
 (M1)
= 98.1(dB) (98.06179...)
A1
[2 marks]

(b) $112 = 10 \log_{10} \left(S \times 10^{12} \right)$ (M1) $0.158 \left(W \, m^{-2} \right) \! \left(0.158489 \ldots \! \left(W \, m^{-2} \right) \right)$ [2 marks]

Total [4 marks]

9. (a) (i) $\mu_1 - \mu_2 = 0$

A1

(ii) $\mu_1 - \mu_2 \neq 0$

A1

Note: Accept equivalent statements in words.

[2 marks]

(b) 0.296 (0.295739...)

A2 [2 marks]

(c) 0.296 > 0.1

R1

fail to reject the null hypothesis, there is no difference between the mean height of male and female students

A1

Note: Award *(R1)* for a correct comparison of their *p*-value to the test level, award *(A1)* for the correct interpretation from that comparison. Do not award *R0A1*.

[2 marks]

Total [6 marks]

10. (a) $A = \int_0^2 (6-3x)(4+x)dx$

A1A1

Note: Award **A1** for the limits x = 0, x = 2. Award **A1** for an integral of f(x).

[2 marks]

(b) 28

A1

[1 mark]

(c) $28 = 0.5 \times a \times 10$

М1

$$5.6\left(\frac{28}{5}\right)$$

A1

[2 marks]

Total [5 marks]

11. volume =
$$240 \left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664... \right)$$

Note: Award *M1* 240×area, award *M1* for correctly substituting area sector formula, award *M1* for subtraction of their area of the sector from area of circle.

12. (a)
$$\frac{4}{18} \left(\frac{2}{9} \right)$$
 A1

(b)
$$-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \dots + 5 \times \frac{7}{18}$$

Note: Award *(M1)* for their correct substitution into the formula for expected value.

$$=1.83\left(\frac{33}{18}, 1.83333...\right)$$

[2 marks]

(c)
$$2 \times \frac{1}{18} \times \frac{3}{18}$$
 (M1)(M1)

Note: Award **(M1)** for $\frac{1}{18} \times \frac{3}{18}$, award **(M1)** for multiplying their product by 2.

$$=\frac{1}{54}\left(\frac{6}{324},0.0185185...,1.85\%\right)$$

[3 marks]

Total [6 marks]

13. (a)
$$\frac{6}{15} \left(0.4, \frac{2}{5} \right)$$

A1

[1 mark]

(b)
$$P(X = 8)$$

(M1)

Note: Award (M1) for evidence of recognizing binomial probability.

eg,
$$P(X = 8)$$
, $X \sim B\left(20, \frac{6}{15}\right)$.

A1

[2 marks]

(c)
$$P(\text{male}) = \frac{9}{15} (0.6)$$

A1

$$P(X \le 9) = 0.128 (0.127521...)$$

(M1)A1

Note: Award *(M1)* for evidence of correct approach *eg*, $P(X \le 9)$.

[3 marks]

Total [6 marks]

14. (a)
$$\frac{\sin C\hat{A}B}{6} = \frac{\sin 15^{\circ}}{4.5}$$
 (M1)(A1)

$$\hat{CAB} = 20.2^{\circ} (20.187415...)$$

Note: Award **(M1)** for substituted sine rule formula and award **(A1)** for correct substitutions.

[3 marks]

A1

(b)
$$C \hat{B} D = 20.2 + 15 = 35.2^{\circ}$$
 (let X be the point on BD where Ollie activates the sensor)

$$\tan 35.18741...^{\circ} = \frac{1.8}{BX}$$
 (M1)

Note: Award **A1** for their correct angle C \hat{B} D. Award **M1** for correctly substituted trigonometric formula.

$$BX = 2.55285...$$
 A1
 $5 - 2.55285...$ (M1)
 $= 2.45 \text{ (m) } (2.44714...)$ A1

[5 marks]

Total [8 marks]



Mathematics: applications and interpretation Standard level Paper 2

Specimen paper

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is [80 marks].



-2-

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of $\in 14000$, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of $14\,\%$ compounded quarterly. No deposit required and repayments are made each quarter.

- (a) (i) Find the repayment made each quarter.
 - (ii) Find the total amount paid for the car.
 - (iii) Find the interest paid on the loan.

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of ϵ 250.

- (b) (i) Find the amount to be borrowed for this option.
 - (ii) Find the annual interest rate, r.

[5]

(c) State which option Bryan should choose. Justify your answer.

[2]

[7]

Bryan's car depreciates at an annual rate of 25% per year.

(d) Find the value of Bryan's car six years after it is purchased.

[3]

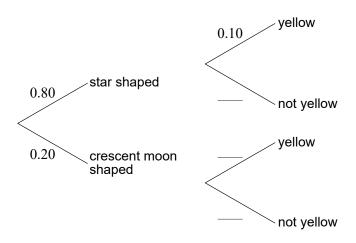
[2]

2. [Maximum mark: 14]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) Using the given information, **copy** and complete the following tree diagram.



- (b) A sweet is selected at random.
 - (i) Find the probability that the sweet is yellow.
 - (ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

(This question continues on the following page)

(Question 2 continued)

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(c) Write down the null hypothesis for this test.

[1]

(d) **Copy** and complete the following table in your answer booklet.

[2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(e) Write down the number of degrees of freedom.

[1]

(f) Find the p-value for the test.

[2]

(g) State the conclusion of the test. Give a reason for your answer.

[2]

3. [Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	В	С	D	Е	F	G	Н
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

- (a) (i) Write down the value of the Pearson's product–moment correlation coefficient, r.
 - (ii) Using the value of r, interpret the relationship between Stan's score and Minsun's score.

[4]

(b) Write down the equation of the regression line y on x.

[2]

- (c) (i) Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.
 - (ii) State whether this estimate is reliable. Justify your answer.

[4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

(d) **Copy** and complete the information in the following table.

[2]

Competitors	A	В	С	D	Е	F	G	Н
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

- (e) (i) Find the value of the Spearman's rank correlation coefficient, r_s .
 - (ii) Comment on the result obtained for r_s .

[4]

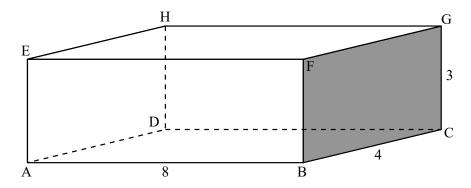
The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

(f) Explain why the value of the Spearman's rank correlation coefficient r_s does not change. [1]

4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\mathrm{cm}$, width $4\,\mathrm{cm}$ and height $3\,\mathrm{cm}$. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm².

[2]

(b) Calculate the length AG.

[2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{\mathrm{d}P}{\mathrm{d}x} = -2x + 220$, $x \ge 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit.

[3]

The profit from the sale of 20000 boxes is \$1700.

(d) Find P(x). [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \, {
m m \, s^{-1}}$, and braking distance, $d \, {
m m}$, of a truck were recorded. This information is summarized in the following table.

Speed, s m s ⁻¹	0	6	10
Braking distance, d m	0	12	60

This information was used to create Model A, where d is a function of s, $s \ge 0$.

Model A:
$$d(s) = ps^2 + qs$$
, where $p, q \in \mathbb{Z}$

At a speed of $6 \,\mathrm{m\,s}^{-1}$, Model A can be represented by the equation $6 \,p + q = 2$.

- (a) (i) Write down a second equation to represent Model A, when the speed is $10\,\mathrm{m\,s}^{-1}$.
 - (ii) Find the values of p and q

[4]

(b) Find the coordinates of the vertex of the graph of y = d(s).

[2]

[3]

- (c) Using the values in the table and your answer to part (b), sketch the graph of y = d(s) for $0 \le s \le 10$ and $-10 \le d \le 60$, clearly showing the vertex.
- (d) Hence, identify why Model A may not be appropriate at lower speeds.

[1]

Additional data was used to create Model B, a revised model for the braking distance of a truck.

Model B:
$$d(s) = 0.95 s^2 - 3.92 s$$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\,\mathrm{m\,s}^{-1}$. [2]

The actual braking distance at $20\,\mathrm{m\,s^{-1}}$ is $320\,\mathrm{m}$.

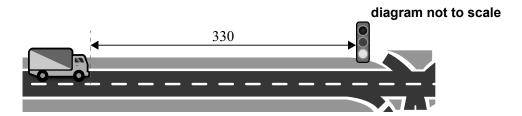
(f) Calculate the percentage error in the estimate in part (e). [2]

(This question continues on the following page)

(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s~{\rm m\,s}^{-1}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\,{\rm m}$.



(g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection.

[3]



Markscheme

Specimen paper

Mathematics: applications and interpretation

Standard level

Paper 2



Instructions to Examiners

Abbreviations

- M Marks awarded for attempting to use a correct Method.
- A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- **R** Marks awarded for clear **Reasoning**.
- **AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg M1, A2.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award MO followed by A1, as A mark(s) depend on the preceding M mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **M2**, **A3**, etc., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working.
 However, if further working indicates a lack of mathematical understanding do not award the final
 A1. An exception to this may be in numerical answers, where a correct exact value is followed by
 an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part,
 and correct FT working shown, award FT marks as appropriate but do not award the final A1 in
 that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 (incorrect decimal value)	Award the final A1 (ignore the further working)
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final A1
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final A1

3 Implied marks

Implied marks appear in **brackets e.g.** (M1), and can only be awarded if **correct** work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks without brackets can only be awarded for work that is seen.

4 Follow through marks (only applied after an error is made)

Follow through (FT) marks are awarded where an incorrect answer from one part of a question is used correctly in subsequent part(s) or subpart(s). Usually, to award FT marks, there must be working present and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then FT marks should be awarded if appropriate.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (e.g. probability greater than 1, use of r > 1 for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question

- If the question becomes much simpler because of the MR, then use discretion to award fewer marks.
- If the *MR* leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- The MR penalty can only be applied when work is seen. For calculator questions with no
 working and incorrect answers, examiners should not infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by METHOD 1, METHOD 2, etc.
- Alternative solutions for part-questions are indicated by EITHER . . . OR.

7 Alternative forms

Unless the question specifies otherwise, accept equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- Rounding errors: only applies to final answers not to intermediate steps.
- Level of accuracy: when this is not specified in the question the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

9 Calculators

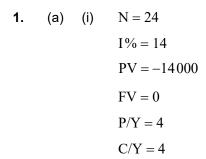
A GDC is required for this examination, but calculators with symbolic manipulation features/CAS functionality are not allowed.

Calculator notation

The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.



(M1)(A1)

Note: Award *M1* for an attempt to use a financial app in their technology, award *A1* for all entries correct. Accept $PV = 14\,000$.

(€)871.82 **A1**

(ii) $4 \times 6 \times 871.82$ (M1)

(€) 20923.68 **A1**

(iii) 20923.68–14000 **(M1)**

(€) 6923.68 **A1**

[7 marks]

(b) (i) $0.9 \times 14\,000 \ (= 14\,000 - 0.10 \times 14\,000)$

(€)12600.00 A1

(ii) N = 72

PV = 12600

PMT = -250

FV = 0

P/Y = 12

C/Y = 12 (M1)(A1)

Note: Award *M1* for an attempt to use a financial app in their technology, award *A1* for all entries correct. Accept PV = -12600 provided PMT = 250.

12.56(%) A1

[5 marks]

continued...

Question 1 continued

(c) **EITHER**

Bryan should choose Option A no deposit is required

A1

A1

R1

R1

Note: Award *R1* for stating that no deposit is required. Award *A1* for the correct choice from that fact. Do not award *R0A1*.

OR

Bryan should choose Option B cost of Option B $(72 \times 250 - 12600 = 5400)$

Note: Award *R1* for a correct comparison of costs. Award *A1* for the correct choice from that comparison. Do not award *R0A1*.

[2 marks]

(d)
$$14000 \left(1 - \frac{25}{100}\right)^6$$
 (M1)(A1)

Note: Award *M1* for substitution into compound interest formula. Award *A1* for correct substitutions.

$$= 2491.70 \text{ (USD)}$$

OR

N = 6

1% = -25

 $PV = \pm 14000$

P/Y = 1

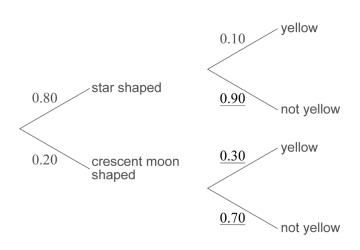
$$C/Y = 1$$
 (A1)(M1)

Note: Award **A1** for $PV = \pm 14000$, **M1** for other entries correct.

[3 marks]

Total [17 marks]

2. (a)



A1A1

Note: Award *A1* for each correct pair of branches. Accept decimal or percentage responses as equivalent forms on branches.

[2 marks]

(b) (i)
$$P(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$$
 M1 = 0.14

(ii)
$$P(\text{Star} \mid Y) = \frac{0.8 \times 0.1}{0.14}$$
 M1
= $0.571 \left(\frac{4}{7}, 0.571428... \right)$

[4 marks]

(c) the colours of the sweets are distributed according to manufacturer specifications

A1 [1 mark]

(d)

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency	12	20	16	16	8	8
						A

Note: Award **A2** for all 6 correct expected values, **A1** for 4 or 5 correct values, **A0** otherwise.

[2 marks]

continued...

Question 2 continued

(g) since 0.469 > 0.05

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications

A1

Note: Award *R1* for a correct comparison of their correct *p*-value to the test level, award *A1* for the correct result from that comparison. Do not award *R0A1*.

[2 marks]

Total [14 marks]

3. (a) (i) 0.909 (0.909181...)

A2

(ii) (very) strong and positive

A1A1

Note: Award A1 for (very) strong A1 for positive.

[4 marks]

(b) y = 1.14x + 0.578 (y = 1.14033...x + 0.578183...)

A1A1

Note: Award **A1** for 1.14x, **A1** for 0.578. Award a maximum of **A1A0** if the answer is not an equation in the form y = mx + c.

[2 marks]

(c) (i) $1.14 \times 10 + 0.578$

М1

12.0 (11.9814...)

A1

(ii) no the estimate is not reliable

A1

outside the known data range

R1

OR

a score greater than 10 is not possible

R1

Note: Do not award A1R0.

[4 marks]

(d)

Competitors	A	В	С	D	Е	F	G	Н
Stan's rank	7	8	6	4	2	4	1	4
Minsun's rank	7	8	6	4.5	3	2	1	4.5

A1A1

Note: Award A1 for correct ranks for Stan. Award A1 for correct ranks for Minsun.

[2 marks]

(e) (i) 0.933 (0.932673...)

A2

(ii) Stan and Minsun strongly agree on the ranking of competitors.

A1A1

A1

Note: Award A1 for "strongly agree", A1 for reference to a rank order.

[4 marks]

(f) decreasing the score to 9.1, does not change the rank of competitor G

[1 mark]

Total [17 marks]

4. (a) $2(8\times4+3\times4+3\times8)$ = 136 (cm²)

A1

[2 marks]

(b)
$$\sqrt{8^2 + 4^2 + 3^2}$$

M1

A1

M1

$$(AG =) 9.43 (cm) (9.4339..., \sqrt{89})$$

[2 marks]

(c)
$$-2x + 220 = 0$$

 $x = 110$

M1 A1

-10-

[3 marks]

(d)
$$P(x) = \int -2x + 220 \, dx$$

M1

Note: Award M1 for evidence of integration.

$$P(x) = -x^2 + 220x + c$$

A1A1

Note: Award **A1** for either $-x^2$ or 220x award **A1** for both correct terms and constant of integration.

$$1700 = -(20)^2 + 220(20) + c$$

М1

$$c = -2300$$

$$P(x) = -x^2 + 220x - 2300$$

A1 [5 marks]

(e)
$$-x^2 + 220x - 2300 = 0$$

M1

$$x = 11.005$$

A1

A1

Note: Award *M1* for their P(x) = 0, award *A1* for their correct solution to x. Award the final *A1* for expressing their solution to the minimum number of boxes. Do not accept $11\,005$, the nearest integer, nor $11\,000$, the answer expressed to 3 significant figures, as these will not satisfy the demand of the question.

[3 marks]

Total [15 marks]

$$10p + q = 6 (100p + 10q = 60)$$

A1

M1

(ii)
$$p = 1, q = -4$$

(a) (i) $p(10)^2 + q(10) = 60$

A1A1

Note: If p and q are both incorrect then award **M1A0** for an attempt to solve simultaneous equations.

[4 marks]

(b)
$$(2, -4)$$

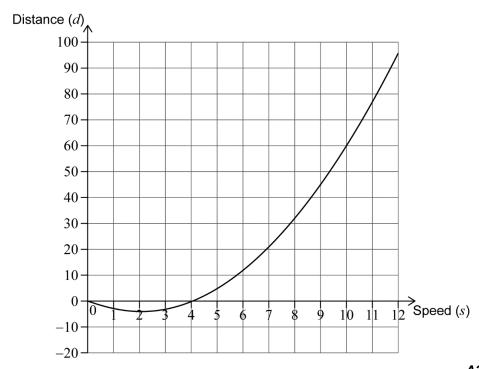
A1A1

Note: Award *A1* for each correct coordinate. Award *A0A1* if parentheses are missing.

[2 marks]

(c)

5.



– 11 –

A3

Note: Award **A1** for smooth quadratic curve on labelled axes and within correct window. Award **A1** for the curve passing through (0,0) and (10,60). Award **A1** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(d) the graph indicates there are negative stopping distances (for low speeds)

R1

Note: Award *R1* for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances...).

[1 mark]

continued...

Question 5 continued

(e) $0.95 \times 20^2 - 3.92 \times 20$ (M1) = 302 (m) (301.6...)

[2 marks]

(f) $\left| \frac{301.6 - 320}{320} \right| \times 100$ M1 = 5.75(%)

[2 marks]

(g) $330 = 1.6 \times s + 0.95 \times s^2 - 3.92 \times s$ **M1A1**

Note: Award *M1* for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330. Award *A1* for a completely correct equation.

 $19.9 (m s^{-1}) (19.8988...)$ A1 [3 marks]

Total [17 marks]