

Name: _____

ACADEMIC MATH 2

UNIT 2: TRANSFORMATIONS

- Use prime notation to distinguish an image from its pre-image. (G-CO.2)
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (G-CO.4)
- Verify experimentally the properties of transformations. (8-G.1, G-SRT.1)
- Compare transformations that preserve distance and angle (rigid motions) to those that do not (e.g. dilation or horizontal stretch). (G-CO.2)

Date	Lesson	Activity
Wednesday, February 4	Translations	
Thursday, February 5	Reflections	
Friday, February 6	Rotations	
Monday, February 9	Dilations	QUIZ
Tuesday, February 10	Compositions of Transformations	Transformation of Functions Lab
Wednesday, February 11	Review	
Thursday, February 12	Unit 2 Test	TEST

Transformation Rules Sheet

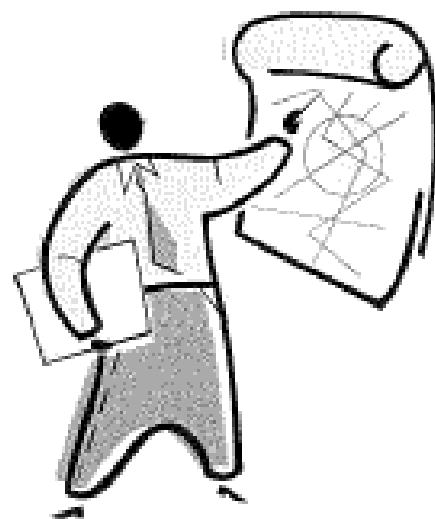
Line Reflections:

$$r_{x\text{-axis}}(x, y) = (x, -y)$$

$$r_{y\text{-axis}}(x, y) = (-x, y)$$

$$r_{y=x}(x, y) = (y, x)$$

$$r_{y=-x}(x, y) = (-y, -x)$$



Point Reflection:

$$R_{180^\circ}(x, y) = (-x, -y)$$

Rotations:

$$R_{90^\circ}(x, y) = (-y, x)$$

$$R_{180^\circ}(x, y) = (-x, -y)$$

$$R_{270^\circ}(x, y) = (y, -x)$$

$$R_{-90^\circ}(x, y) = (y, -x)$$

Translation:

$$T_{a,b}(x, y) = (x + a, y + b)$$

Dilation:

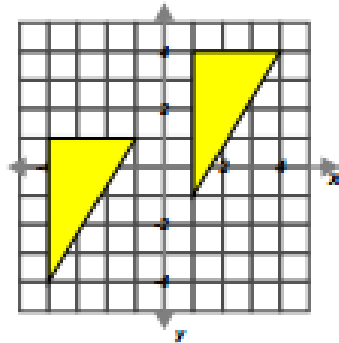
$$D_k(x, y) = (kx, ky)$$

Transformation Terms

Transformation - Transformation means to change. Specific types of transformations or changes that can be made to objects on the coordinate plane include, translations, rotations, reflections and dilations.

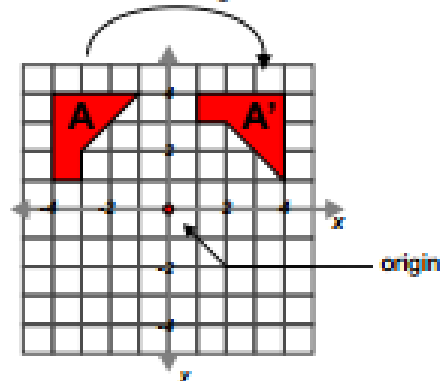
Translation - An object that is *translated* on the coordinate plane is commonly referred to as *sliding* an object on the coordinate plane. An object that is translated on the coordinate plane must not be turned or flipped. The object will maintain the same orientation after it is moved. For example, if an arrow pointing upwards is translated, it will still be pointing upwards after it is translated. All translated objects will remain congruent.

Example Of A Translation



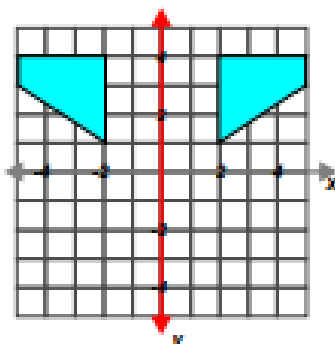
Rotation - A *rotation* is the same thing as a *turn*. How the object is turned on the coordinate plane will depend on what is identified as the center of rotation. Sometimes an object is rotated around the origin of a coordinate plane. This means that the center of rotation is located at (0,0). All rotated objects will remain congruent.

90° Clockwise Rotation Around The Origin

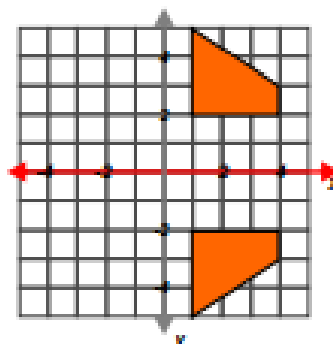


Reflection - *Reflecting* an object is the same thing as *flipping* an object. Where the reflected object will end up depends on where the line of reflection is located. Sometimes the line of reflection is defined as the x or y-axis. If the line of reflection is the x or y-axis there are some specific rules that may be followed. When an object is reflected across the x-axis, keep the x-values the same for any ordered pairs and make the y-values opposite of what they are. For a reflection across the y-axis, keep the y-value the same and make the x-value of the ordered pair opposite. A way to determine where the reflected object will end up is by simply folding the coordinate plane along the line of reflection. Wherever the object touches after the coordinate plane is folded is where it will end up. This will work even if the line of reflection is not the x or y-axis. All reflected objects will remain congruent.

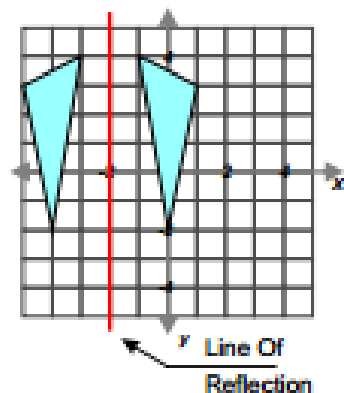
Reflection Across The Y-Axis



Reflection Across The X-Axis



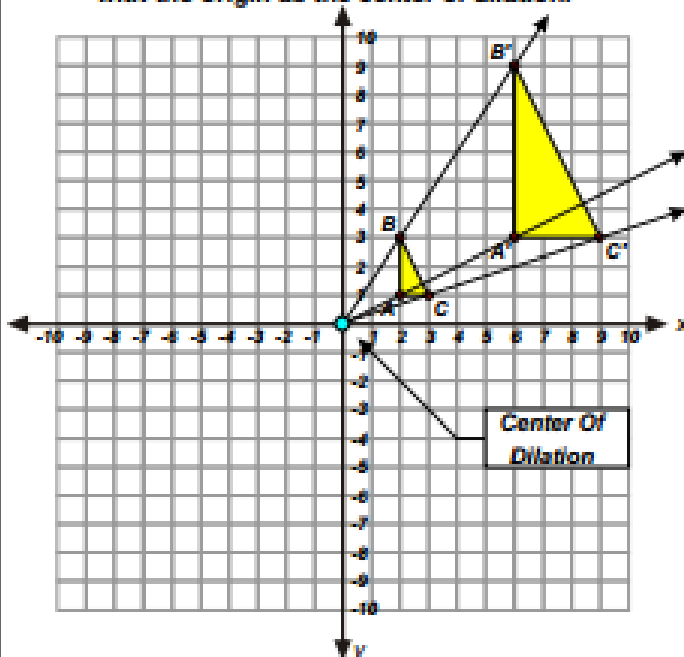
Reflection Across $x = -2$



Dilations When The Origin Is The Center Of Dilation

Dilation - A dilation will transform a polygon by making it larger or smaller by a scale factor that is $\neq 0$. A dilation greater than 1 means the image will be enlarged. A dilation less than one is an image that is reduced. Any dilated polygon will remain similar to the original. For example, if the length of one side of a polygon was half of what it was, all other sides must also be half of the original length. Also remember that all corresponding angles of similar objects will remain congruent.

$\triangle A'B'C'$ is a dilation of $\triangle ABC$ by a scale factor of 3 with the origin as the center of dilation.



Checking For Accuracy

By connecting all corresponding points with a straight line you will notice that all lines will converge at a single point. This point is called the center of dilation. In the example given notice that all three lines converge at the origin.

If it is given in any problem that the center of dilation is the origin, you may check the accuracy of your dilation by connecting corresponding points (A to A', B to B', etc.) with a straight line going to the origin. All lines should intersect at the origin. If they do not your dilation must be incorrect.

When the center of dilation is the origin you may compare corresponding points by the values of their ordered pairs. All corresponding ordered pairs should have changed by the same scale factor.

original location		dilated location
A (2,1)	x3	A'(6,3)
B (2,3)	x3	B'(6,9)
C (3,1)	x3	C'(9,3)

← scale factor

Notice that all original values became three times larger. This should be true because dilated figures are similar to each other. If one number triples, they should all triple. If one number doubles, all of them should double and so on.

Although all dilated shapes including those when the origin is NOT the center of dilation increase or decrease in size by a scale factor, it is only possible to compare scale factor using ordered pairs when the center of dilation IS located at the origin.

Dilations

When The Center Of Dilation Is A Vertex Of The Original Figure

$\triangle A'B'C'$ is a dilation of $\triangle ABC$ by a scale factor of 3 with point A(-7,-7) as the center of dilation.

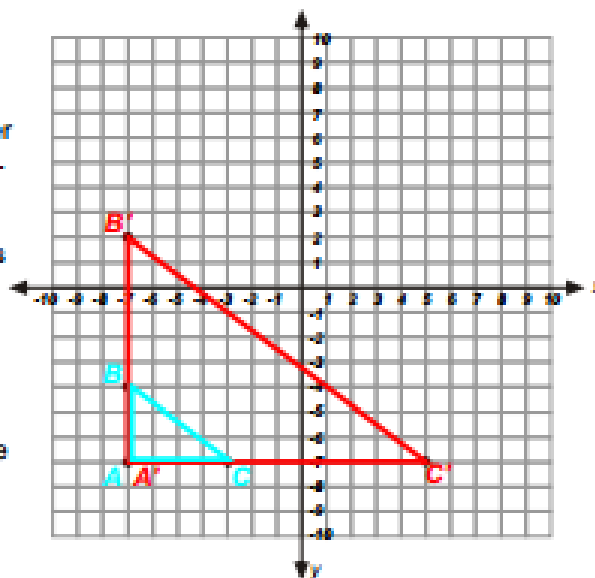
A (-7,7) A (-7,7)
 B (-7,-4) B (-7,2)
 C (-3,-7) C (5,-7)

You can not determine what the factor of dilation is by comparing the coordinates when the center of dilation is NOT the origin.

You must compare the length of corresponding line segments to determine the scale factor.

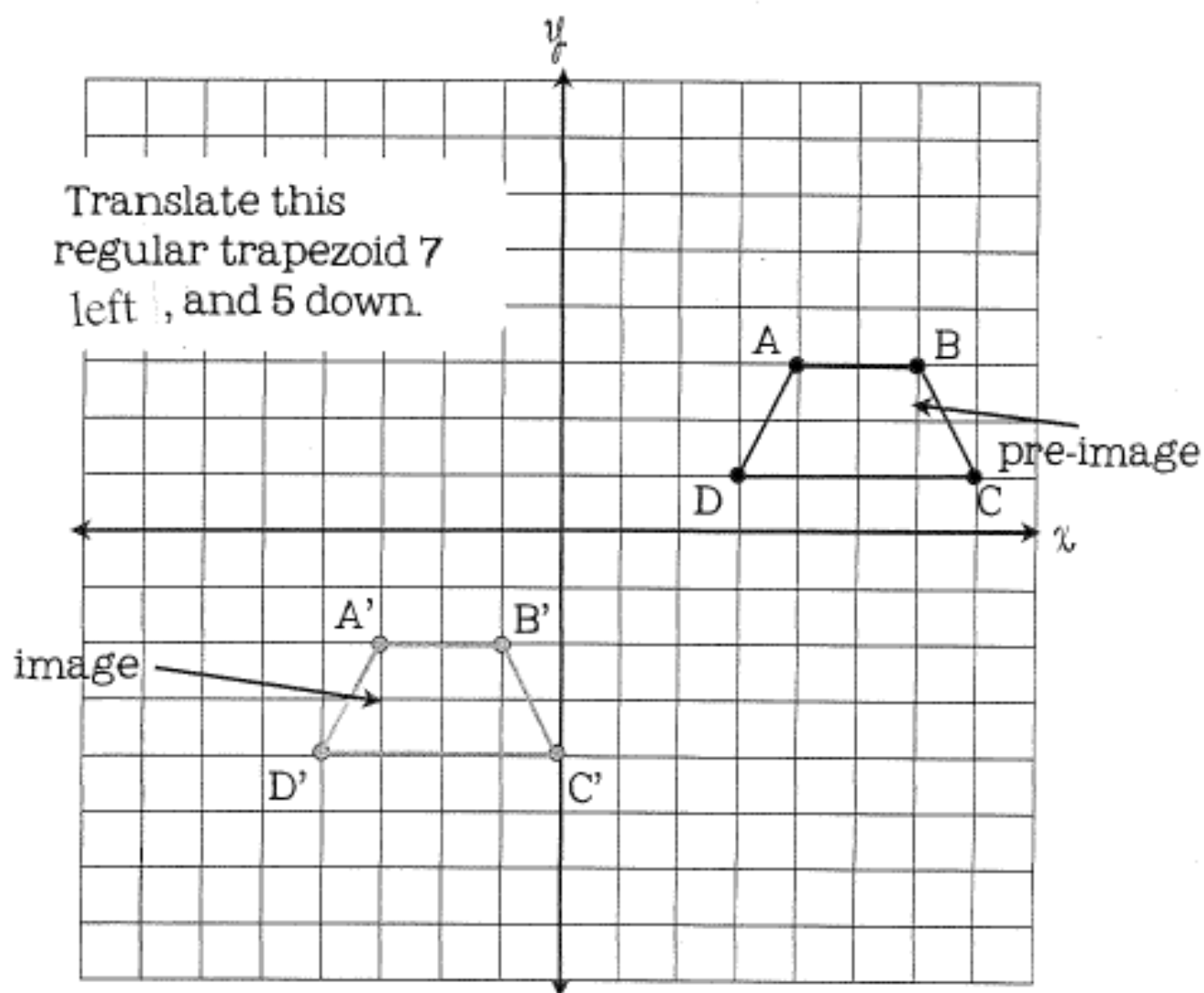
\overline{AC} = 4 units and $\overline{A'C'}$ = 12 units
 \overline{AB} = 3 units and $\overline{A'B'}$ = 9 units
 \overline{BC} = 5 units and $\overline{B'C'}$ = 15 units

When comparing corresponding sides we can see that the length of each line segment became three times larger than what it was, therefore $\triangle ABC$ was dilated by a scale factor of 3.

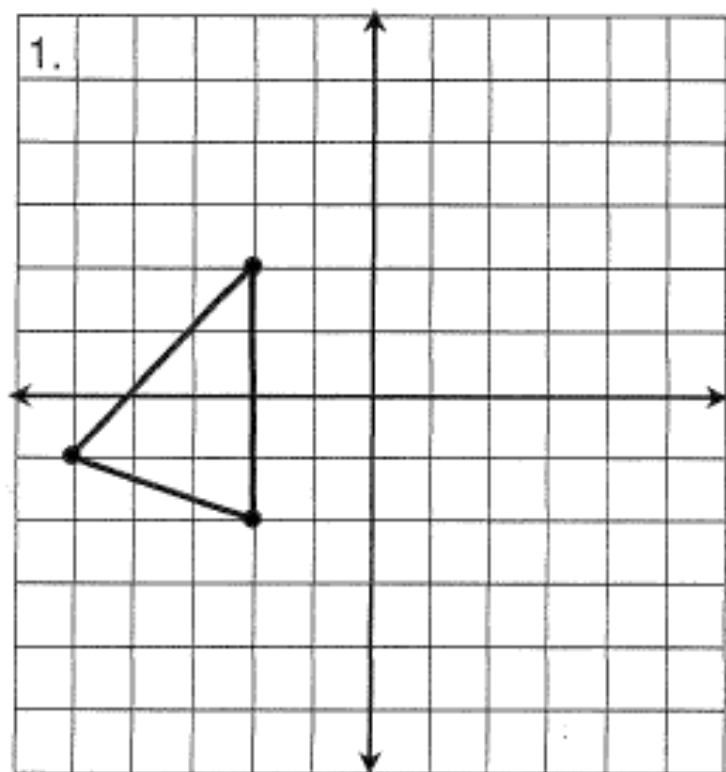


Translation

A translation moves a pre-image point by point to slide the figure into a new location. The shape at the new location is called the image, and is noted with the ' symbol called prime. An original triangle $\triangle ABC$ would be called $\triangle A'B'C'$ after the movement happens. The prime symbol helps to avoid confusion as to which is the original figure and which is the new one.

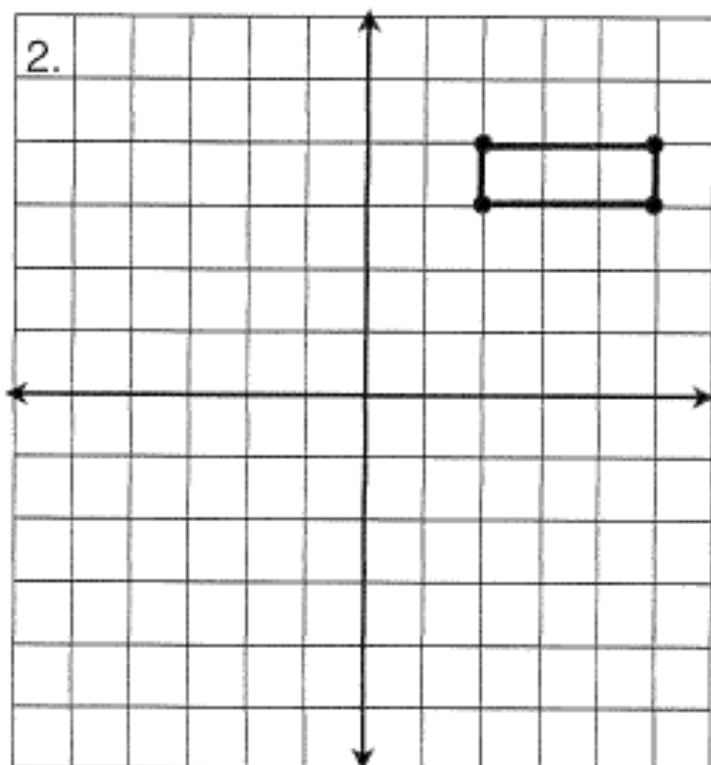


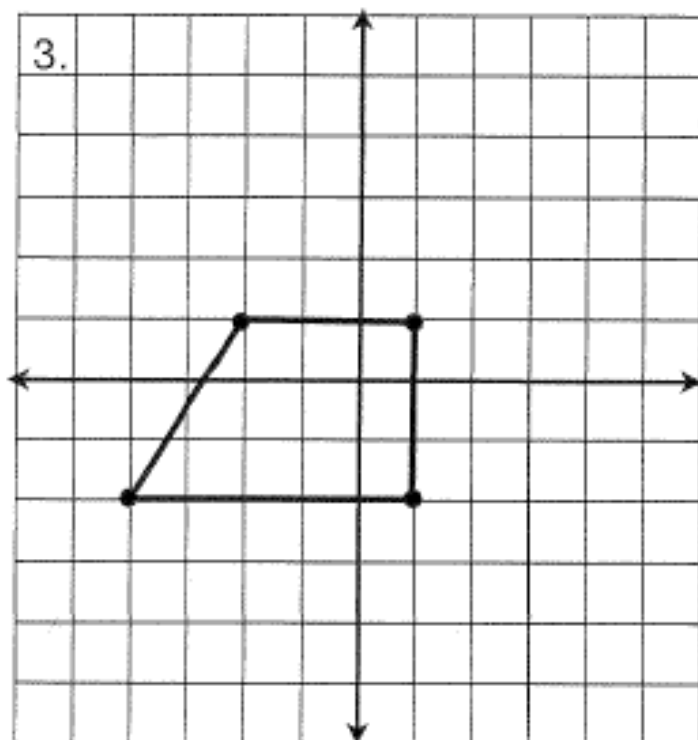
Translation Practice



1. Label the x and y axes.
2. Label each of the points in the pre-image with a letter. (Remember to move around the shape in alphabetical order.)
3. Translate the shape right 6, down 2.
4. Mark the image with the same letters, but use the prime symbol.

1. Label the x and y axes.
2. Label each of the points in the pre-image with a letter. (Remember to move around the shape in alphabetical order.)
3. Translate the shape left 4, down 6.
4. Mark the image with the same letters, but use the prime symbol.





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

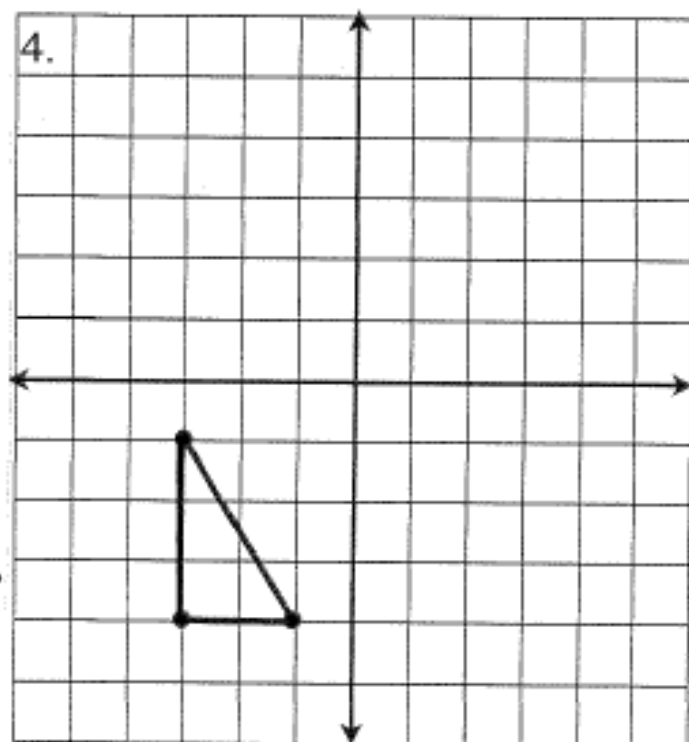
3. Translate the shape right 2, up 2. Write the rule associated with the transformation.

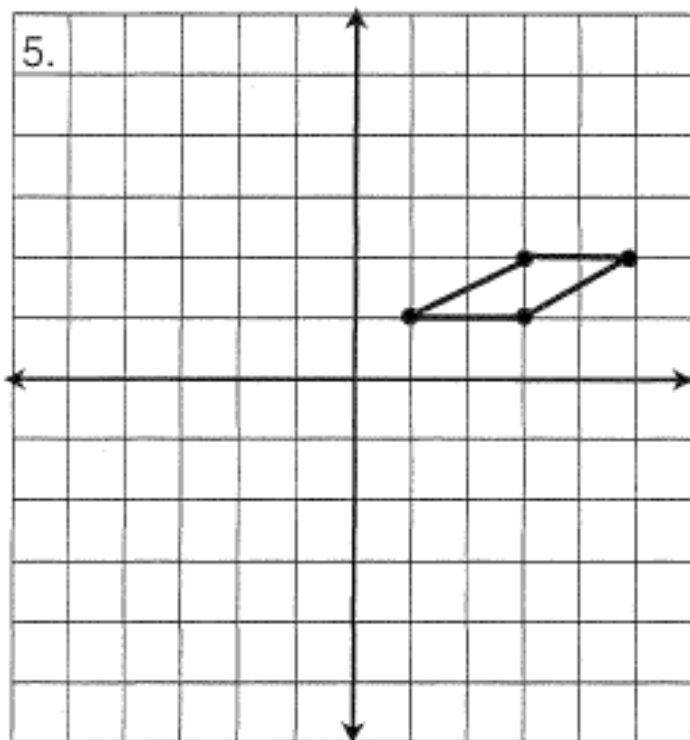
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Translate the shape right 0, up 5. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

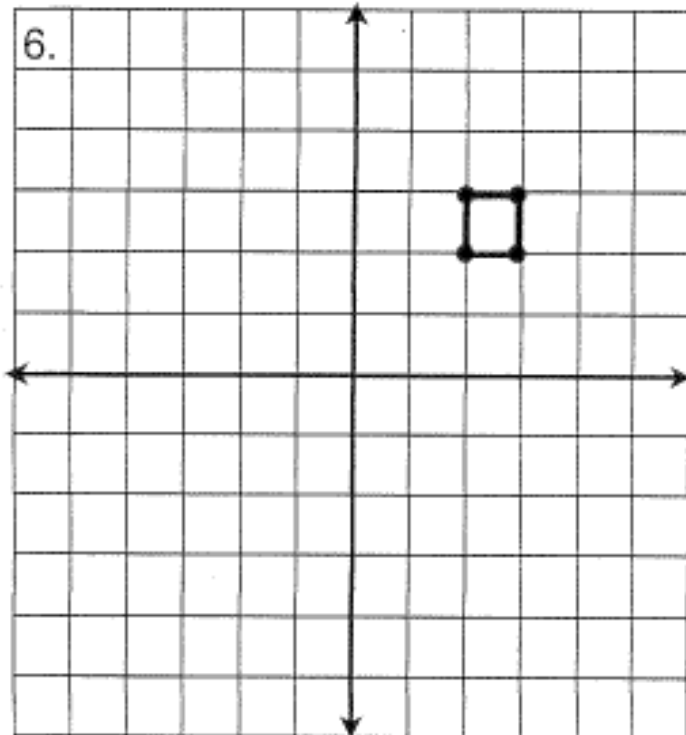
3. Translate the shape left 3, down 0. Write the rule associated with the transformation.

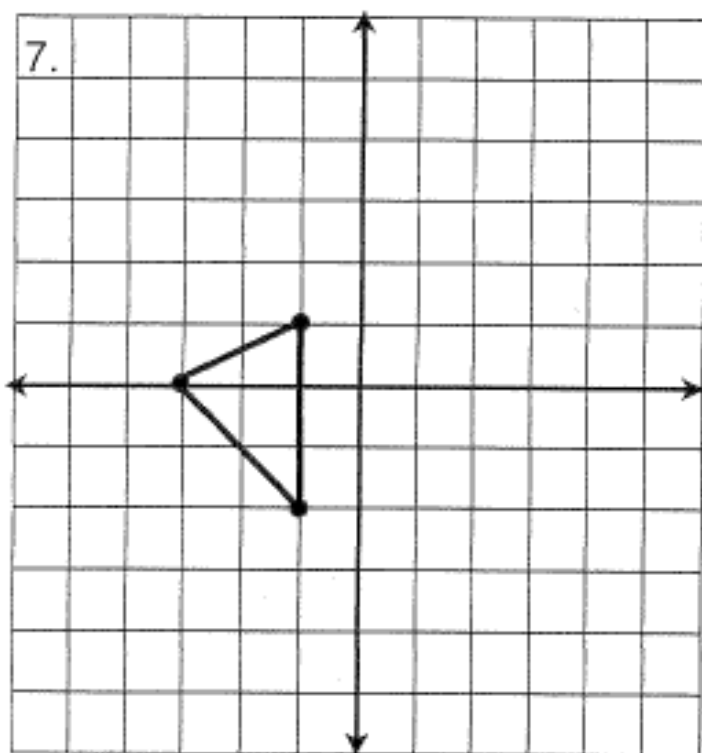
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Translate the shape left 4, down 1. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

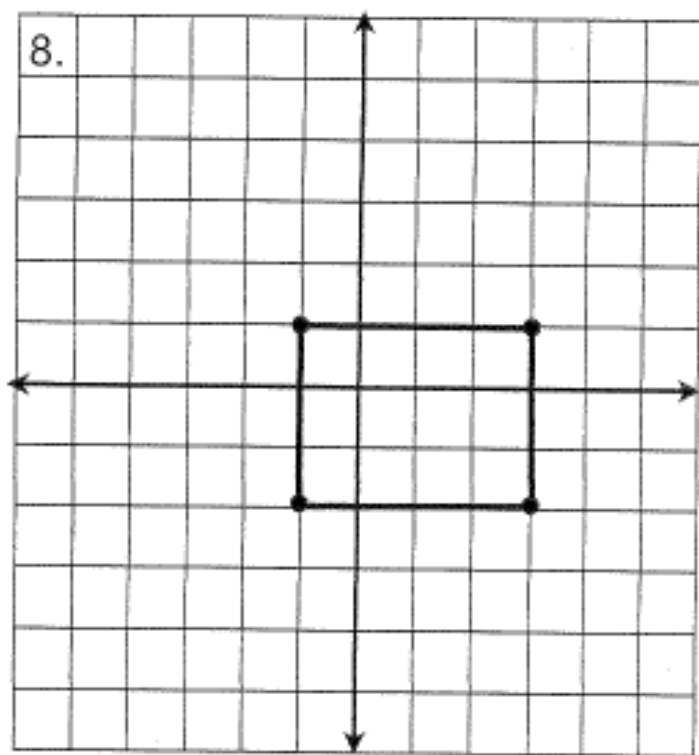
3. Translate the shape right 3, up 4. Write the rule associated with the transformation.

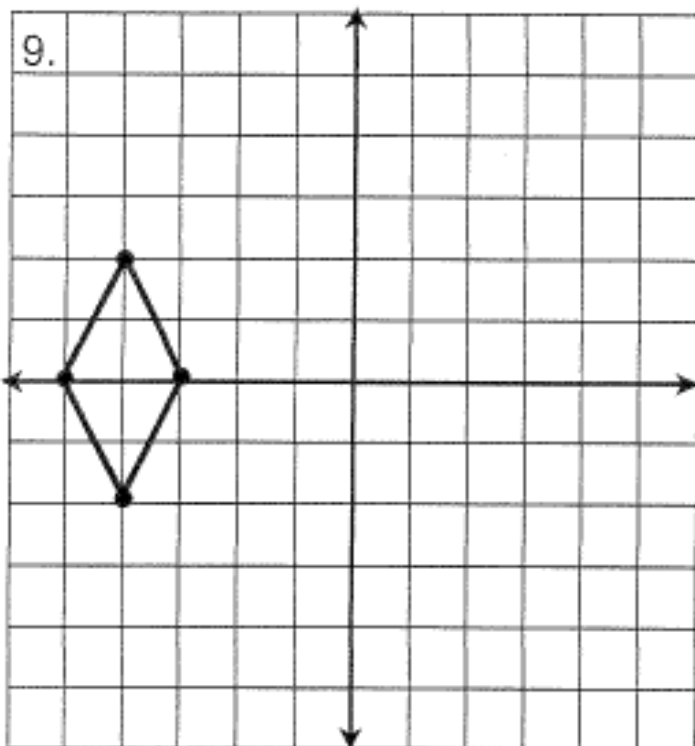
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Translate the shape left 4, up 2. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

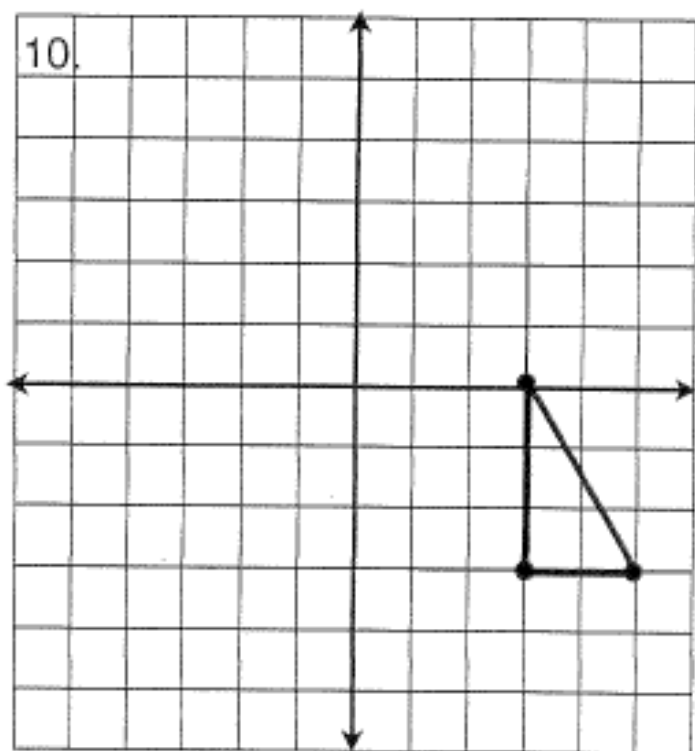
3. Translate the shape right 6, down 2. Write the rule associated with the transformation.

4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

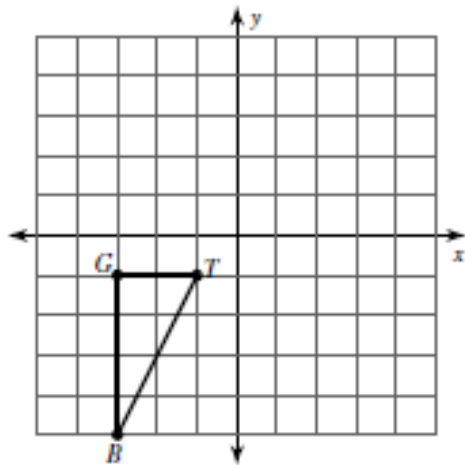
3. Translate the shape left 6, up 5. Write the rule associated with the transformation.

4. What are the points of the image?

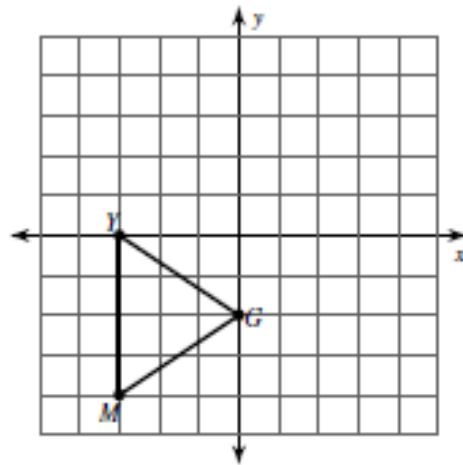


Graph the image of the figure using the transformation given.

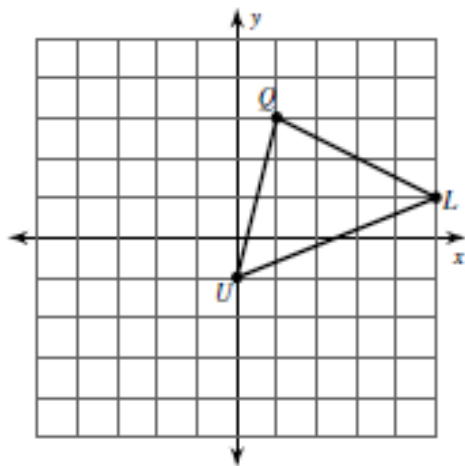
1) translation: 5 units right and 1 unit up



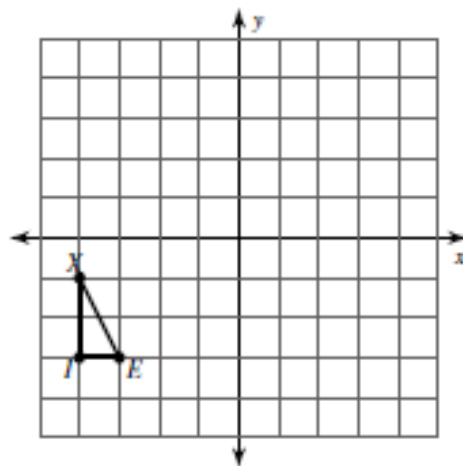
2) translation: 1 unit left and 2 units up



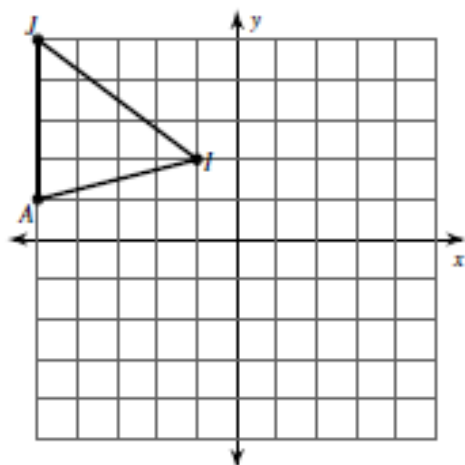
3) translation: 3 units down



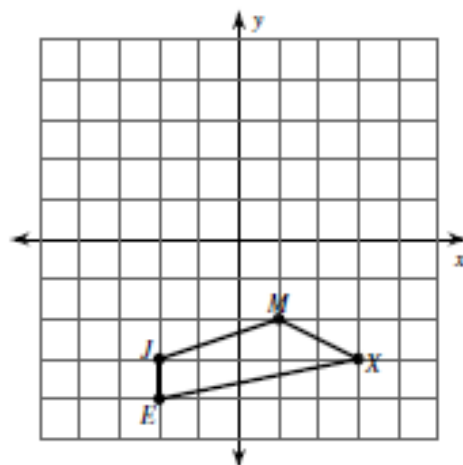
4) translation: 5 units right and 2 units up



5) translation: 4 units right and 4 units down

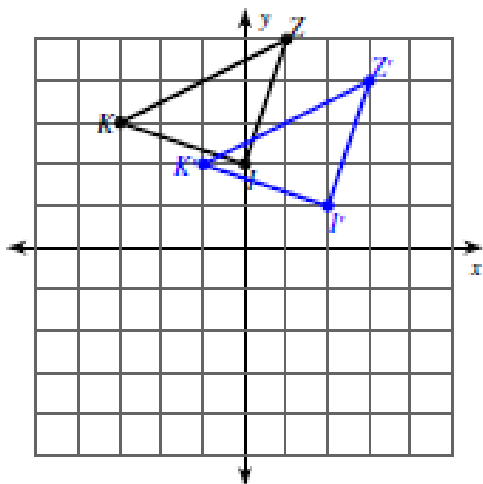


6) translation: 2 units right and 3 units up

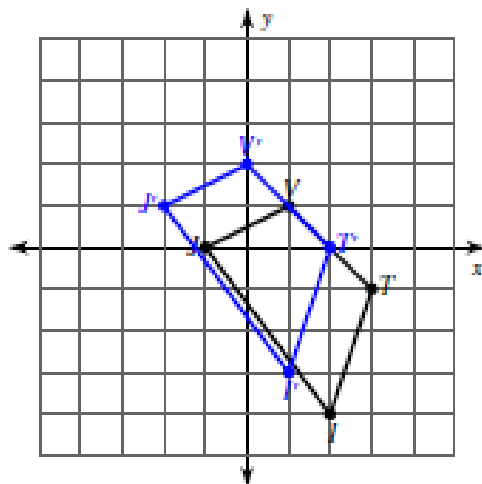


Write a rule to describe each transformation.

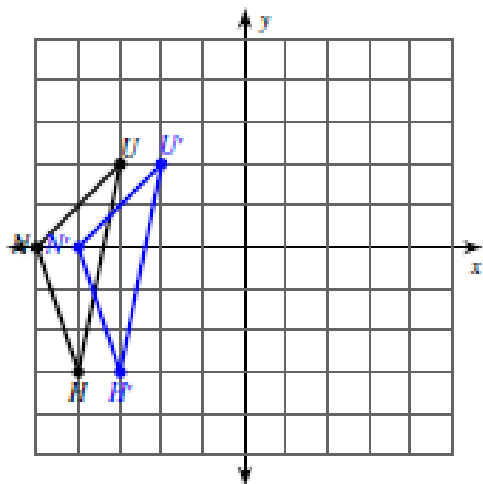
7)



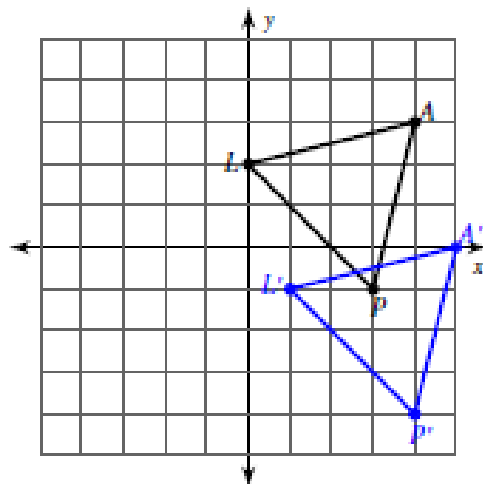
8)



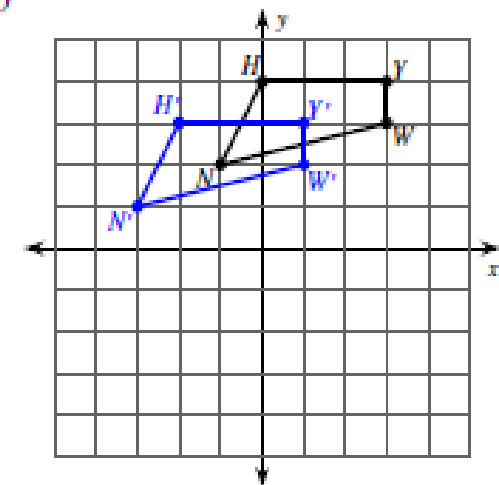
9)



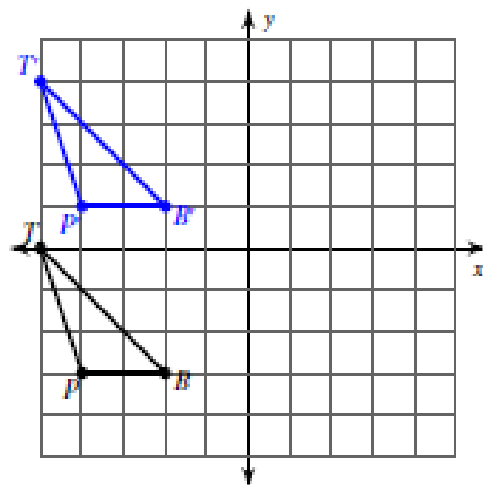
10)



11)

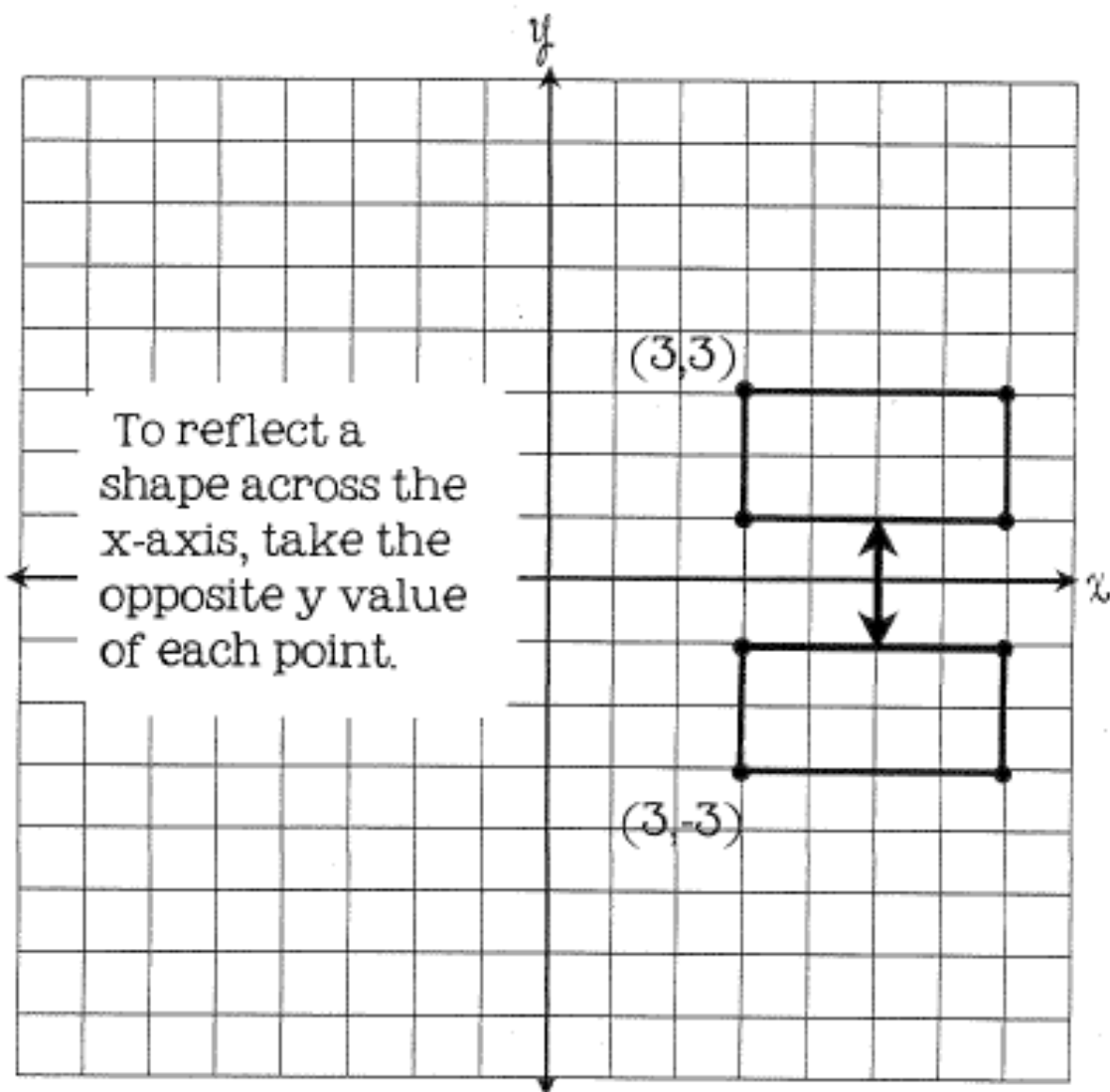


12)

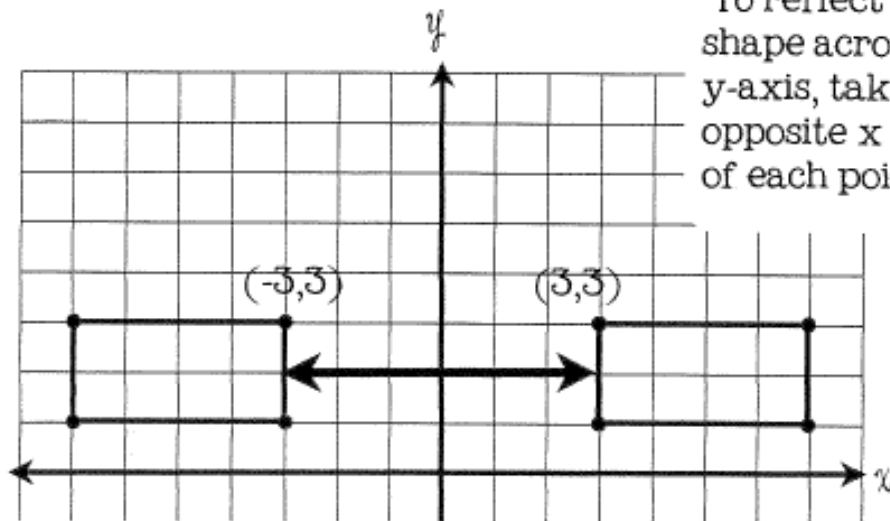


Reflection

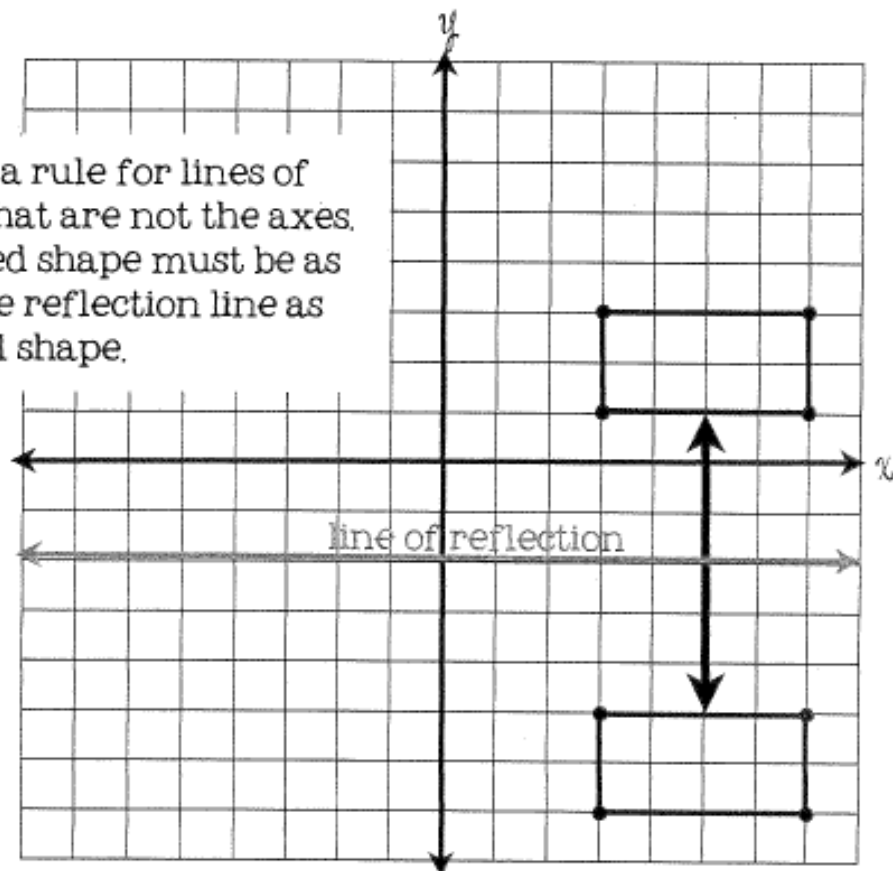
Either single points or entire shapes can be flipped, or reflected, over a line. Sometimes the x and y axes are used as lines of reflection. Sometimes, the lines must be drawn in to solve the reflection. For example, if the line of reflection is at $x = 2$, a vertical line would be drawn at $+2$ and the reflection would take place across that line.



To reflect a shape across the y-axis, take the opposite x value of each point.



There isn't a rule for lines of reflection that are not the axes. The reflected shape must be as far from the reflection line as the original shape.



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SUMMARY: REFLECTIONS

X-axis

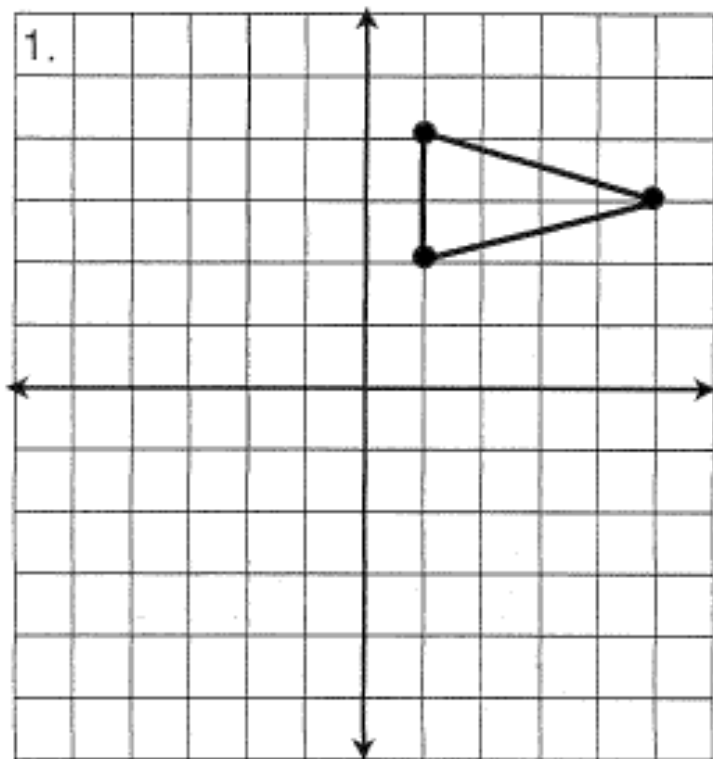
$$Y=X$$

Y-axis

$$Y=-X$$

over a HORIZONTAL/VERTICAL LINE:

Reflection Practice



1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

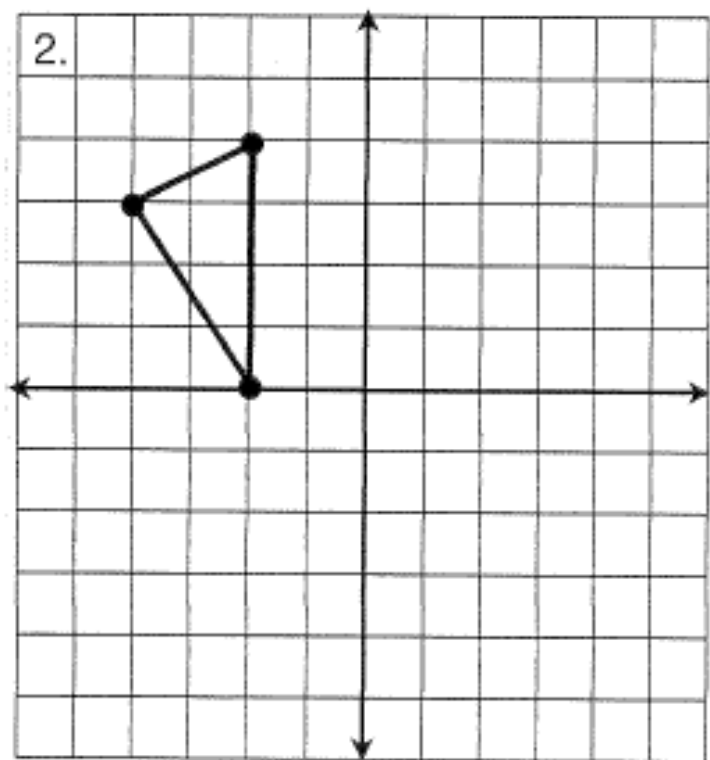
3. Reflect the shape across the x-axis. Write the rule associated with the transformation.

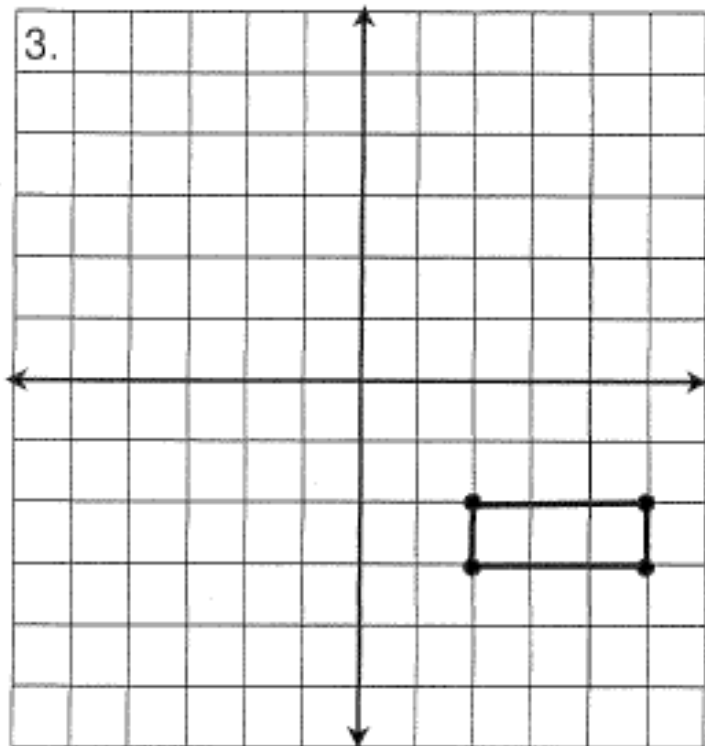
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Reflect the shape across the x-axis. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

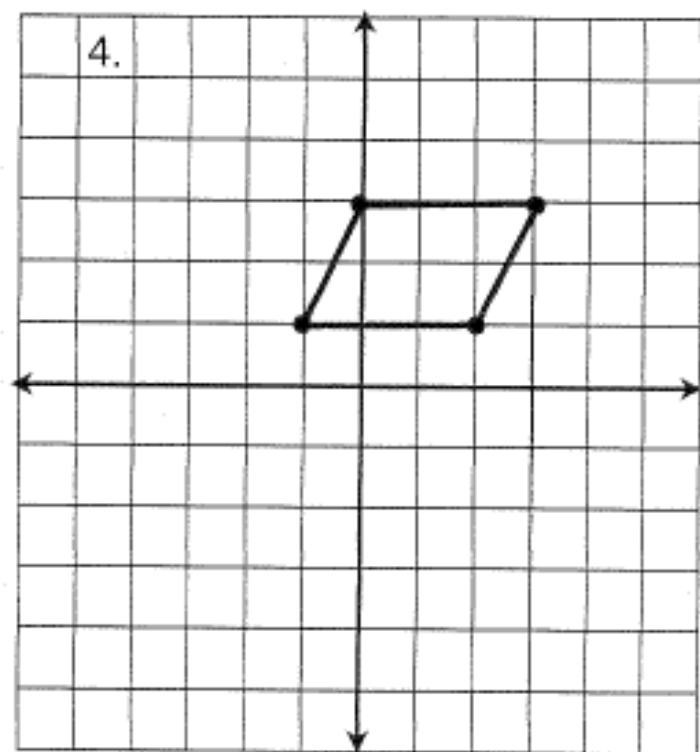
3. Reflect the shape across the $y = x$. Write the rule associated with the transformation.

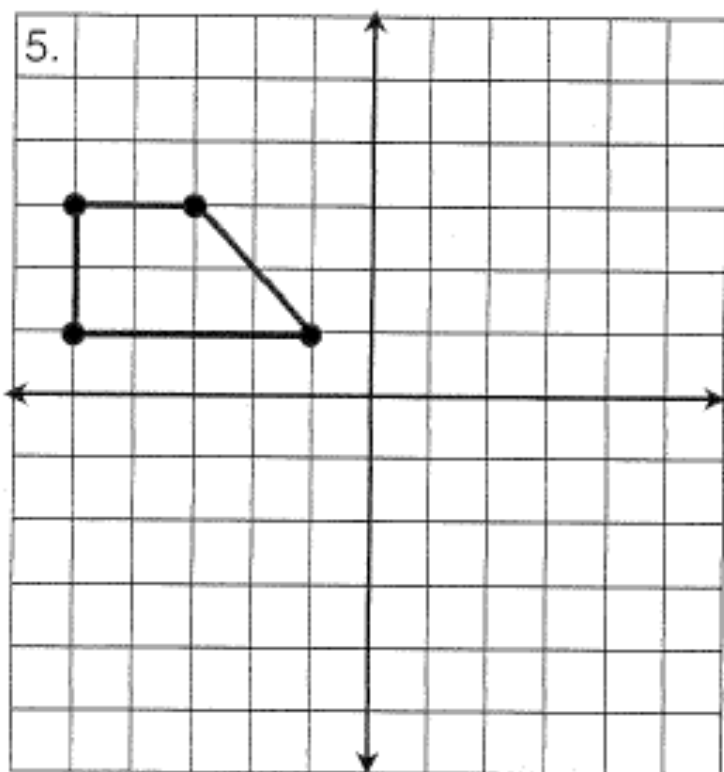
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Reflect the shape across the $y = x$. Write the rule associated with the transformation.

4. What are the points of the image?



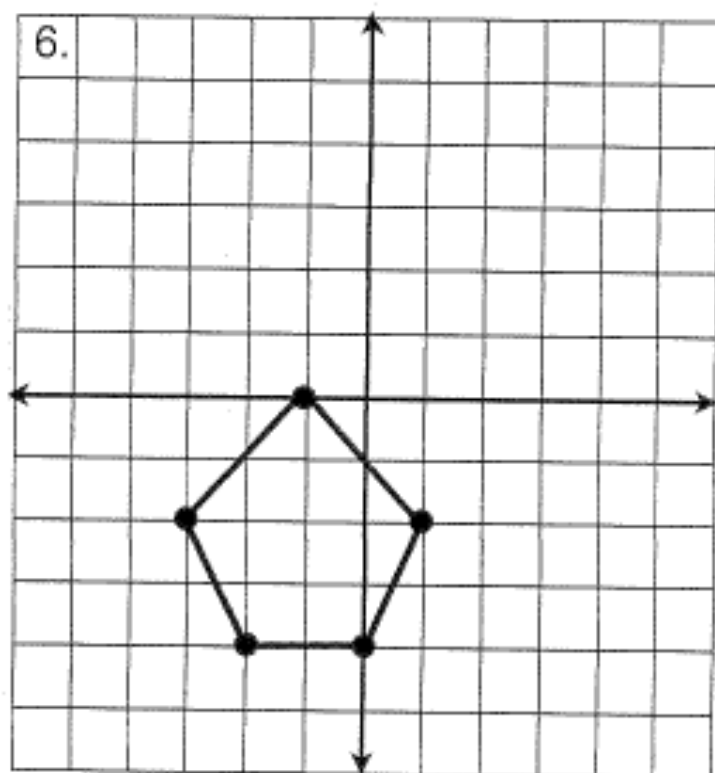


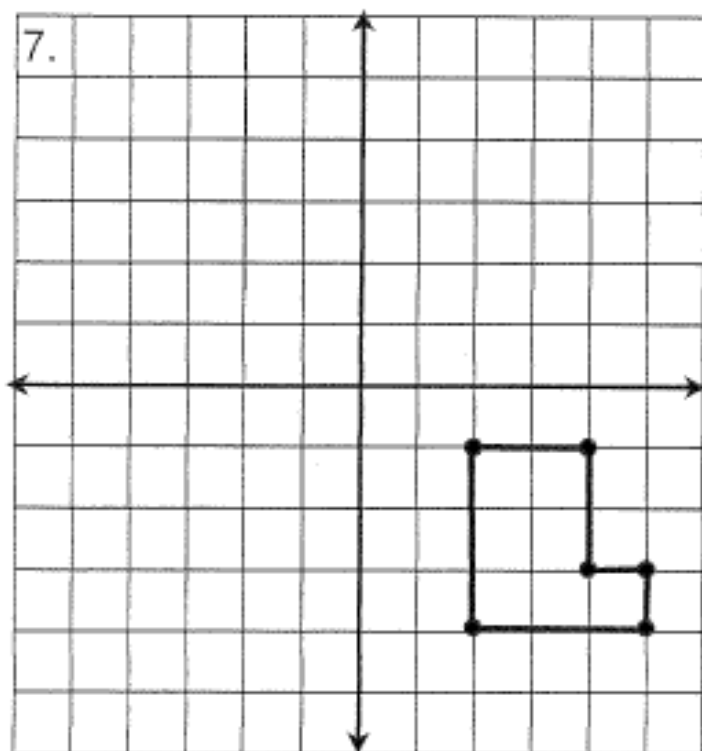
1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Reflect the shape across the y-axis. Write the rule associated with the transformation.

4. What are the points of the image?

1. Label the x and y axes.
 2. Label each of the points in the pre-image. List the points in the pre-image
- _____
3. Reflect the shape across the y-axis. Write the rule associated with the transformation.
- _____
4. What are the points of the image?
- _____





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

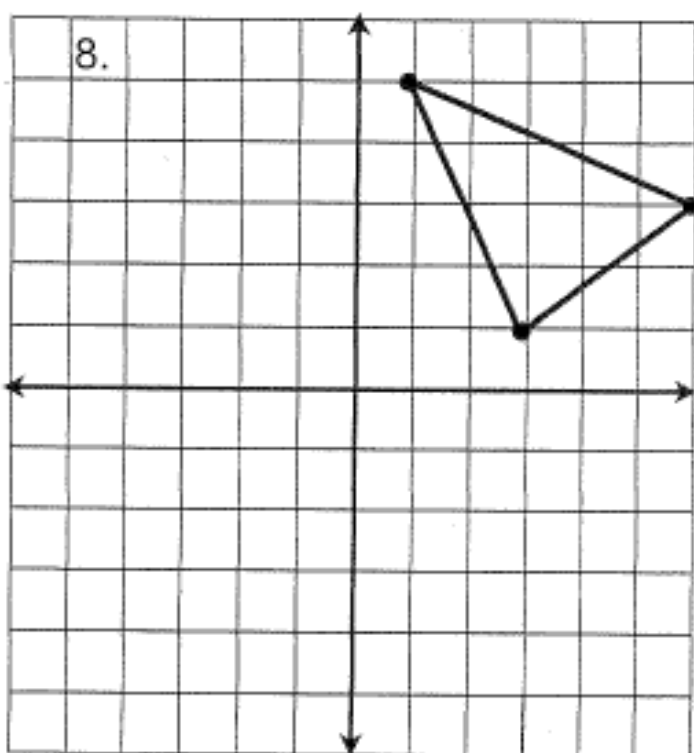
3. Reflect the shape across the $y = -x$. Write the rule associated with the transformation.

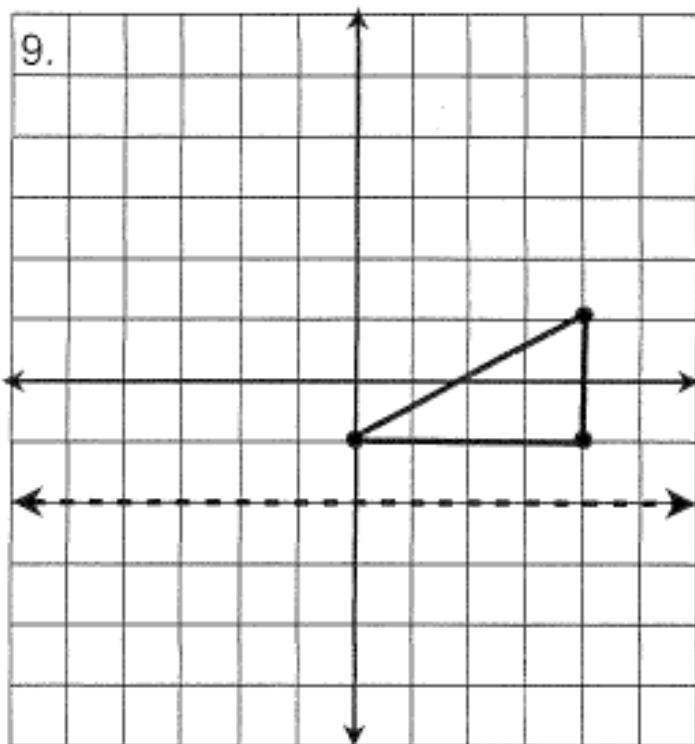
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Reflect the shape across the $y = -x$. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

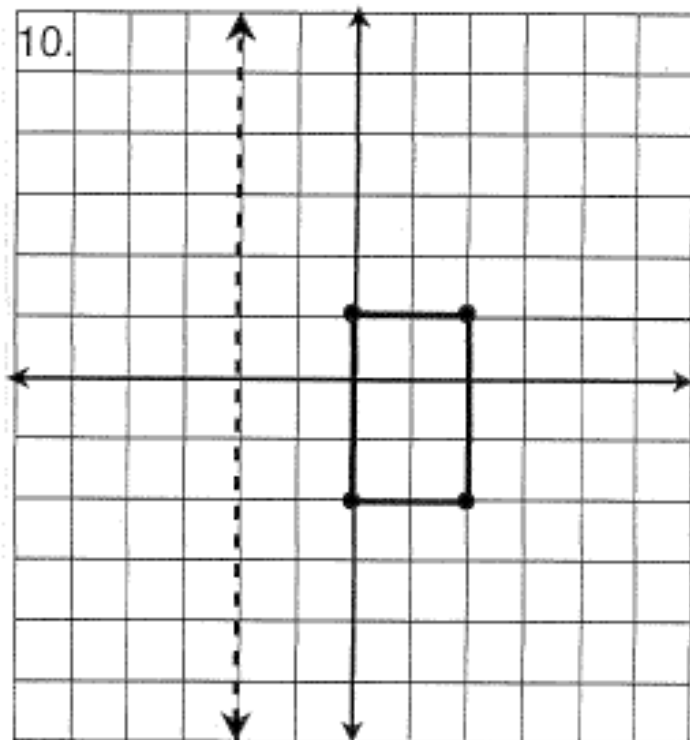
3. Reflect the shape across the line of reflection.
What are the points of the image?

4. Reflect the image across $x = -2$.
What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Reflect the shape across the line of reflection.
What are the points of the image?

4. Reflect the image across $y = 1$.
What are the points of the image?



Practice:

Directions: Write what the coordinates will be by reflecting each of the given points across the x-axis. Use the reflection rules above to help you answer the questions. When reflecting across the x-axis, the x-value remains the same and change the sign of the y-value.

1) A (4,5) A' () 2) D (-3,-5) D' () 3) G (-6,-8) G' ()

4) M (-2,8) M' () 5) N (7,0) N' () 6) E (-5,4) E' ()

Directions: Write what the coordinates will be by reflecting each of the given points across the y-axis. Use the reflection rules above to help you answer the questions. When reflecting across the y-axis, the y-value remains the same and change the sign of the x-value.

7) J (-2,-4) J' () 8) Y (-5,7) Y' () 9) S (4,6) S' ()

10) Q (4,7) Q' () 11) R (-12,3) R' () 12) W (3,8) W' ()

Directions: Write what the coordinates will be by reflecting each of the given points across the line $y=x$. Use the reflection rules above to help you answer the questions. When reflecting across the line $y=x$, the x-value and y-value switch spots.

13) H (4,5) H' () 14) X (-3,-4) X' () 15) B (-6,-6) B' ()

16) A (2,4) A' () 17) J (11,5) J' () 18) P (-2,-4) P' ()

Directions: Write what the coordinates will be by reflecting each of the given points across the line $y=-x$. Use the reflection rules above to help you answer the questions. When reflecting across the line $y=-x$, the x-value and y-value switch spots and change signs.

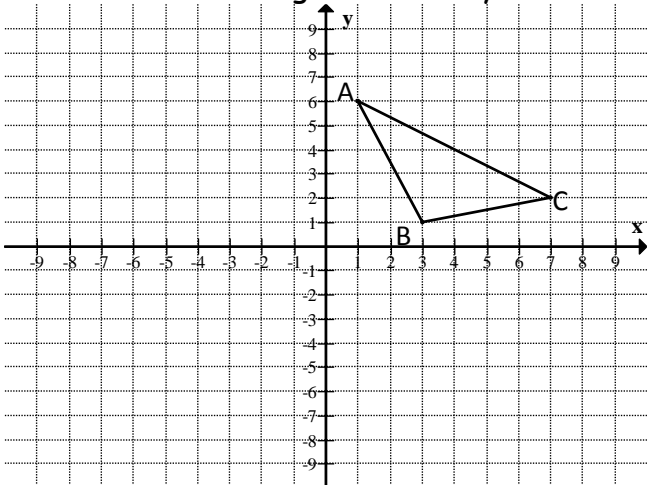
19) V (-8,-5) V' () 20) C (-6,-2) C' () 21) D (3,-1) D' ()

22) O (2,2) O' () 23) U (-4,5) U' () 24) K (9,-6) K' ()

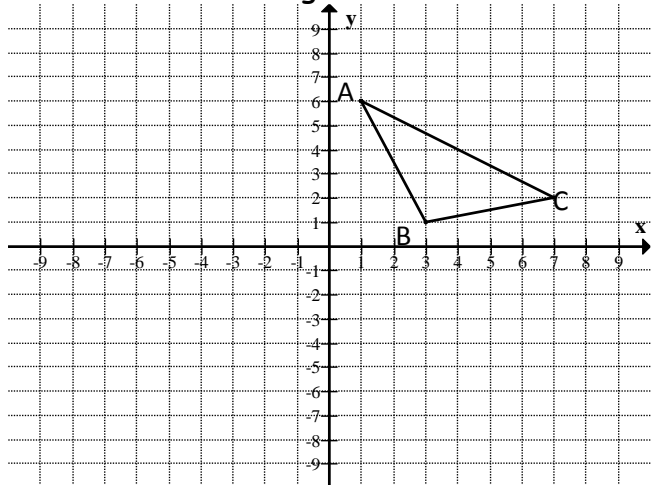
Reflections Worksheet

For #1-5, draw the triangle after each transformation and give the coordinates of A', B' and C'.

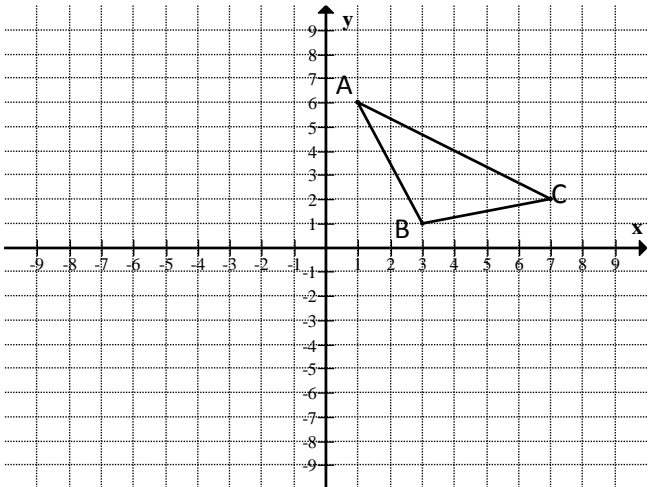
1. Reflect the triangle over the y-axis.



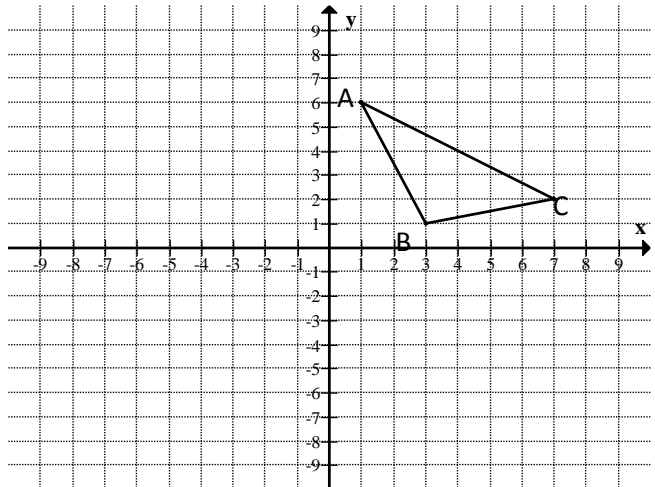
2. Reflect the triangle over the x-axis.



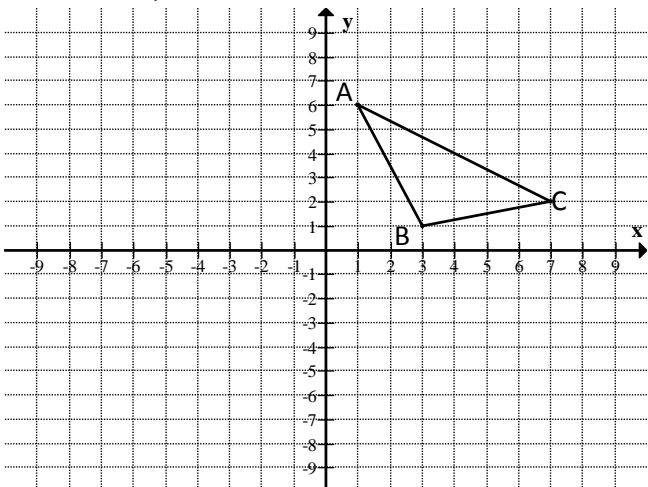
3. Reflect the triangle over $y = x$.



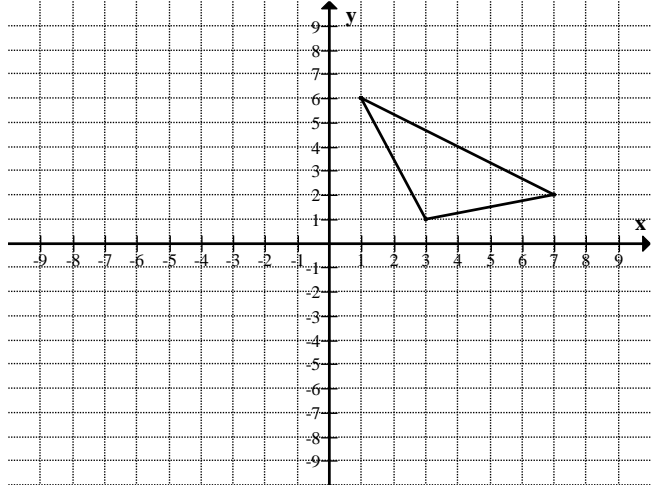
4. Reflect the triangle over $y = -x$.



5. Reflect the triangle over the x-axis and then over $y = x$.

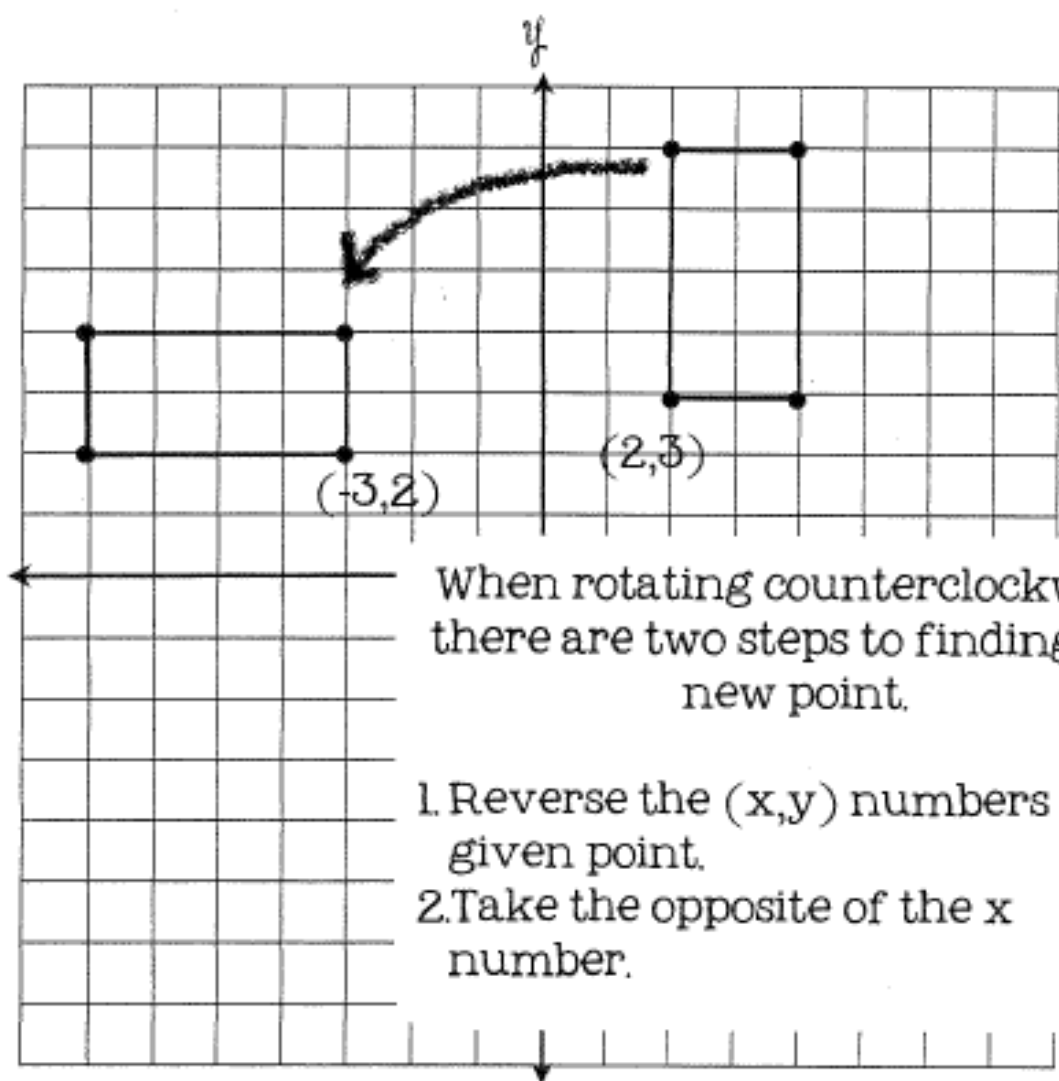


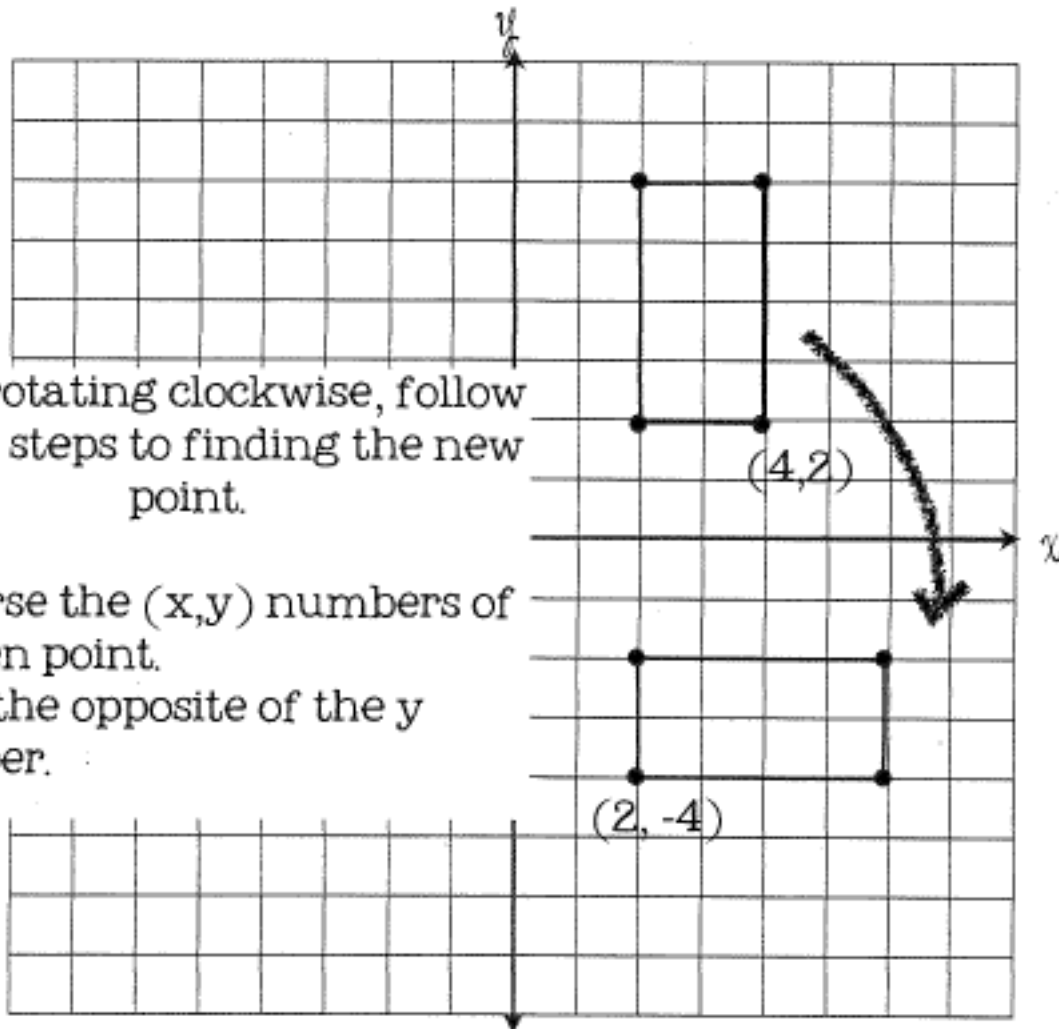
6. Reflect the triangle over the line $x = -3$



Rotation

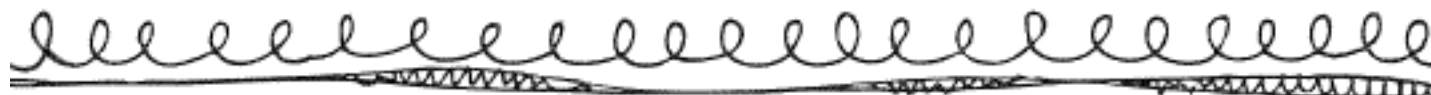
The rotation, or turn, is the most difficult of the transformations. When performing a rotation on a coordinate plane, it is usually called "about the origin." This means that the point $(0,0)$ is the center point of the rotation. Directional words like clockwise and counterclockwise are also used. Unlike the reflection and translation, the rotation isn't a movement that is easily visualized. The rule for the rotation is also more difficult and can be confusing.





When rotating clockwise, follow similar steps to finding the new point.

1. Reverse the (x,y) numbers of a given point.
2. Take the opposite of the y number.



Rotating a figure 90° , puts it into the adjacent quadrant.

Rotating a figure 180° , puts it into the opposite quadrant. (To rotate 180° , complete the rotation twice.)



SUMMARY: ROTATIONS

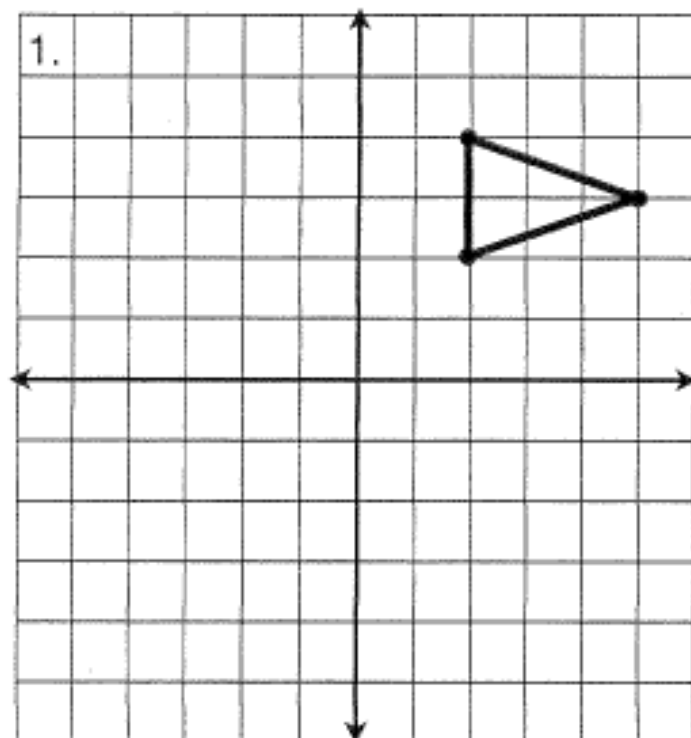
90° CCW (270° CW):

270° CCW (90° CW):

180° :

360° :

Rotation Practice



1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

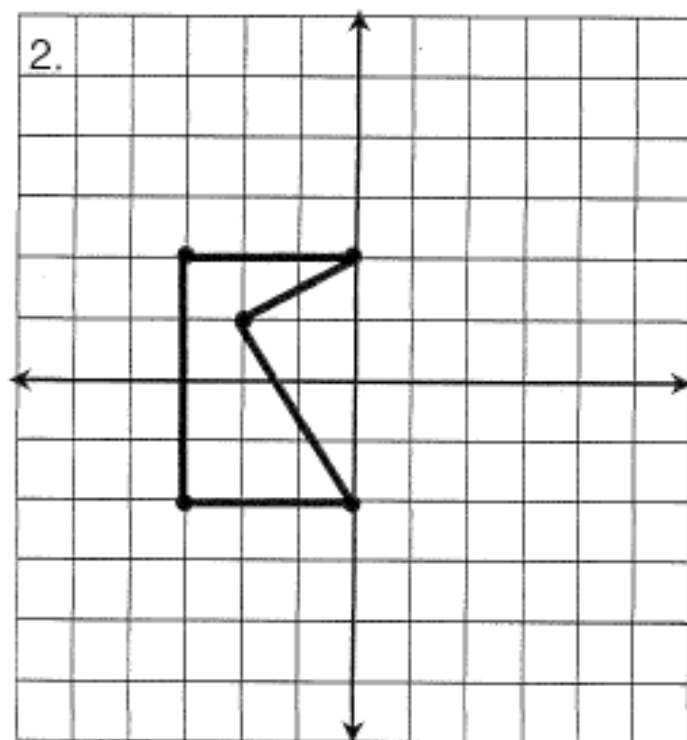
3. Rotate the figure 90° counterclockwise. Write the rule associated with the transformation.

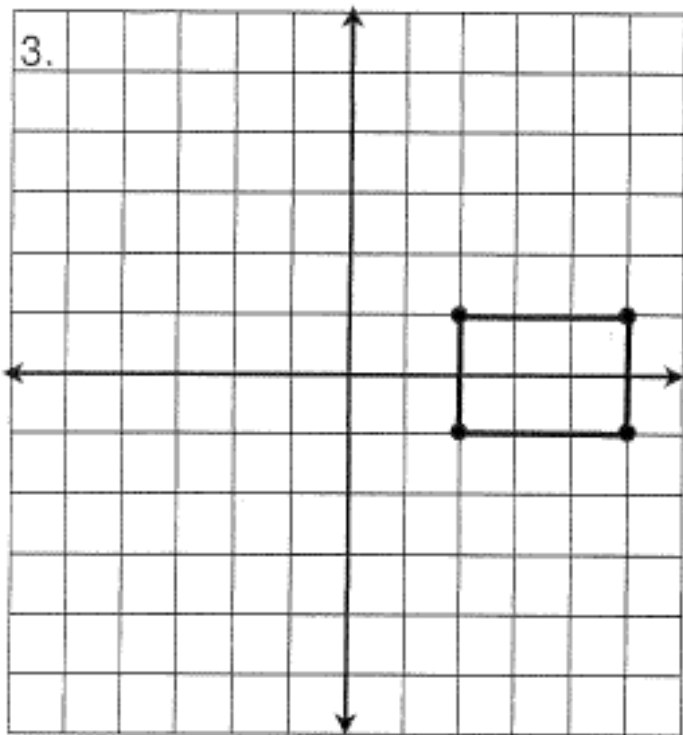
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Rotate the figure 90° clockwise. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

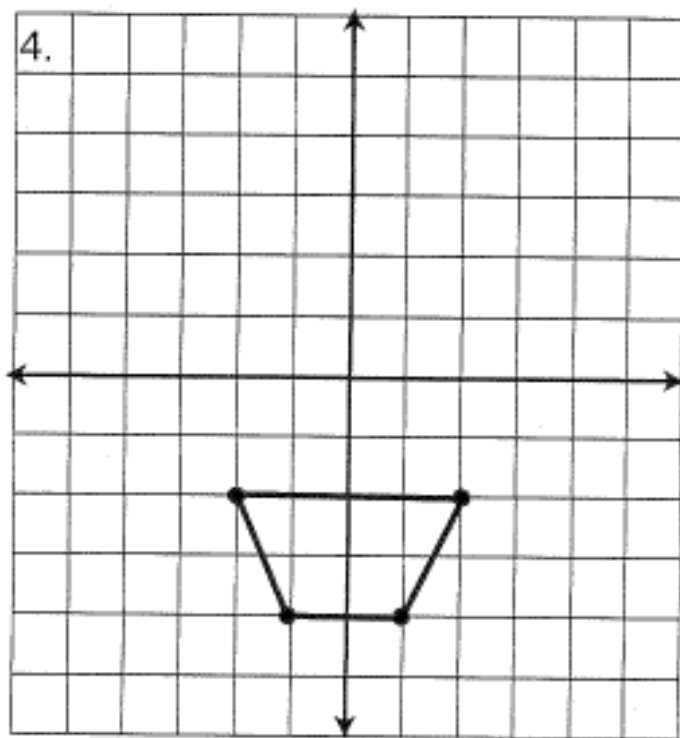
3. Rotate the figure 270° counterclockwise. Write the rule associated with the transformation.

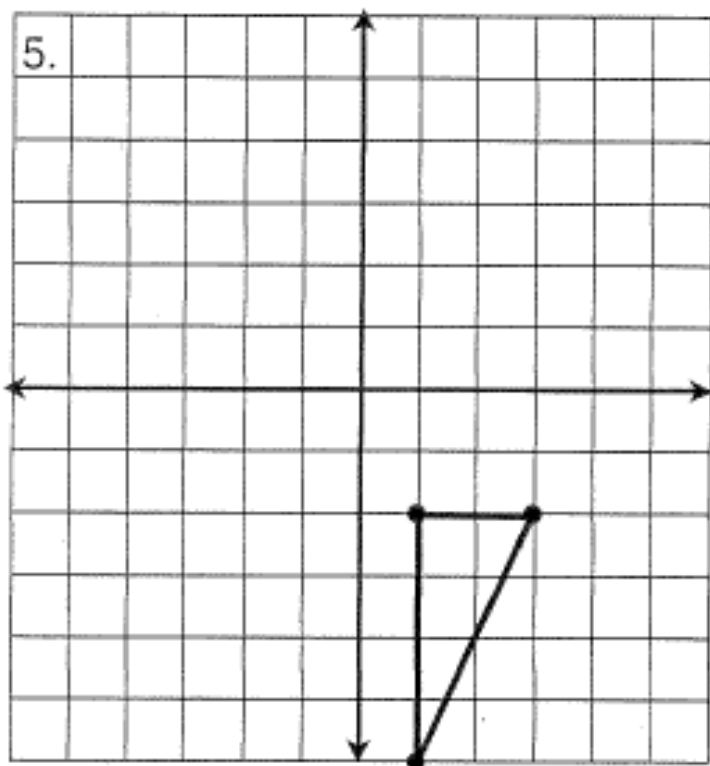
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Rotate the figure 270° clockwise. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

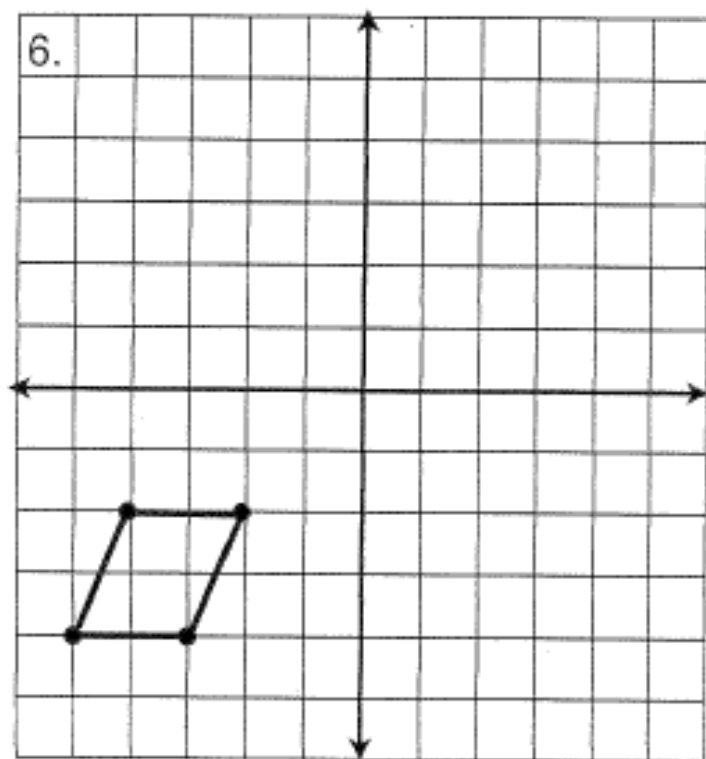
3. Rotate the figure 90° counterclockwise. Write the rule associated with the transformation.

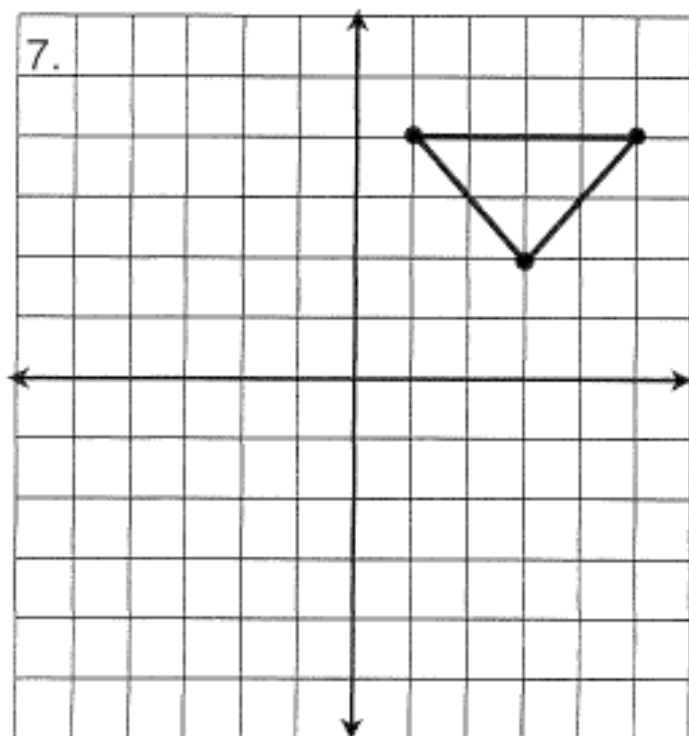
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Rotate the figure 90° clockwise. Write the rule associated with the transformation.

4. What are the points of the image?





1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

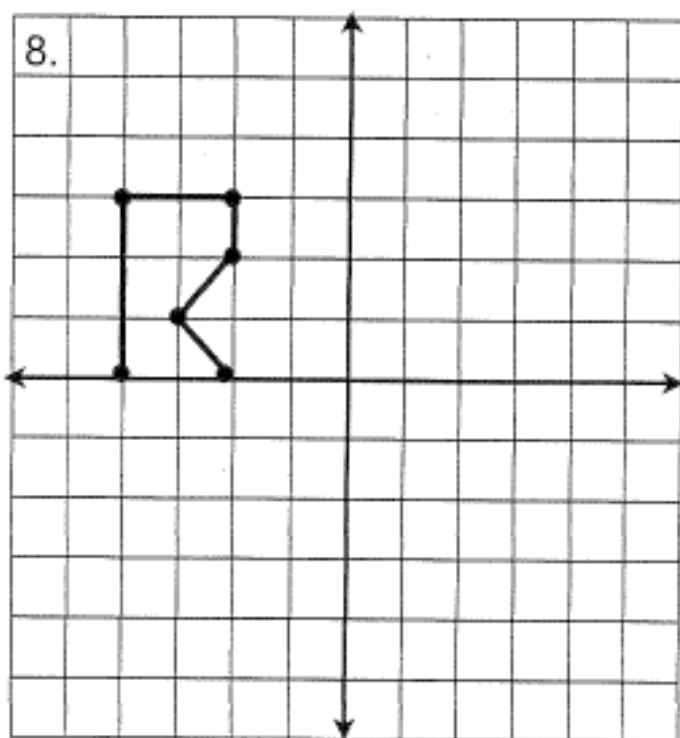
3. Rotate the figure 270° counterclockwise. Write the rule associated with the transformation.

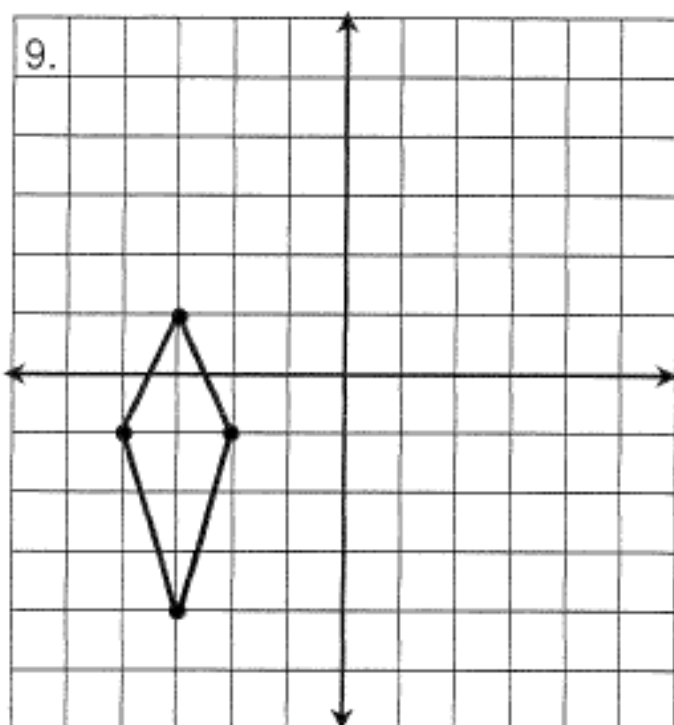
4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Rotate the figure 270° clockwise. Write the rule associated with the transformation.

4. What are the points of the image?



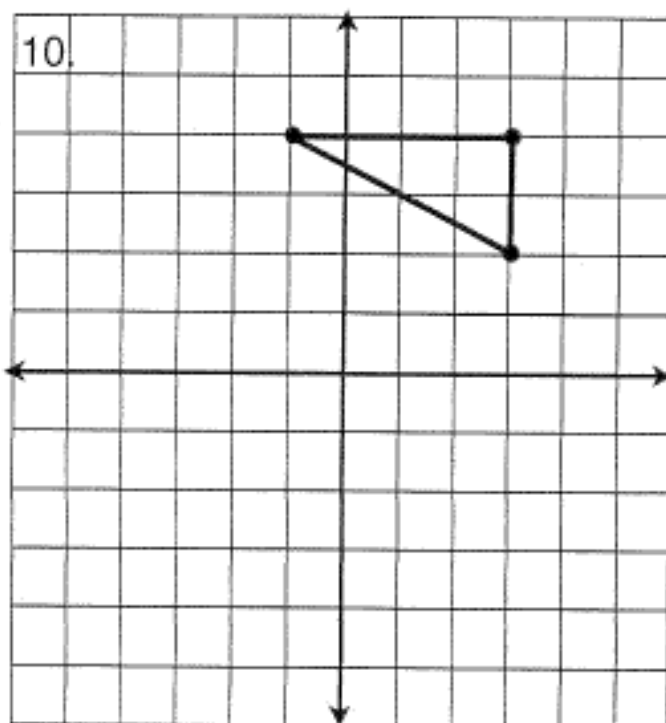


Label the x and y axes.
 Label each of the points in the pre-image. List the points in the pre-image

Rotate the figure 180° counterclockwise.
 Write the rule associated with the transformation.

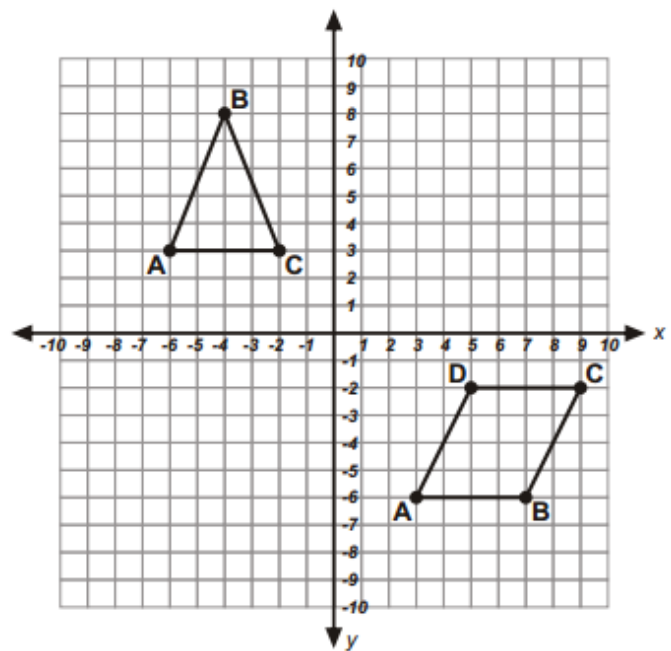
What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image
3. Rotate the figure 180° clockwise. Write the rule associated with the transformation.
4. What are the points of the image?



- 1) List the coordinates of triangle ABC. After listing the coordinates, rotate triangle ABC 90 degrees clockwise around the origin and list the new coordinates. Draw the new location of triangle ABC on the coordinate plane.

Coordinates Before Rotation	Coordinates After Rotation
A _____	A' _____
B _____	B' _____
C _____	C' _____

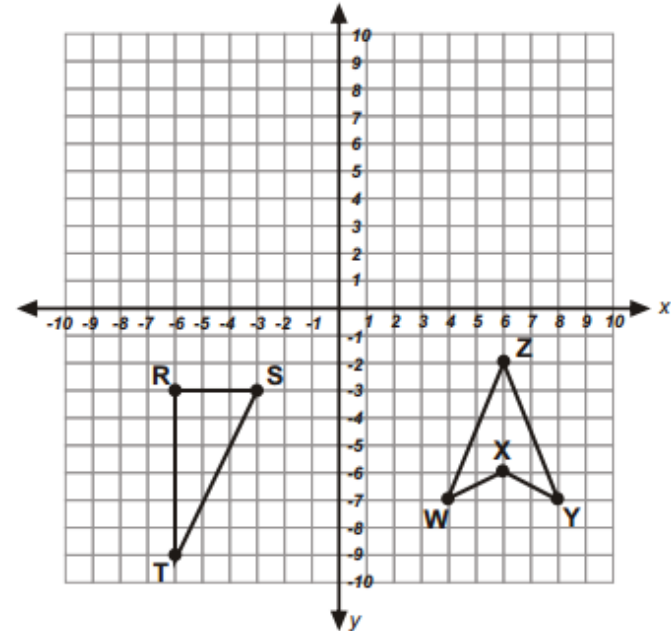


- 2) Rotate parallelogram ABCD 90 degrees clockwise around the origin. List the coordinates before and after the rotation and draw the rotated shape on the coordinate plane.

Coordinates Before Rotation	Coordinates After Rotation
A _____	A' _____
B _____	B' _____
C _____	C' _____
D _____	D' _____

- 3) Rotate object WXYZ 180 degrees around the origin. List the coordinates before and after the rotation and draw the rotated shape on the coordinate plane.

Coordinates Before Rotation	Coordinates After Rotation
W _____	W' _____
X _____	X' _____
Y _____	Y' _____
Z _____	Z' _____



- 4) Rotate triangle RST 180 degrees around the origin. List the coordinates before and after the rotation and draw the rotated shape on the coordinate plane.

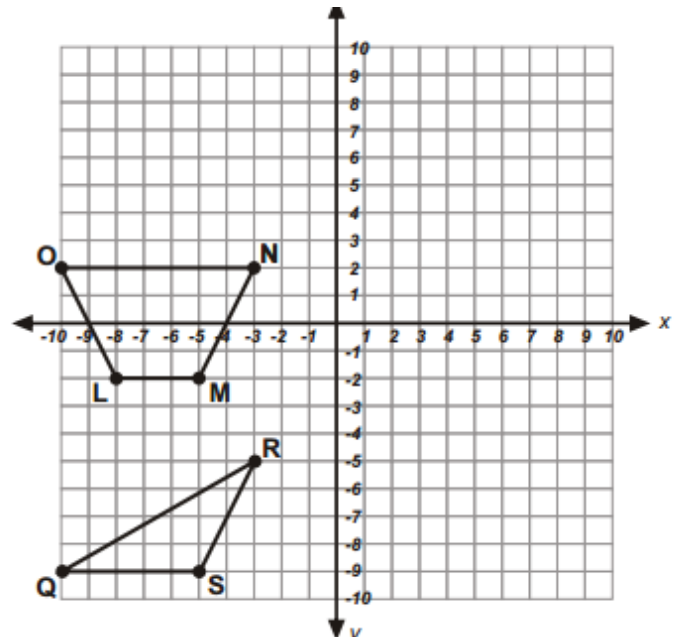
Coordinates Before Rotation	Coordinates After Rotation
R _____	R' _____
S _____	S' _____
T _____	T' _____

- 5) Rotate triangle QRS 90 degrees counter-clockwise around the origin. List the coordinates before and after the rotation and draw the rotated shape on the coordinate plane.

Coordinates Before Rotation	Coordinates After Rotation
Q _____	Q' _____
R _____	R' _____
S _____	S' _____

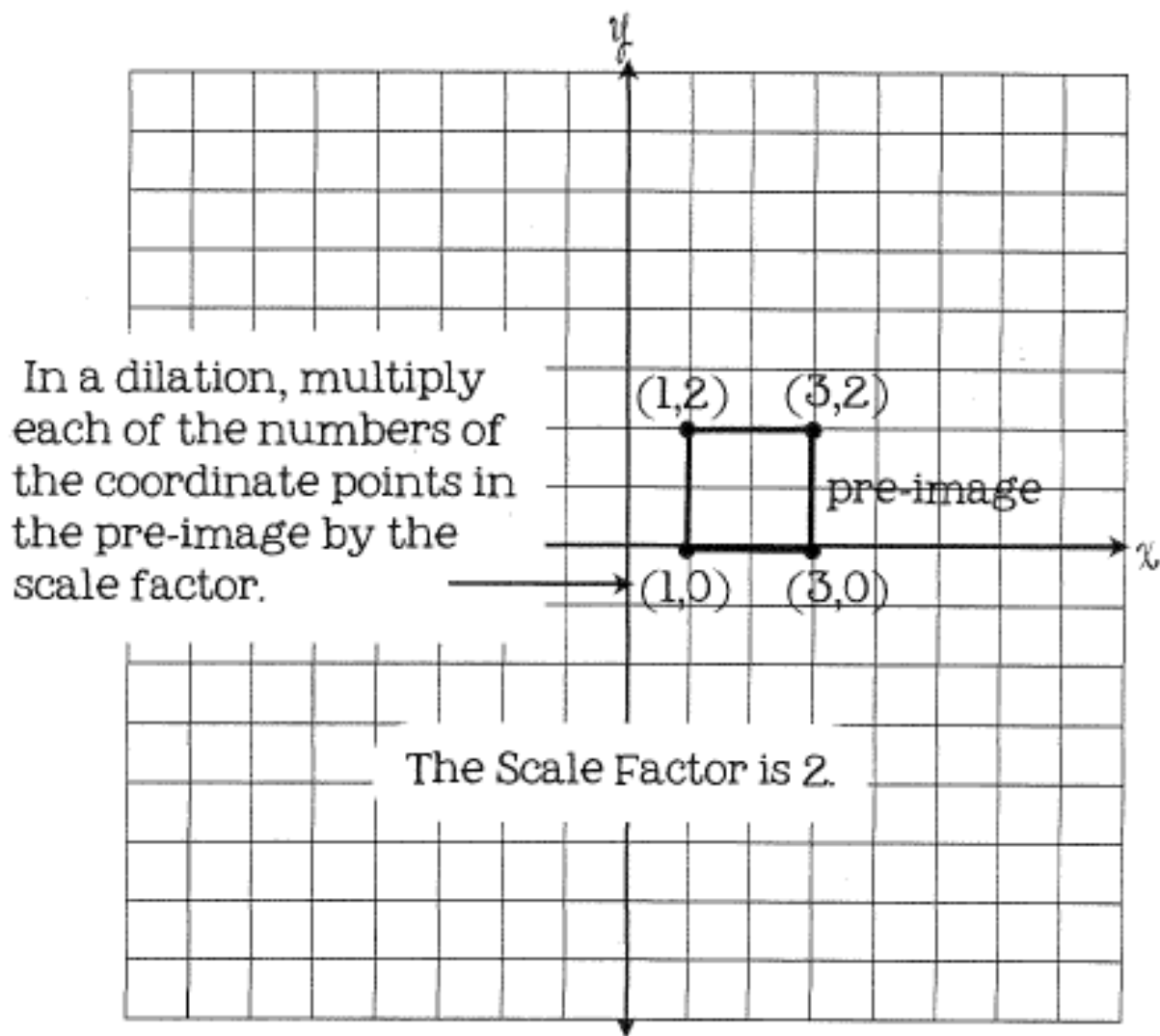
- 6) Rotate trapezoid LMNO 180 degrees around the origin. List the coordinates before and after the rotation and draw the rotated shape on the coordinate plane.

Coordinates Before Rotation	Coordinates After Rotation
L _____	L' _____
M _____	M' _____
N _____	N' _____
O _____	O' _____

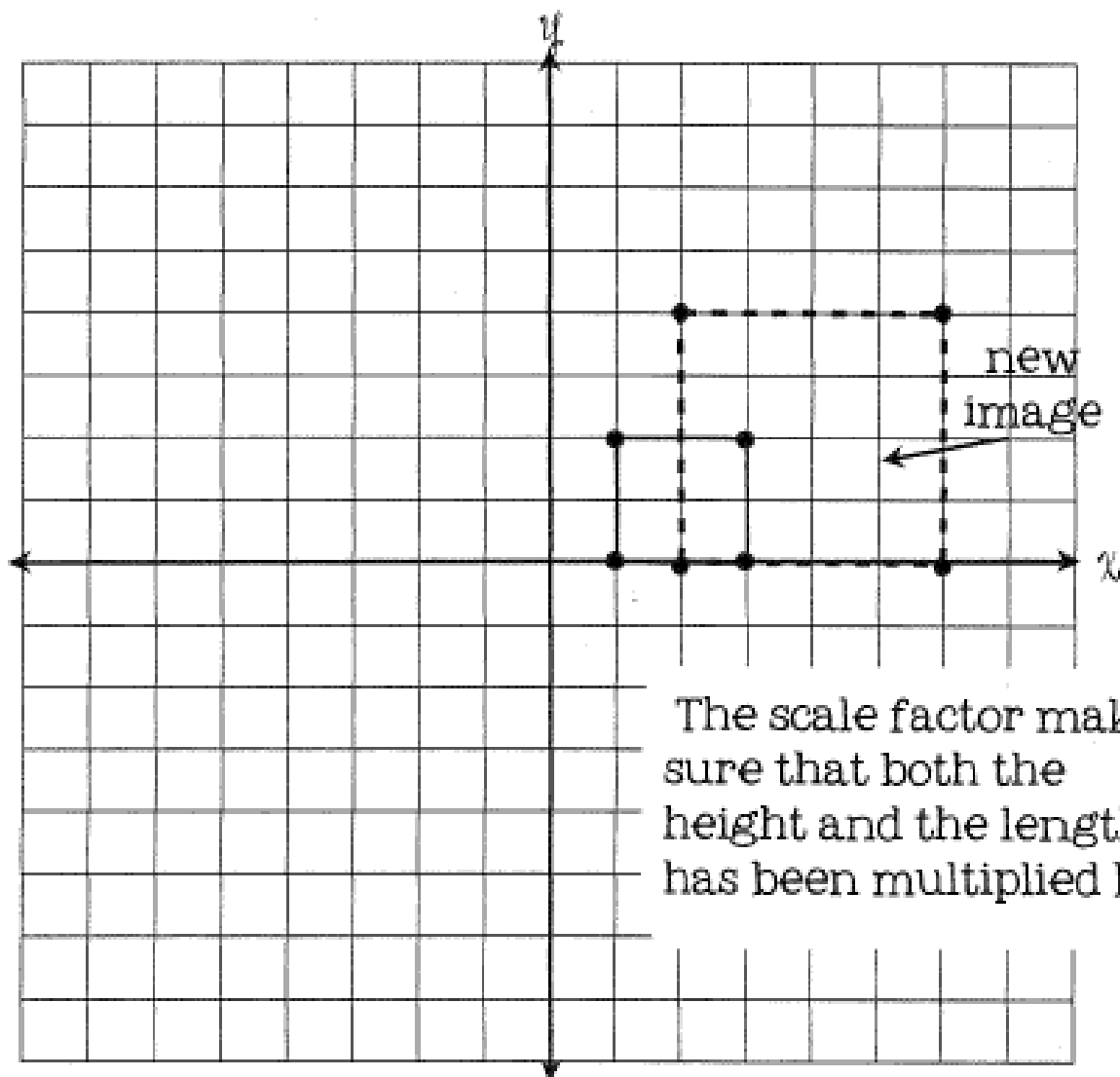


Dilation

A dilation takes a shape on the coordinate plane and enlarges it, just as an eye doctor makes your pupils larger with dilation drops. To complete a dilation in math, we work with scale factors. A scale factor makes sure that as a shape is enlarged, it is done so proportionally. To complete a shape's dilation, we change each individual point by multiplying the x and y values by the scale factor.



One point in the pre-image is (1,0).
To use the scale factor of 2, multiply 1 x 2 to create the
new image's x value, and multiply 0 x 2 to create the
new image's y value.



Multiply by the scale factor of 2.

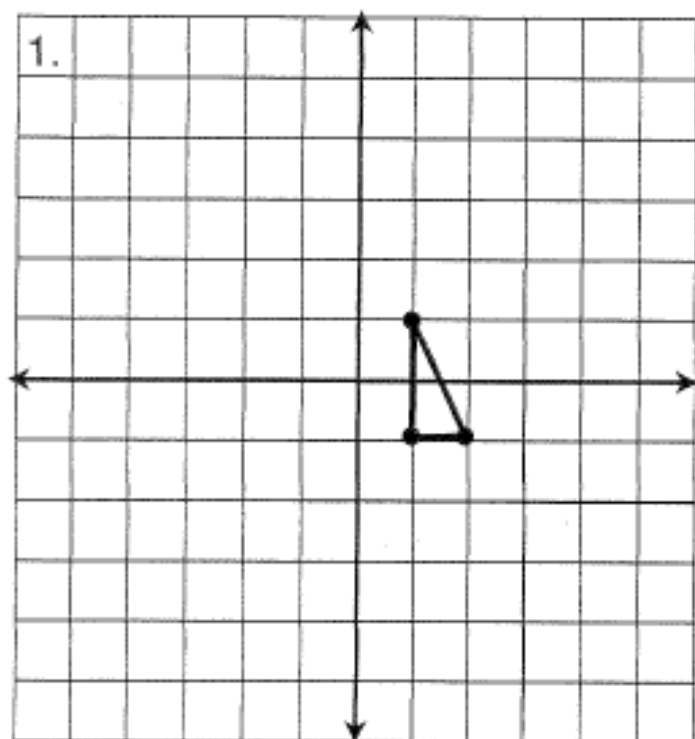
$$(1,0) \longrightarrow (2,0)$$

$$(3,0) \longrightarrow (6,0)$$

$$(1,2) \longrightarrow (2,4)$$

$$(3,2) \longrightarrow (6,4)$$

Dilation Practice



1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

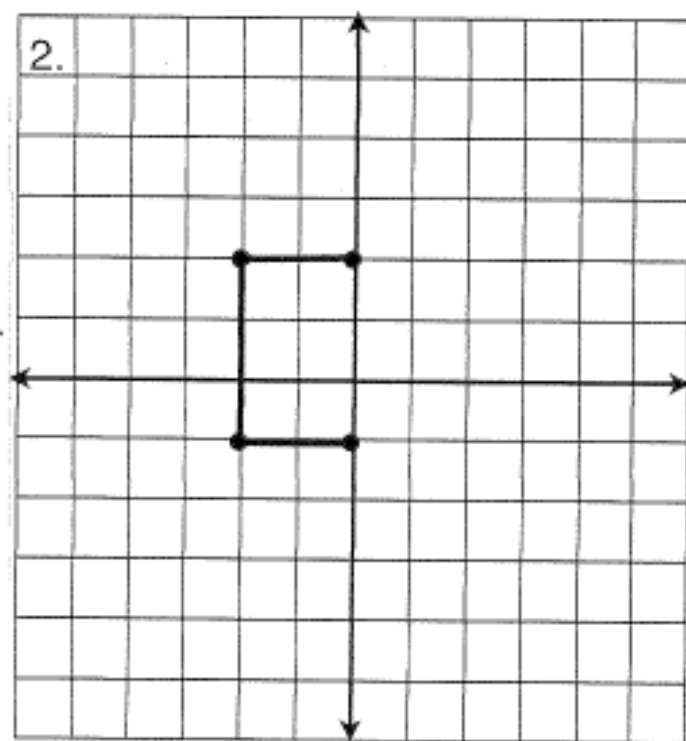
3. Dilate the figure. The scale factor is 2. Write the rule associated with the transformation.

4. What are the points of the image?

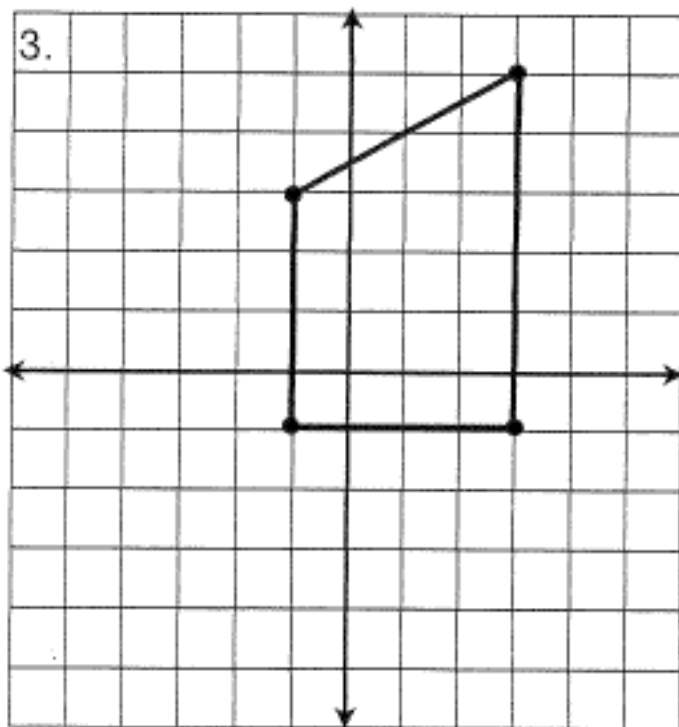
1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Dilate the figure. The scale factor is 3. Write the rule associated with the transformation.

4. What are the points of the image?



Dilations that use a fraction as a scale factor are known as reductions.



1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

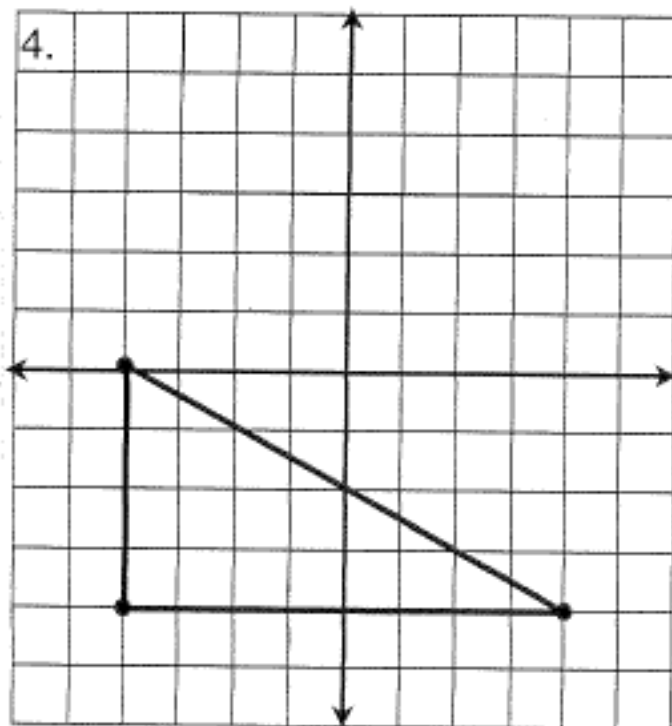
3. Dilate the figure. The scale factor is $\frac{1}{2}$. Write the rule associated with the transformation.

4. What are the points of the image?

1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

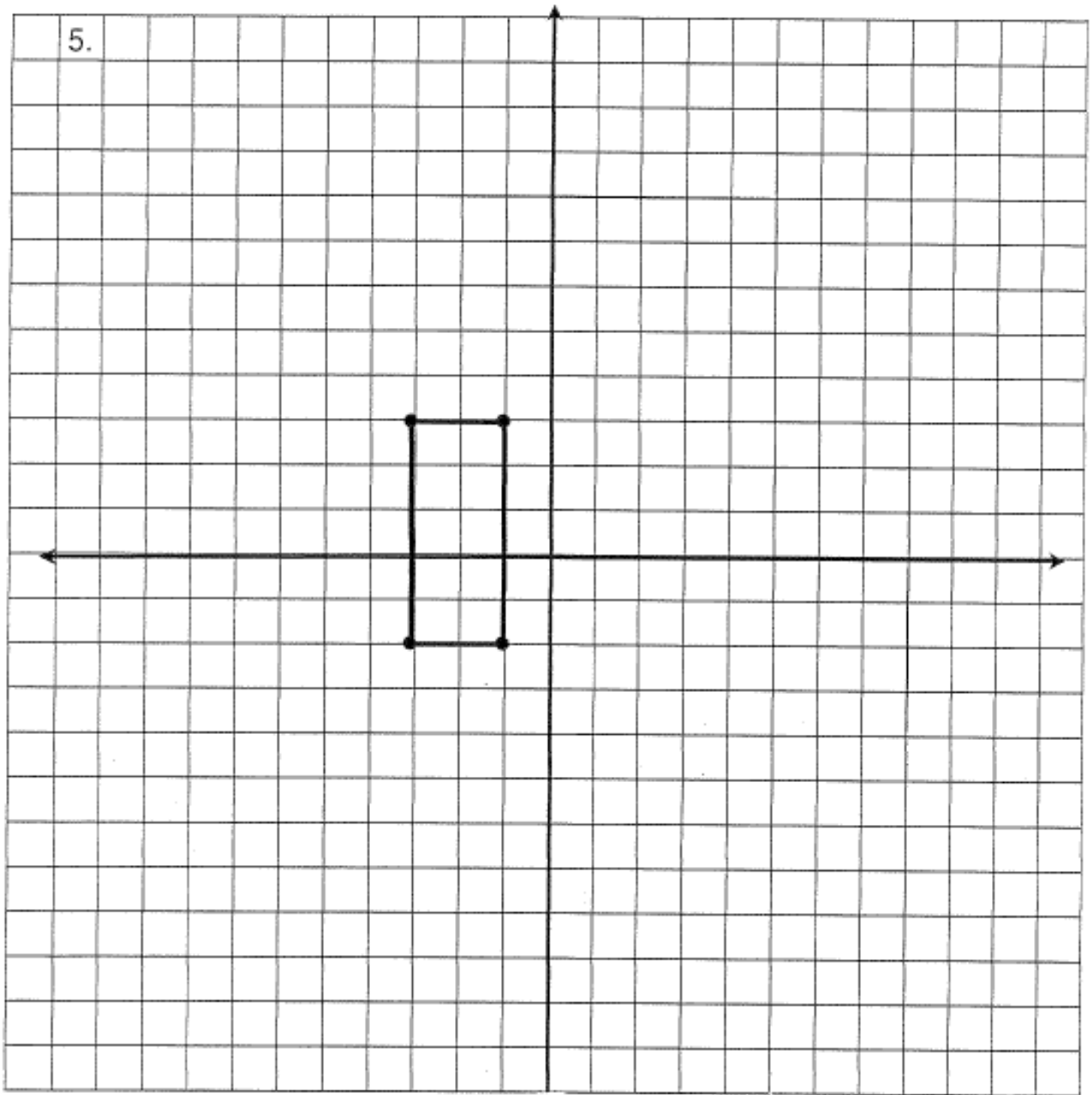
3. Dilate the figure. The scale factor is $\frac{1}{4}$. Write the rule associated with the transformation.

4. What are the points of the image?



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5.

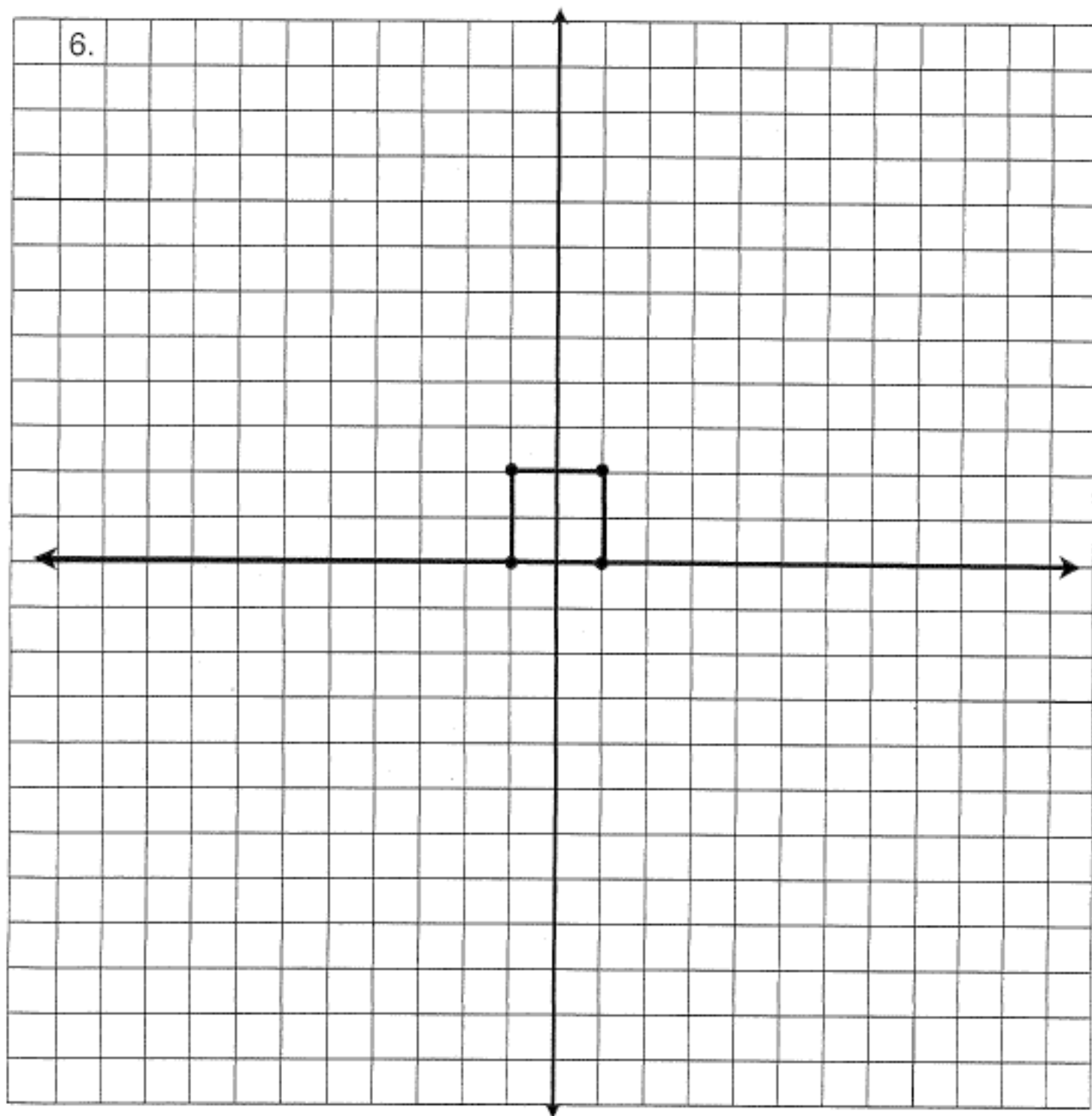


1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Dilate the figure. The scale factor is 3. Write the rule associated with the transformation.

4. What are the points of the image?

6.

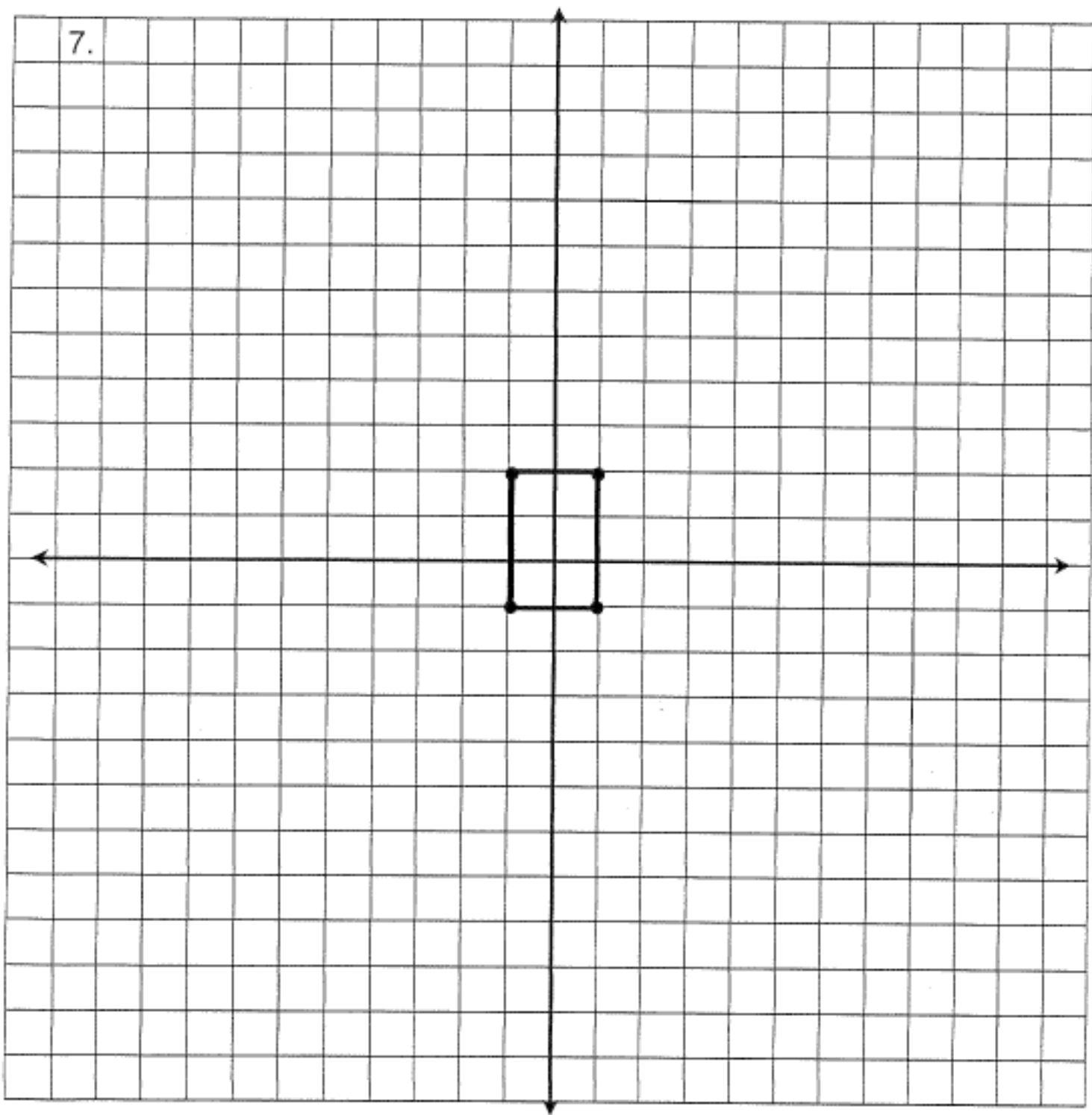


1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image

3. Dilate the figure. The scale factor is 4. Write the rule associated with the transformation.

4. What are the points of the image?

7.

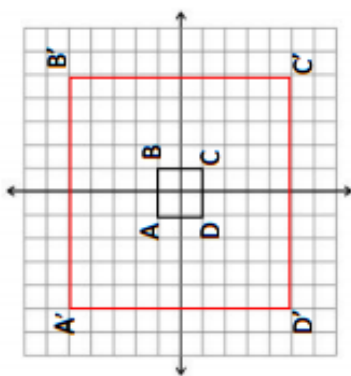


1. Label the x and y axes.
2. Label each of the points in the pre-image. List the points in the pre-image.

3. Dilate the figure. The scale factor is 5. Write the rule associated with the transformation.

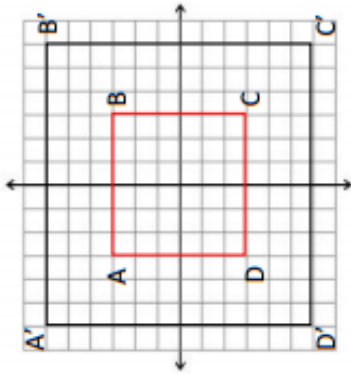
4. What are the points of the image?

Match each graph containing the image and pre-image with its corresponding transformation.



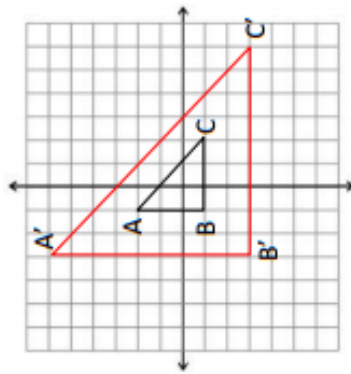
Dilate
 $r=1/3$
 Purple

Dilate
 $r=2$
 Brown

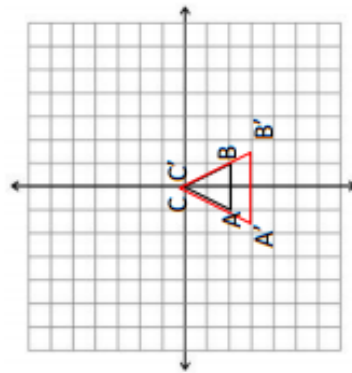
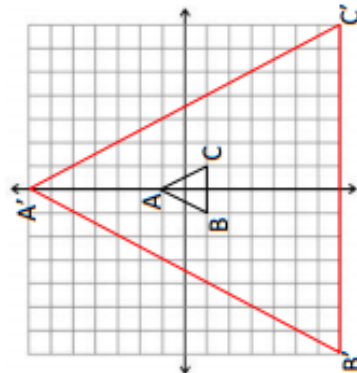
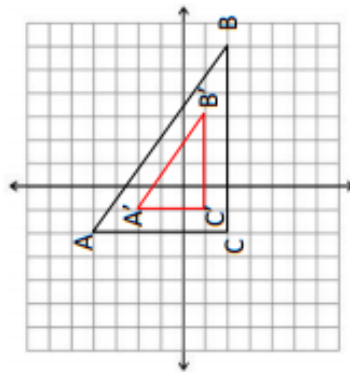


Dilate
 $r=1/2$
 Red

Dilate
 $r=1.5$
 Green

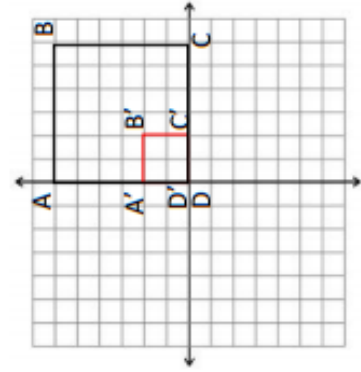


Dilate
 $r=5$
 Orange



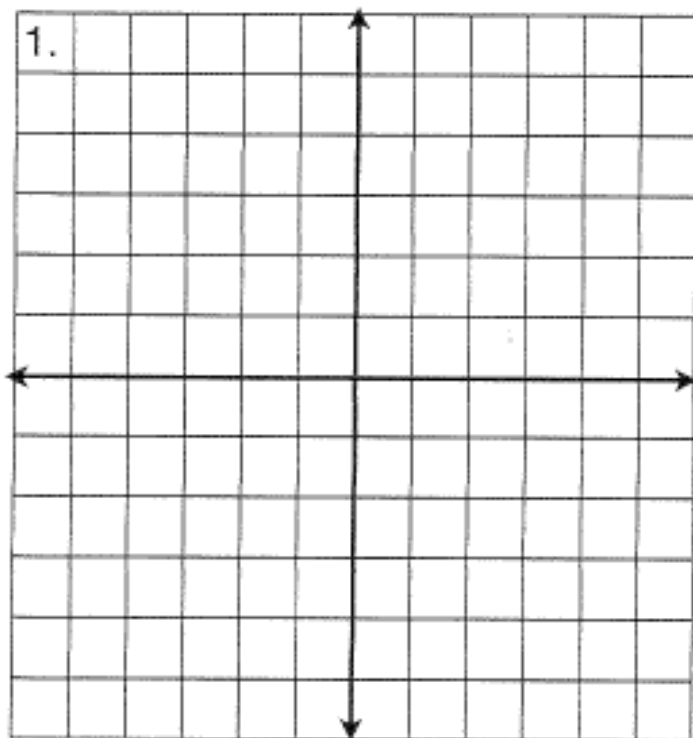
Dilate
 $r=7$
 Blue

Dilate
 $r=3$
 Yellow



Multiple Transformations

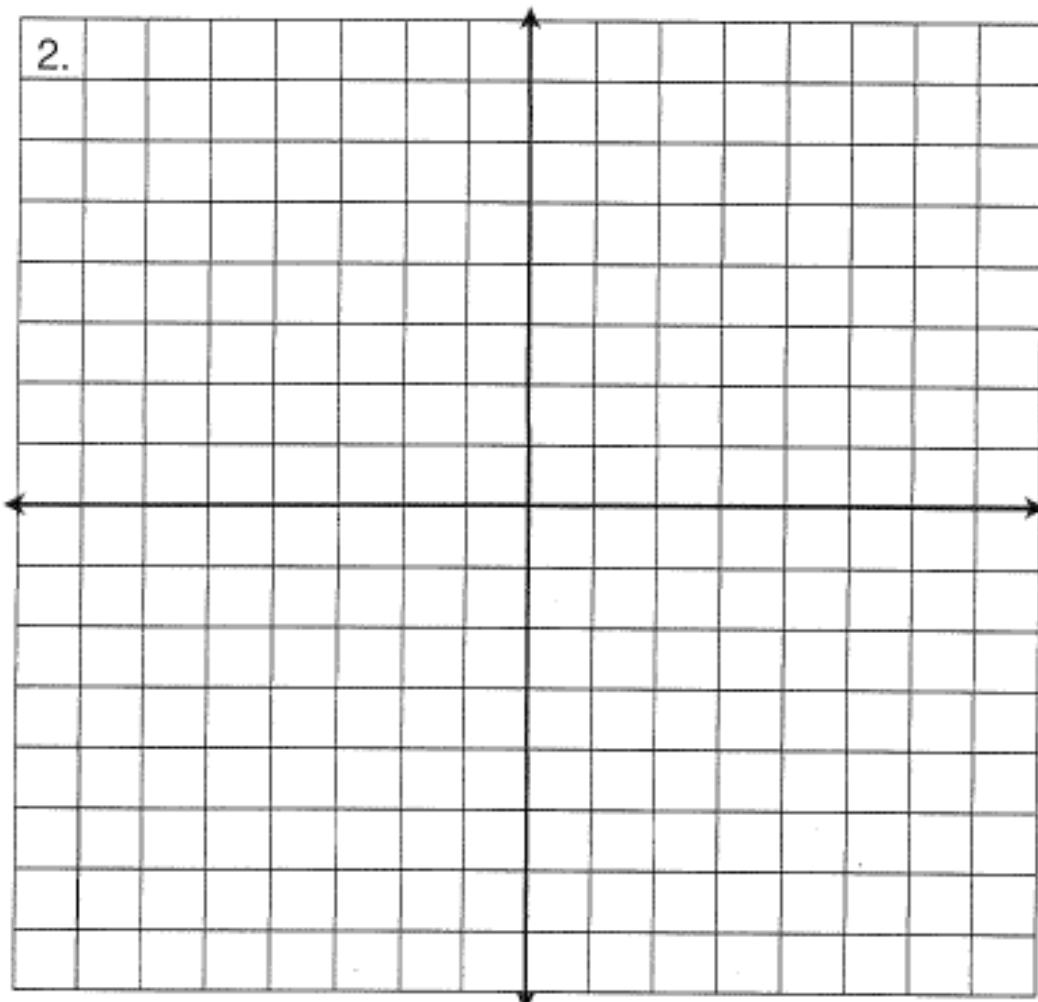
Use all you have learned to solve the following problems about transformations on the coordinate plane.



1. Label the x and y axes.
2. Create the pre-image at $(0,2)$, $(3,2)$, $(0,0)$, and $(3,0)$. Label the figure.
3. Translate the pre-image left 3 and down 3. Outline this transformation with a different color. Write the rule associated with the transformation.

4. Rotate the image 90° counterclockwise. Outline this transformation with a different color. Write the rule associated with the transformation.

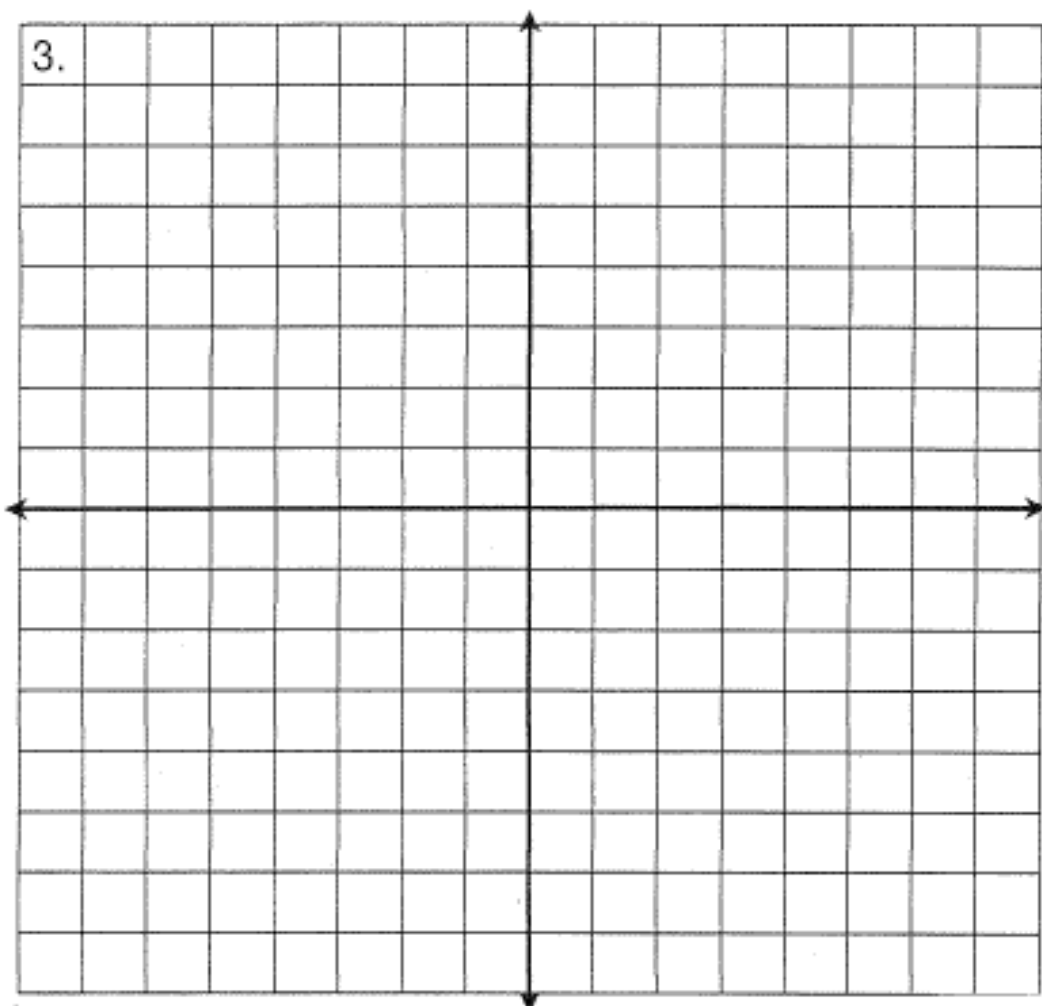
5. Write a rule for the transformation from the pre-image to the final image.



1. Label the x and y axes.
2. Create the pre-image at $(-2,-2)$, $(-2,0)$, and $(-5,-1)$.
Label the figure.
3. Reflect the image across the y-axis. Outline this transformation with a different color.
Write the rule associated with the transformation.

4. Translate the new image left 3 and up 5.
Outline this transformation with a different color.
Write the rule associated with the transformation.

5. Write a rule for the transformation from the pre-image to the final image.



1. Label the x and y axes.
2. Create the pre-image at $(1, 2)$, $(2,2)$, $(1,1)$, and $(2,1)$.
Label the figure.
3. Dilate the image by a scale factor of 2. Outline this transformation with a different color.
Write the rule associated with the transformation.

4. Reflect the new image across the x-axis.
Outline this transformation with a different color.
Write the rule associated with the transformation.

5. Write a rule for the transformation from the pre-image to the final image.
