Matching Differential Equations

Match each differential equation in the left column with the correct separation of variables in the right column. Several of the differential equations cannot be solved by separation so answer choice B can be used more than once.

$$\underline{\qquad} \quad \mathbf{1.} \quad \frac{dy}{dx} = x - 2y + 12$$

2.
$$\frac{dy}{dx} = e^{y-x}$$

$$\underline{\qquad} \quad \mathbf{3.} \quad \frac{dy}{dx} = x^2 + 12xy$$

$$4. \frac{dy}{dx} = xy + 12x$$

5.
$$\frac{dy}{dx} = -2y + 12$$

$$\mathbf{6.} \quad \frac{dy}{dx} = \frac{x+12}{2y}$$

$$8. \ \frac{dy}{dx} = x - 12y$$

$$\mathbf{9.} \quad \frac{dy}{dx} = \frac{x}{12y}$$

$$\mathbf{A.} \quad \frac{1}{v+12} \ dy = x \ dx$$

B. non-separable

C.
$$\frac{1}{y-6} dy = -2 dx$$

$$\mathbf{D.} \ e^{-y} \ dy = e^{-x} \ dx$$

E.
$$12y \, dy = x \, dx$$

F.
$$2y dy = (x + 12) dx$$

$$\mathbf{G.} \ e^{-y} \ dy = e^x \ dx$$

AP CALCULUS STUDENT HANDOUT

Procedures for Solving Differential Equations

Instructions:

- 1. First, sort the cards into four stacks according to the shape in the upper left hand corner.
- 2. For each stack, put the cards in the correct order for solving a differential equation.
- 3. Watch out! In each set of cards, there is one card that shows a false step, based on a common mistake that students might make in solving that differential equation. Set that card aside, insert the card showing the correct step instead, and then describe the mistake or misunderstanding that leads to that false step.

Symbol	Card Sequence	False step card & mistake
		Card
		Mistake:
•		
		Card
		Mistake:
		Card
		Mistake:
4		Card
_		Mistake:
		Card
		Mistake:



© 2017 College Board Page 2

Differential Equations Error Analysis

In each differential equation solution below, there is a major mistake! Unfortunately, that means that every step *after* the mistake occurred is *also* wrong. Your task is to identify the precise step in which the mistake occurs. Correct the error and then correct all of the steps that follow the error so that the problem is worked correctly.

Step 1
$$\int -y^{-2} dy = \int \frac{x}{2} dx$$

Step 2
$$y^{-1} = \frac{x^2}{4} + c_1$$

Step 3
$$y = \frac{4}{x^2} + c_2$$

Corrected Work:

Problem 2	$\frac{dy}{dx} = e^{x-y}$
Step 1	$\int e^{-y} dy = \int e^x dx$
Step 2	$-e^{-y} = e^x + c_1$
Step 3	$e^{-y} = -e^x + c_2$
Step 4	$\ln e^{-y} = \ln(-e^x + c_2)$
Step 5	$-y = \ln(-e^x + c_2)$
Step 6	$y = -\ln(-e^x + c_2)$

Corrected Work:

$$\frac{dy}{dx} = \frac{xy}{2}$$

$$\int \frac{1}{y} \, dy = \int \frac{x}{2} \, dx$$

$$ln|y| = \frac{x^2}{4}$$

$$e^{ln|y|} = e^{\frac{x^2}{4}}$$

$$|y| = e^{\frac{x^2}{4}}$$

$$y = \pm e^{\frac{x^2}{4}} + c_1$$

Corrected Work:

$$\frac{dy}{dx} = \frac{-2x}{y}$$

$$\int y \ dy = \int -2x \ dx$$

$$\frac{y^2}{2} = -x^2 + c_1$$

$$y^2 = -2x^2 + c_2$$

$$y = \sqrt{-2x^2 + c_2}$$

Corrected Work:



Problem 5 $\frac{dy}{dt} = -2v - 32$

Step 1
$$\int 2v \ dy = \int -32 \ dt$$

Step 2
$$v^2 = -32t + c_2$$

Step 3
$$v = \pm \sqrt{-32t + c_2}$$

Corrected Work:

