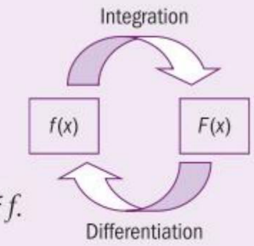
- If $f(t) \geq 0$ is a continuous function, the area enclosed between the graph of f and the x-axis over the interval $a \leq t \leq x$ can be found with the definite integral $\int_a^x f(t)\,dt$. Also $\int_a^x f(t)\,dt = F(x) - F(a)$ where $F'(x) = f(x)$.

  If $F(x)$ is a function where $F'(x) = f(x)$, we say that $F(x)$ is an **antiderivative** of f. The process of finding an antiderivative is called **antidifferentiation.**



- If $F'(x) = f(x)$ then $\int f(x)\,dx = F(x) + c$ where $c \in \mathbb{R}$.

  The expression $\int f(x)\,dx$ is called an **indefinite integral** and $\int f(x)\,dx$ is read as "the integral of f with respect to x".

  HINT
  c is called the **constant of integration**.

- $\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c$ where a and n are constants, $a \neq 0$ and $n \neq -1$.

  This is an integration rule and is called the **power rule.**

Developing inquiry skills

Write down any further inquiry questions you could ask and investigate how you could find the areas of irregular shapes and curved shapes.

Chapter review

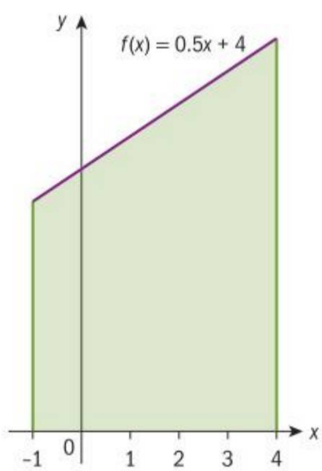
Click here for a mixed review exercise

1 For each of the following shaded regions:

i Write down a definite integral that represents the area of the region.

ii Hence or otherwise, find the area of these shaded regions.

a



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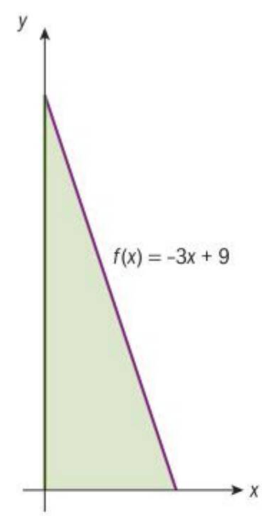
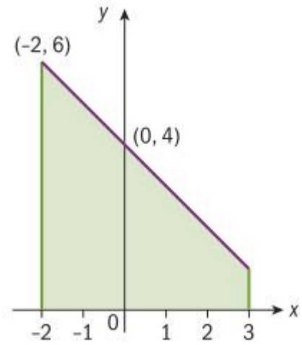


2 For each of the following regions:

i Write down a definite integral that represents the area of the region.

ii Hence or otherwise, find the area of these shaded regions.

a



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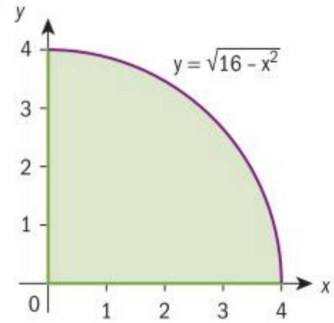
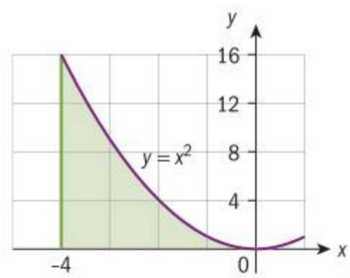


3 For each of the following regions:

i Write down a definite integral that represents the area of the region.

ii Find the area of the region.

a



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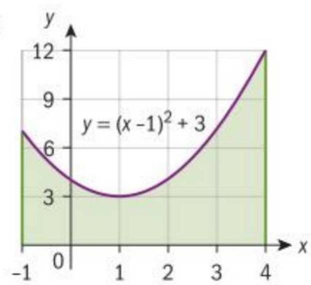

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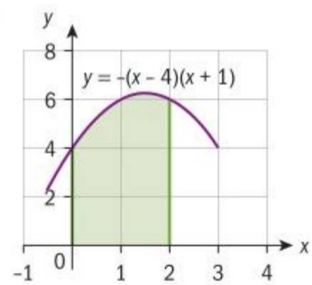

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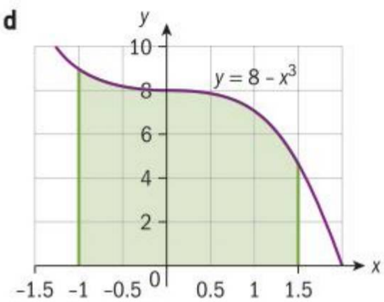

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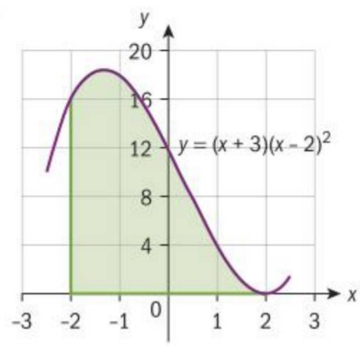

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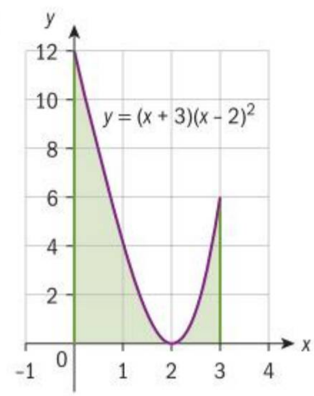

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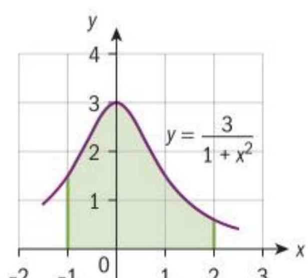

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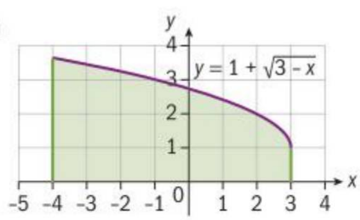
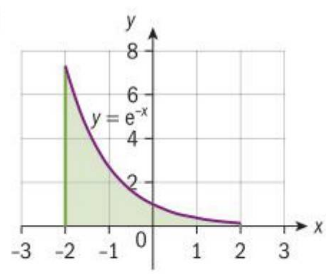


i



4 For each of the following definite integrals:

i On a diagram shade the region that they represent.

ii Find their value.

a $\int_0^4 \sqrt{x}\,dx$ b $\int_{-2}^2 x^2 dx$

c $\int_{2.5}^3 -(x-2)(x-4)\,dx$

d $\int_2^4 -(x-2)(x-4)\,dx$

e $\int_2^5 \frac{10}{x+1}\,dx$ f $\int_{-1}^1 \left(3^x+2\right)dx$

g $\int_{-2}^3 (x^2-2x+3)\,dx$

5 Consider the region A enclosed between the graph of $y=-(x+1)(x-4)$ and the x-axis.

a Write down a definite integral that represents the area of A.

b Find the value of this area.

6 Consider the curve $y = x(x-4)^2$. Let A be the region enclosed between this curve and the x-axis.

a Write down the x-intercepts of this curve.

b Write down a definite integral that represents the area of A.

c Find the value of this area.

7 Consider the curve $y = x^3$. Let A be the region enclosed between this curve, the x-axis and the vertical line $x = 2$.

a Write down the x-intercept of this curve.

b Sketch the curve and clearly label A.

c Write down a definite integral that represents the area of A.

d Find the value of this area.

8 Consider the graph of the function $f(x) = (x+2)^2 + 1$.
The region bounded by the graph of f, the x-axis, the y-axis and the vertical line $x = b$ with $b > 0$ has an area equal to 42.

a Sketch the region.

b Find the value of b.

9 The table below shows the coordinates (x, y) of five points that lie on a curve $y = f(x)$.

x	1	3.25	5.5	7.75	10
$y=f(x)$	3	9	7	12	5

Estimate the area under the curve over the interval $1 \le x \le 10$.

10 Estimate the area under the graph of $f(x) = \sqrt{x-2}$ over the interval $2 \le x \le 4$ using five trapezoids. Give your answer correct to four significant figures.

11 Consider the graph of the function $f(t) = -t^2 + 2t$ where $f(t) \ge 0$.

a Draw the graph of the function f and shade the area enclosed between this graph and the t-axis over the interval $0 \le t \le x$.

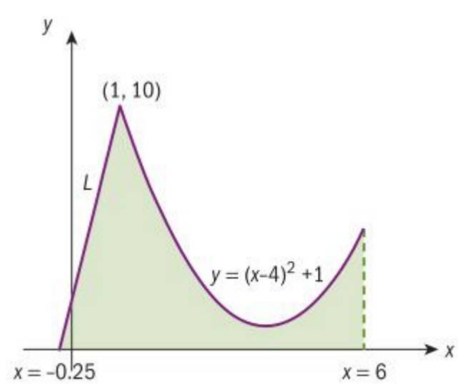
b Find an expression for the area under the graph of f over the interval $0 \le t \le x$.

12 Consider the graph of the function $f(t) = 4t$ over the interval $3 \le t \le x$. Find $\int_3^x 4t\,dt$.

13 Calculate $\int (2+x)\,dx$.

14 Find $\int (1 + x - \frac{x^3}{4})\,dx$.

15 Line L passes through the points $(-0.25, 0)$ and $(1, 10)$. Consider the region bounded by the graph of line L for $-0.25 \le x \le 1$, the curve $y = (x-4)^2 + 1$ for $1 \le x \le 6$ and the x-axis. The region is shown below.

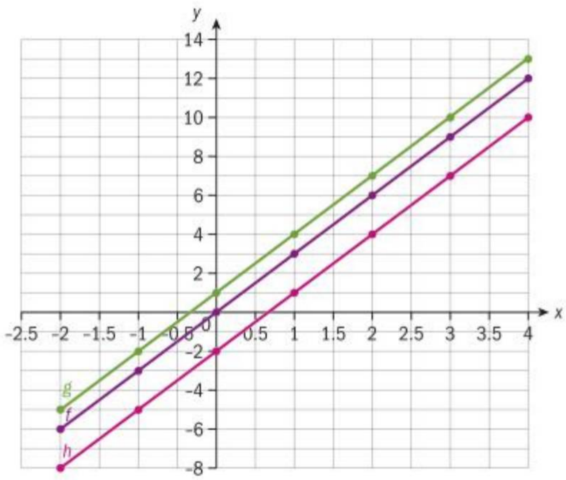


a Find the area under the graph of L for $-0.25 \le x \le 1$.

b Write down an expression for the area under the curve $y = (x - 4)^2 + 1$ for $1 \le x \le 6$.

c Hence, find the area of the shaded region.

16 The diagram shows the graph of three linear functions, g, f and h.



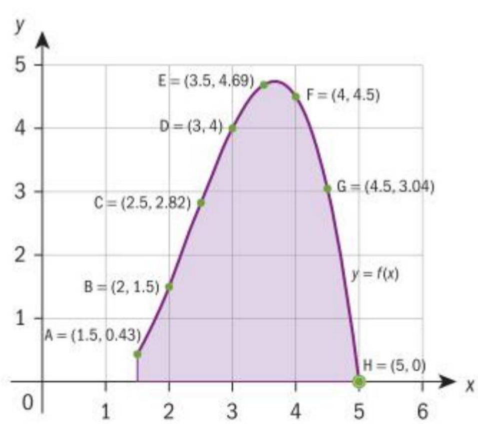
a Find the equation of each of these functions.

The three functions are antiderivatives of $y = t(x)$.

b Find the equation of $y = t(x)$.

c Find $\int t(x)\,dx$.

17 Estimate the area under the graph of $y = f(x)$ in the interval $1.5 \le x \le 5$ using the data points given in the diagram.



18 a Estimate the area A under the graph of $f(x) = e^{-x^2}$ over the interval $0 \le x \le 1$ using five trapezoids. Give your answer correct to four significant figures.

b i Write down a definite integral that represents A.

ii Hence, find the actual area. Give your answer correct to four significant figures.

c Find the percentage error made with the estimation found in part **a**.

Exam-style questions

19 P1: Find $\int (5 - 12x^2 + 4x^3)\,dx$, simplifying your answer as far as possible. (4 marks)

20 P1: The derivative of the function f is given by $f'(x) = \frac{3}{2}x^2 + x + 3$ and the curve $y = f(x)$ passes through the point $\left(-1, \frac{13}{2}\right)$.

Find an expression for f. (6 marks)

21 P2: a Find the coordinates of the points of intersection of the graphs of $y = 6x - x^2$ and $y = 10 - x$. (4 marks)

b On the same axes, sketch the graphs of $y = 6x - x^2$ and $y = 10 - x$. (2 marks)

c Find the exact value for the area bounded by the two curves. (7 marks)

22 P1: A particle P is travelling in a straight line. After t seconds, the particle has velocity $v = 0.6t^2 + 4t + 1$ m/s for $t \ge 0$.

a Find an expression for the displacement of the particle from the origin after t seconds. (2 marks)

b Hence, find the distance travelled by the particle during the third second of motion. (3 marks)

23 P1: Consider the curve $y = -\frac{x^2}{10}(x-10)$.

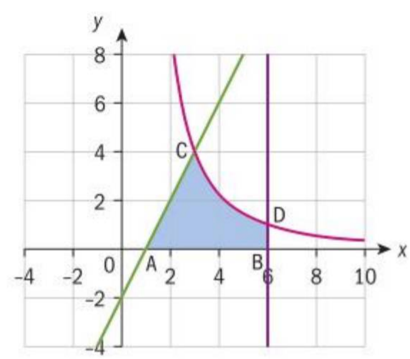
The area A is defined as the region bounded by the curve and the x-axis.

- **a** Sketch the curve, clearly showing the area defined as A. (3 marks)
- **b** Write down a definite integral that represents A. (1 mark)
- **c** Find the exact value of A. (5 marks)

24 P2: a By using technology, find the coordinates of the points of intersection of the graphs of $y = x^3$ and $y = \sqrt[3]{x}$. (4 marks)

- **b** On the same axes, sketch the graphs of $y = x^3$ and $y = \sqrt[3]{x}$. (2 marks)
- **c** Find an exact value for the total area bounded by the two curves. (6 marks)

25 P1: The diagram below shows an area bounded by the x-axis, the line $x = 6$, the line $y = 2x - 2$ and the curve $y = \frac{36}{x^2}$.



- **a** Using technology or otherwise, find the coordinates of points A, B, C and D. (4 marks)
- **b** Show that the shaded area is exactly 10 units^2. You must show all of your working. (6 marks)

26 P2: Consider the area enclosed by the curve $y = 5 - \frac{x^3}{25}$ and the positive x- and y-axis.

- **a** Sketch the curve, shading the area described above. (3 marks)
- **b** Using the trapezium rule with five strips, determine an approximation for the shaded area. (5 marks)
- **c** Explain why your answer to part **b** will be an underestimate. (2 marks)
- **d** Using integration, determine the exact value of the shaded area. (4 marks)
- **e** Find the percentage error of your approximation, compared with the exact value. (2 marks)

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