WORKSHEET: Newton's Law of Cooling

Newton's Law of Cooling models how an object cools. In words, the rate of change of temperature of a cooling body is proportional to the difference between the temperature of the body and the ambient temperature. We can express this as a differential equation:

$$\frac{dT}{dt} = -k(T - T_a)$$

where T_a is the ambient temperature.

By generating a slope field for this differential equation, we saw that a solution to the differential equation is a vertical shift of the exponential decay equation:

$$T(t) = Ae^{-kt} + T_a \qquad \text{for} \quad k > 0.$$

Here the proportionality constant k will be specific to each problem, depending on the object which is cooling. The coefficient A will be the initial difference in temperatures. Indeed, we have that $T(0) = A + T_a$, so that $A = T(0) - T_a$.

1. A cup of coffee is made with boiling water at a temperature of $100°C$ in a room with ambient temperature $20°C$. After 4 minutes the coffee has cooled to $90°C$.

1a. What is the equation $T(t)$ for the temperature of the coffee?

1b. What is the temperature of the coffee after 8 minutes?

1c. Does the coffee cool more in the first 4 minutes or the second 4 minutes? Why does this make sense in terms of the differential equation?

1d. What is the rate of change of the temperature when $T = 80°C$?

2. A detective is called to the scene of a crime where a dead body has been found. She arrives at 10pm and immediately records the temperature of the body to be $80°F$. One hour into her investigation she measures the temperature of the body to be $76°F$. She notes that the thermostat is programmed at a constant $68°F$. Assuming that the victim's body temperature was normal ($98.6°F$) prior to death, when did the death occur?